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Lorentz and CPT violation

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Abstract. During the past two decades there has been a growing interest in the possibility that Lorentz and/or CPT might not be exact symmetries of Nature. In this short review, we present the current state of affairs, addressing both theoretical and experimental/observational issues. We pay particular attention to the role that has been played by the so-called Standard Model Extension.

1. CPT and Lorentz invariance violation

Invariance under Lorentz and CPT symmetry is a fundamental ingredient of both quantum field theory and General Relativity. This has been the major reason physicist have been, and to a large degree still are, reluctant to consider any violation of these symmetries. Nevertheless, in the last two decades, there has been growing interest in the possibility that Lorentz symmetry may not be an exact symmetry of Nature, or, at least, in testing whether or not this is the case.

Roughly, there are two major reasons as to why this has happened.

On the theoretical side, it turns out that many candidate theories of quantum gravity involve Lorentz invariance violation (LIV) as a possible effect. For example, the possibility of four-dimensional LIV has been investigated in string theory [1], non-commutative geometry [2], loop quantum gravity [3] and warped brane worlds [4]. Also other ideas, including emergent gauge bosons [5, 6] and emergent gravity [7] include LIV. We will discuss some of them in the next section.

On the experimental and phenomenological side, a very important development has been the formulation of low-energy effective field theories with LIV. In particular, the so-called Standard Model Extension (SME) has prompted much interest in the experimental testing of Lorentz and CPT symmetry. It includes all possible terms in which the Standard Model fields are coupled in a Lorentz-covariant way to constant tensor coefficients, while maintaining consistency requirements like gauge invariance, causality, etc. The value of these coefficients can in principle be measured (or bound) in experiments. This has allowed a systematic search for a large range of possible Lorentz-violating effects. In section 4 we will outline the main features of the SME.

CPT invariance is an issue that is closely related to Lorentz invariance. Indeed, the CPT theorem [8] states that any Lorentz-invariant, local quantum field theory with a hermitian Hamiltonian must have CPT symmetry. Conversely it can be shown [9] that any unitary interacting theory that violates CPT necessarily violates Lorentz invariance.¹ Thus violation of

¹ See the talk by Nick Mavromatos for a scenario with CPT violation that bypasses the assumptions of this theorem.



CPT is always accompanied by Lorentz violation. However, the reverse is not true: it is possible to have Lorentz violation while maintaining CPT invariance!

2. Models with Lorentz invariance violation

Let us consider some types of explicit models that exhibit LIV. Roughly we can consider two types. First of all, there are fundamental models that are basically Lorentz invariant by construction, but where LIV arises at low energy, usually as a small effect. There are also models that are not Lorentz invariant by construction, but where Lorentz invariance arises as an approximate symmetry at low energy. In the next two subsections we will consider examples of both categories.

2.1. Fundamental models with Lorentz invariance violation

2.1.1. Spontaneous symmetry breaking with Lorentz invariance violation. Consider a model with a Dirac spinor ψ and a vector field B^μ which includes an axial spinor-vector coupling

$$\mathcal{L} \supset \lambda B^\mu \bar{\psi} \gamma_5 \gamma_\mu \psi. \quad (1)$$

Suppose moreover that the dynamics of the vector field is such that the latter acquires a vacuum expectation value $\langle B^\mu \rangle = b^\mu \neq 0$. This could happen in the presence of an effective nonderivative potential for the vector field

$$V_{eff}(B^\mu B_\mu) \quad (2)$$

in which V_{eff} has a minimum for nonzero argument. When expanding the fields around this vacuum the kinetic term of the fermion will acquire the additional term

$$\mathcal{L} \supset \lambda b^\mu \bar{\psi} \gamma_5 \gamma_\mu \psi \quad (3)$$

which evidently violates Lorentz invariance.

The example just described is an example of a so-called bumblebee model [5]. Note that the Lagrangian itself is Lorentz invariant, while the LIV arises by spontaneous symmetry breaking. Usually it is assumed that the vacuum expectation value b^μ is a very small effect. More general models containing Lorentz tensors and fermions assume the presence of couplings of the form:

$$\mathcal{L} \supset \lambda m_{\text{pl}}^{-k} T \cdot \bar{\psi} \Gamma (i\partial)^k \psi \quad (4)$$

with m_{pl} some fundamental mass scale (presumably the Planck mass) and T a higher rank tensor contracted with a fermion bilinear containing a combination of spacetime derivatives and Dirac gamma matrices. If T acquires some vacuum expectation value, (4) generates the following Lorentz-violating contribution to the fermion inverse propagator:

$$\Delta K(p) = \lambda m_{\text{pl}}^{-k} \langle T \rangle \cdot \Gamma p^k. \quad (5)$$

One could wonder whether models that include non-derivative potentials like (2) are realistic, as they seem to be incompatible with gauge invariance. However, there are indications that such potentials might arise in the context of string field theory [1]. Various authors have also shown effective potentials that could allow for spontaneous Lorentz violation can arise in the context of fermion models, where fermion bilinears carrying Lorentz indices acquire nonzero vacuum expectation values [6].

2.1.2. Cosmologically varying scalars. Rather than having a Lorentz-violating expectation value arise through the expectation value of a vector field, the same could be due to a vacuum expectation value of a scalar that has a (slow) variation as a function of spacetime [10]. In other words, a preferred direction is selected through a non-zero value for the gradient of a scalar. For instance, one can take an axion coupling to the electromagnetic field:

$$\mathcal{L} \supset a(x)F\tilde{F}. \quad (6)$$

Integrating (6) by parts yields the contribution

$$\mathcal{L}' \supset -k^\mu A^\nu \tilde{F}_{\mu\nu} \quad (7)$$

which violates Lorentz invariance as well as CPT. As we will see below, the Lagrangian term (7) is incorporated in the SME.

2.1.3. Noncommutative geometry. Consider spacetime where the coordinates are taken to be noncommuting quantities [2]:

$$[x_\alpha, x_\beta] = i \frac{1}{\Lambda_{NC}} \theta_{\alpha\beta}. \quad (8)$$

Here $\theta_{\alpha\beta}$ is a tensor-valued set of coefficients of $\mathcal{O}(1)$, while Λ_{NC} denotes the noncommutative energy scale.

Noncommutative quantum field theories can be constructed by taking an ordinary quantum field theory and replacing the ordinary multiplication of fields with Moyal products:

$$f \star g(x) = \exp\left(\frac{1}{2}i\theta^{\mu\nu}\partial_{x^\mu}\partial_{y^\nu}\right) f(x)g(y) \Big|_{x=y}. \quad (9)$$

The definition of gauge transformations must be adapted analogously.

It is possible to re-express resulting noncommutative field theory in terms of a conventional one, by use of the so-called Seiberg-Witten map [13]. It expresses the non-commutative fields in terms of ordinary gauge fields. For instance, by applying this map to non-commutative Quantum Electrodynamics one obtains the following Lorentz-violating expression, at lowest nontrivial order in $1/\Lambda_{NC}$:

$$\begin{aligned} S = & \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q}{8}\frac{\theta^{\alpha\beta}}{\Lambda_{NC}} \left[-iF_{\alpha\beta}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu\psi \right. \\ & \left. + 2iF_{\alpha\mu}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\beta\psi + 2mF_{\alpha\beta}\bar{\psi}\psi - 4F_{\alpha\mu}F_{\beta\nu}F^{\mu\nu} + F_{\alpha\beta}F_{\mu\nu}F^{\mu\nu} \right]. \quad (10) \end{aligned}$$

The effect of the Lorentz-violating contributions is amenable to experimental detection, or can be bounded. They are an example of (non-minimal) Standard Model Extension terms.

A study of the perturbative dynamics of noncommuting field theories has revealed that high energies of virtual particles in loops produce non-analyticity at low momentum, questioning the consistency of the low energy effective action [11]. Supersymmetric gauge theories exhibit better behaviour in the infrared [12]. This issue has been the object of extensive recent study.

2.1.4. LIV from topology. Another interesting idea that has been explored [14] is the possibility that LIV arise from nontrivial spacetime topology. Consider a spacetime in which one of the dimensions is compact with a large radius R . Then the vacuum fluctuations along this dimension have periodic boundary conditions. This defines a preferred direction in the vacuum. Applying this to electrodynamics once again yields the lagrangian term given by eq. (7), where k^μ points in the direction of the compact dimension and has a size of order R^{-1} [14].

2.2. Models with approximate Lorentz invariance at low energy

A second class of models are those that have no Lorentz invariance built in at the fundamental level at all, but nevertheless exhibits approximate Lorentz invariance at low energy.

An example is given by the recently proposed *Horava-Lifshitz gravity* [15], which has received much attention in the literature. Here the fundamental role of local Lorentz invariance is abandoned and instead it is assumed that this appears only at low energies as an approximate symmetry. Spacetime is endowed with a preferred foliation by 3-dimensional spacelike surfaces, which defines the splitting of the coordinates into space and time, which are thus assumed to be on a fundamentally different footing. The Hilbert-Einstein action of general relativity (GR) is completed with higher spatial derivatives of the metric which improve the UV behavior of the graviton propagator, allowing for a power-counting renormalizable theory. However, the time derivatives of the action are maintained at second order, thus avoiding any problems with ghosts. A required key property is thus that the theory flow to GR in the infrared limit.

It has been pointed out that the original proposal by Horava contains an extra propagating scalar degree of freedom with pathological behavior. An extended model has been proposed where this problem is claimed to be cured [16]. In the infrared limit this the model reduces to a Lorentz-violating scalar-tensor gravity theory. Other "projectable" versions of Horava-Lifshitz gravity have also been considered. It should be noted that this remains very much an open field.

3. Kinematic frameworks

Historically most relativity tests have only considered kinematic aspects, that is, they investigated deviations of single-particle motion from the expected Lorentz-invariant dispersion relation. In this section we will consider various tests of relativity of this kind that have been proposed and used in the past.

3.1. Modified dispersion relations

One of the most simple approaches is to assume that Lorentz-violating effects modify the usual relativistic dispersion relation $E^2 = p^2 + m^2$ to the more general relation $E^2 = F(p, m)$. It is natural to expand $F(p, m)$ in a Taylor series:

$$E^2 = m^2 + p^2 + m_{\text{pl}} f_i^{(1)} p^i + f_{ij}^{(2)} p^i p^j + \frac{f^{(3)}}{m_{\text{pl}}} p^i p^j p^k + \dots \quad (11)$$

with dimensionless coefficients $f^{(n)}$, that depend on the particle species. The order n of the first nonzero coefficient depends on the underlying fundamental theory. The coefficients $f^{(n)}$, while arbitrary, are presumably such that Lorentz violation is a small effect. Note that this will be automatically the case for $n \geq 3$ whenever $p \ll m_{\text{pl}}$ if the $f^{(n)}$ are of order 1, due to the inverse powers of the Planck mass accompanying those. Thus no special suppression mechanism is necessary for those terms. This will be discussed in more detail below. It might be noted that terms with odd powers of p tend to have problems with coordinate invariance, causality and/or positivity [17].

Much of the relevant literature assumes rotational invariance, turning (11) into

$$E^2 = m^2 + p^2 + m_{\text{pl}} f^{(1)} |p| + f^{(2)} p^2 + \frac{f^{(3)}}{m_{\text{pl}}} |p|^3 + \dots \quad (12)$$

For an example of an application to the photon sector, see [18].

It has been suggested that stochastic or foamy spacetime structure can lead to modifications of spacetime structure that modify over time. In such frameworks the particle dispersion is taken to fluctuate according to a model-dependent probability distribution [19].

3.2. The Robertson-Mansouri-Sexl framework

This method assumes a preferred frame where the speed of light is isotropic. The Lorentz transformation to other frames is generalized with respect to the conventional boosts [20]:

$$t' = a^{-1}(t - \vec{\epsilon} \cdot \vec{x}) \quad (13)$$

$$\vec{x}' = d^{-1}\vec{x} - (d^{-1} - b^{-1})\frac{\vec{v}(\vec{v} \cdot \vec{x})}{v^2} - a^{-1}\vec{v}t \quad (14)$$

with a, b, c, d, e functions of the relative speed v . Without Lorentz violation and Einstein clock synchronization we have $a = b^{-1} = \sqrt{1 - v^2}$, $d = 1$, and $\vec{\epsilon} = \vec{v}$. Modifying the values of the parameters results in a variable speed of light, assuming experiments that use a fixed set of rods and clocks.

The Robertson-Mansouri-Sexl framework is incorporated in the SME.

3.3. The c^2 model

The c^2 model is a test model developed for application to studies of Lorentz invariance, as a limiting case of the $\text{THE}\epsilon\mu$ formalism. It is described by a lagrangian that considers the motion of test particles in an electromagnetic field [21]. It assumes a preferred frame in which the limiting speed of particles is considered to be 1, but the speed of light $c \neq 1$.

Also this framework can be incorporated in the SME [42].

3.4. Doubly Special Relativity

Doubly Special Relativity (DSR) is a recently developed idea [22]. It is assumed that the Lorentz transformations act in a modified way on the physical four-momentum such that both c as well as a special energy scale E_{DSR} are invariant. The physical energy/momentum are taken to be given by

$$E = \frac{\epsilon}{1 + \lambda_{\text{DSR}}\epsilon}, \quad p = \frac{\pi}{1 + \lambda_{\text{DSR}}\pi}, \quad (15)$$

where $\lambda_{\text{DSR}} = E_{\text{DSR}}^{-1}$, in terms of the pseudo energy/momentum ϵ and π , which transform normally under Lorentz boosts. The dispersion relation becomes

$$E^2 - p^2 = \frac{m^2(1 - \lambda_{\text{DSR}}E)^2}{(1 - \lambda_{\text{DSR}}m)^2}. \quad (16)$$

Also this framework can be incorporated in the SME [23].

It should be noted that the physical meaning of the quantities E and p , and of DSR itself, has been questioned.

4. Effective field Theory

What should be a suitable dynamical framework for describing LIV? We can list a few general criteria that it should satisfy:

- (i) *Observer coordinate independence*: The physics it describes should be independent of "observer" coordinate transformation, that is, changes of coordinates used by the observer to describe the same physical situation;
- (ii) *Realism*: the framework must incorporate known physics, while allowing for a suitable parametrization of LIV effects;
- (iii) *Generality*: the framework should ideally be as general as possible, to maximize reach.

A framework that satisfies all of these criteria is the *Standard Model Extension* (SME) [24]. It is an effective field theory which incorporates:

- (i) The Standard Model of particle physics as well as General Relativity;
- (ii) Any scalar term formed by contracting operators for Lorentz violation with tensor-valued coefficients controlling size of the LIV effects;
- (iii) Possibly additional requirements like gauge invariance, locality, stability, and renormalizability can be included as well.

The SME includes, in principle, terms of any mass dimension (starting at dimension three). Imposing power-counting renormalizability limits one to terms of dimension four. This is usually referred to as the minimal SME (mSME). The mSME has a finite number of LIV parameters, while the number of LIV parameters in the full SME is in principle unlimited. The SME leads not only to breaking of Lorentz symmetry, but also to that of CPT, for about half of its terms.

Example: the free fermion sector of the SME:

$$\mathcal{L} = \bar{\psi}(i\Gamma^\mu\partial_\mu - M)\psi \quad (17)$$

$$\Gamma^\mu = \gamma^\mu + c^{\mu\nu}\gamma_\nu - d^{\mu\nu}\gamma^\nu\gamma_5, \quad M = m + \not{a} - \not{b}\gamma_5 + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu} \quad (18)$$

A separate set of coefficients exists for every elementary particle.

As the SME is to be considered an effective field theory, one can relax the requirement of renormalizability. This means, that the coefficients of the mSME become generalized to higher mass dimensions. For instance, the $c^{\mu\nu}$ coefficients in (18) can be generalized to include arbitrary higher dimensions as follows:

$$c^{\mu\nu} \rightarrow \hat{c}^{\mu\nu} \equiv i^{d-4} \sum_{d=4}^{\infty} c^{(d)\mu\nu\alpha_1\dots\alpha_{(d-4)}} \partial_{\alpha_1} \dots \partial_{\alpha_{(d-4)}} \quad (19)$$

The coefficients $c^{(d)}$ have mass dimension $4 - d$. Thus they can naturally be expected to be of order m_{pl}^{4-d} , so that their contribution to $\hat{c}^{\mu\nu}$ is naturally suppressed at low energies.

Table 1. The $SU(3) \times SU(2) \times U(1)$ fields and coupling constants.

Leptons	$L_A = \begin{pmatrix} \nu_A \\ l_A \end{pmatrix}_L$,	$R_A = (l_A)_R$	$(A = e, \mu, \tau)$
Quarks	$Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L$,	$U_A = (u_A)_R$, $D_B = (d_B)_R$	$(A = u, c, t)$ $(B = d, s, b)$
Gauge fields	G_μ ,	W_μ ,	B_μ
Higgs doublet	ϕ		
Gauge couplings	g_3 ,	g ,	g'
Yukawa couplings	G_L ,	G_U ,	G_D

Let's consider more in detail at the construction of the mSME. In table 1 we define the $SU(3) \times SU(2) \times U(1)$ Standard Model fields and coupling constants. The Standard Model consists of:

$$\mathcal{L}_{\text{lepton}} = \frac{1}{2}i\bar{L}_A\gamma^\mu \overleftrightarrow{D}_\mu L_A + \frac{1}{2}i\bar{R}_A\gamma^\mu \overleftrightarrow{D}_\mu R_A \quad (20)$$

$$\mathcal{L}_{\text{quark}} = \frac{1}{2}i\bar{Q}_A\gamma^\mu \overleftrightarrow{D}_\mu Q_A + \frac{1}{2}i\bar{U}_A\gamma^\mu \overleftrightarrow{D}_\mu U_A + \frac{1}{2}i\bar{D}_A\gamma^\mu \overleftrightarrow{D}_\mu D_A \quad (21)$$

$$\mathcal{L}_{\text{Yukawa}} = -(G_L)_{AB}\bar{L}_A\phi R_B - (G_U)_{AB}\bar{Q}_A\phi^c U_B - (G_D)_{AB}\bar{Q}_A\phi D_B \quad (22)$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \frac{\lambda}{6}(\phi^\dagger\phi)^2 \quad (23)$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\text{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}\text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}(B_{\mu\nu}B^{\mu\nu}) \quad (24)$$

In the minimal Standard Model Extension, to this are added:

- *Fermion sector*

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} = \frac{1}{2}i(c_L)_{\mu\nu AB}\bar{L}_A\gamma^\mu \overleftrightarrow{D}^\nu L_B + \frac{1}{2}i(c_R)_{\mu\nu AB}\bar{R}_A\gamma^\mu \overleftrightarrow{D}^\nu R_B \quad (25)$$

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-odd}} = -(a_L)_{\mu AB}\bar{L}_A\gamma^\mu L_B - (a_R)_{\mu AB}\bar{R}_A\gamma^\mu R_B \quad (26)$$

$$\begin{aligned} \mathcal{L}_{\text{quark}}^{\text{CPT-even}} &= \frac{1}{2}i(c_Q)_{\mu\nu AB}\bar{Q}_A\gamma^\mu \overleftrightarrow{D}^\nu Q_B + \frac{1}{2}i(c_U)_{\mu\nu AB}\bar{U}_A\gamma^\mu \overleftrightarrow{D}^\nu U_B \\ &\quad + \frac{1}{2}i(c_D)_{\mu\nu AB}\bar{D}_A\gamma^\mu \overleftrightarrow{D}^\nu D_B \end{aligned} \quad (27)$$

$$\mathcal{L}_{\text{quark}}^{\text{CPT-odd}} = -(a_Q)_{\mu AB}\bar{L}_A\gamma^\mu Q_B - (a_R)_{\mu AB}\bar{U}_A\gamma^\mu U_B - (a_D)_{\mu AB}\bar{D}_A\gamma^\mu D_B \quad (28)$$

- *Higgs sector*

$$\mathcal{L}_{\text{Higgs}}^{\text{CPT-even}} = \frac{1}{2}(k_{\phi\phi})^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi - \frac{1}{2}(k_{\phi B})^{\mu\nu}\phi^\dagger\phi B_{\mu\nu} - \frac{1}{2}(k_{\phi W})^{\mu\nu}\phi^\dagger W_{\mu\nu}\phi \quad (29)$$

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}}^{\text{CPT-even}} &= -\frac{1}{2}(H_L)_{\mu\nu AB}\bar{L}_A\phi\sigma^{\mu\nu}R_B - \frac{1}{2}(H_U)_{\mu\nu AB}\bar{Q}_A\phi\sigma^{\mu\nu}U_B \\ &\quad - \frac{1}{2}(H_D)_{\mu\nu AB}\bar{Q}_A\phi\sigma^{\mu\nu}D_B \end{aligned} \quad (30)$$

- *Gauge sector*

$$\mathcal{L}_{\text{gauge}}^{\text{CPT-even}} = -\frac{1}{2}(k_G)^{\kappa\lambda\mu\nu}\text{Tr}(G_{\kappa\lambda}G_{\mu\nu}) - \frac{1}{2}(k_W)^{\kappa\lambda\mu\nu}\text{Tr}(W_{\kappa\lambda}W_{\mu\nu}) - \frac{1}{2}(k_B)^{\kappa\lambda\mu\nu}B_{\kappa\lambda}B_{\mu\nu} \quad (31)$$

$$\begin{aligned} \mathcal{L}_{\text{gauge}}^{\text{CPT-odd}} &= (k_3)_\kappa\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(G_\lambda G_{\mu\nu} + \frac{2}{3}ig_3G_\lambda G_\mu G_\nu) + (k_1)_\kappa\epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu} + (k_0)_\kappa B^\kappa \\ &\quad + (k_2)_\kappa\epsilon^{\kappa\lambda\mu\nu}\text{Tr}(W_\lambda W_{\mu\nu} + \frac{2}{3}ig_3W_\lambda W_\mu W_\nu \end{aligned} \quad (32)$$

In much the same way conventional quantum electrodynamics (QED) can be obtained from the usual standard model, an extended LIV version of QED can be obtained from the SME. After electroweak symmetry breaking, one sets to zero the gluon fields, the weak bosons and the Higgs field. The (uncharged) neutrinos can be disregarded, leaving only the photon, the charged leptons and the quarks. Considering here the simplified case of only photons and a single fermion one obtains

$$\begin{aligned} \mathcal{L}_{\text{fermion}} &= \frac{1}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &\quad - a_\mu\bar{\psi}\gamma^\mu\psi - b_\mu\bar{\psi}\gamma_5\gamma^\mu\psi + \frac{1}{2}ic_{\mu\nu}\bar{\psi}\gamma^\mu \overleftrightarrow{D}^\nu \psi + \frac{1}{2}id_{\mu\nu}\bar{\psi}\gamma_5\gamma^\mu \overleftrightarrow{D}^\nu \psi \\ &\quad - \frac{1}{2}H_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}. \end{aligned} \quad (33)$$

For every fermion there is a separate set of coefficients a_μ , b_μ , $c_{\mu\nu}$, $d_{\mu\nu}$ and $H_{\mu\nu}$. Sometimes one also considers the additional terms

$$\mathcal{L}_{\text{fermion}}^{\text{extra}} = \frac{1}{2}ie_\mu\bar{\psi} \overleftrightarrow{D}^\mu \psi - \frac{1}{2}if_\mu\bar{\psi}\gamma_5 \overleftrightarrow{D}^\mu \psi + \frac{1}{4}ig_{\lambda\mu\nu}\bar{\psi}\sigma^{\lambda\mu} \overleftrightarrow{D}^\nu \psi \quad (34)$$

that are compatible with renormalizability but that cannot be obtained as a reduction from the SME. We note here that for phenomenological purposes protons and neutrons are treated as fundamental constituents with their own set of

The SME fields can also be coupled to gravity [25]. As an example, consider the lepton sector. In the Standard Model it couples through the Lagrangian density

$$\mathcal{L}_{\text{lepton}} = \frac{1}{2}ie e^\mu_a\bar{L}_A\gamma^a \overleftrightarrow{D}_\mu L_A + \frac{1}{2}ie e^\mu_a\bar{R}_A\gamma^a \overleftrightarrow{D}_\mu R_A. \quad (35)$$

Here e_a^μ is the vierbein, which is used to convert local Lorentz indices to curved spacetime indices: $b_\mu = e_\mu^a b_a$. The flat-space LIV lepton sectors can similarly be coupled to gravity, for example:

$$\mathcal{L}_{\text{lepton}}^{\text{CPT-even}} = -\frac{1}{2}i(c_L)_{\mu\nu AB} e e^\mu_a \bar{L}_A \gamma^a \overleftrightarrow{D}^\nu L_B - \frac{1}{2}i(c_R)_{\mu\nu AB} e e^\mu_a \bar{R}_A \gamma^a \overleftrightarrow{D}^\nu R_B. \quad (36)$$

Also for the pure gravity sector itself a LIV extension can be defined. The minimal sector is described by the Lagrangian density

$$\mathcal{L}_{e,\omega}^{\text{LV}} = e(k_T)^{\lambda\mu\nu} T_{\lambda\mu\nu} + e(k_R)^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} + e(k_{TT})^{\alpha\beta\gamma\lambda\mu\nu} T_{\alpha\beta\gamma} T_{\lambda\mu\nu} + e(k_{DT})^{\kappa\lambda\mu\nu} D_\kappa T_{\lambda\mu\nu}. \quad (37)$$

Here $R_{\kappa\lambda\mu\nu}$ is the Riemann curvature tensor, while $T_{\lambda\mu\nu}$ is the torsion tensor. In the Riemannian limit (no torsion) the action becomes a generalization of the Einstein-Hilbert action:

$$S_{e,\omega,\Lambda} = \frac{1}{2\kappa} \int d^4x e \left[(1-u)R - 2\Lambda + s^{\mu\nu} R_{\mu\nu} + t^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu} \right]. \quad (38)$$

As has been alluded to above, different types of *scaling behaviour* can be expected for the LIV coefficients of the SME.

- For the minimal SME, with terms of mass dimension less than or equal to four, one expects the model to be renormalizable with coefficients running logarithmically as a function of energy. Indeed, a renormalization group study that has been done to one loop and to first order in LIV coefficients for the QED sector of the mSME bears this out [26]. Thus no suppression with power of energy scale occurs for these coefficients, and so any suppression that occurs should be present already at very high energy scales, creating a naturalness problem. A toy model study with a scalar field with Planck scale Lorentz-violating cutoff at high energy yields percent-level LIV at low energy [27].
- Higher (≥ 5) dimension operators can be expected to scale with inverse powers of the Planck mass. However, there are higher dimension operators that mix with dimension three or four operators, eliminating the suppression mechanism. Studies have been done identifying dimension five operators that do not exhibit such mixing [28]. It has been claimed that in a Lorentz-violating minimal supersymmetric extension of the Standard Model can only contain higher-dimension operators, but this depends on whether or not the superalgebra sector involving Q and P_μ operators is perturbed [29].

5. Phenomenology

5.1. Free particles: modified dispersion relations

The SME implies in general modified dispersion relations of the form (11), where the coefficients $f^{(n)}$ are determined by the relevant parameters of the SME effective lagrangian. One important possible consequence of modified dispersion relations is the occurrence of shifted reaction thresholds. In particular, normally allowed processes may become forbidden, while normally forbidden processes may become allowed in certain regions of phase space. For example, it is possible within the SME that charged massive particles exhibit Čerenkov radiation [30, 31]. Normally this can only happen in a refractive optical medium, but if the dispersion relation of the massive particle allows a maximum speed above the (possibly modified) speed of light, *vacuum* Čerenkov radiation can take place. Nonobservance of vacuum Čerenkov radiation for LEP electrons has lead to bounds on SME parameters in QED sector [31]. Another example is photon decay into, for instance, a positron-electron pair which can become a kinematically allowed process. Nonobservance of photon decay in Tevatron photons has lead to bounds on SME parameters [32].

5.2. Mesons

Meson systems have long provided tests for CP and CPT invariance [33]. In the context of the SME they provide a test for the a_μ coefficients. To see this, consider the Schrödinger equation for a neutral meson system:

$$i\partial_t\Psi = \Lambda\Psi \quad (39)$$

where Ψ is a linear combination of the neutral meson and the anti-meson states (K , D , B_d or B_s). $\Lambda = M - i\Gamma/2$ is the effective 2×2 Hamiltonian, with eigenvalues $\lambda_S \equiv m_S - \frac{i}{2}\gamma_S$ and $\lambda_L \equiv m_L - \frac{i}{2}\gamma_L$. It is possible to show the following simple relation with the SME coefficients a_μ :

$$\Delta\Lambda \approx \beta^\mu \Delta a_\mu, \quad \Delta\Lambda \equiv \Lambda_{11} - \Lambda_{22}, \quad \beta^\mu \equiv (\gamma, \gamma\vec{\beta}). \quad (40)$$

One commonly introduces the dimensionless parameter $\xi = \Delta\Lambda/\Delta\lambda \approx 2\delta$ that parametrizes CPT violation. Note that ξ depends explicitly on the meson four-velocity.

Experimental sensitivities have been obtained for Δa_μ in the K system of order 10^{-17} to 10^{-20} GeV [34],² in the D system of order 10^{-15} GeV [35], and in the B_d system of order 10^{-15} GeV [36].

5.3. Neutrinos

The SME suggests many possible observable consequences in the neutrino sector [37]. For example, there are LIV terms in the SME that provoke neutrino oscillations. This in turn yields very precise tests of LIV.

At leading order, LIV in the neutrino sector is described by the following effective two-component Hamiltonian acting on neutrino- antineutrino state vector:

$$h_{\text{eff}} = |\vec{p}| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{2|\vec{p}|} \begin{pmatrix} \tilde{m}^2 & 0 \\ 0 & (\tilde{m}^2)^* \end{pmatrix} + \frac{1}{|\vec{p}|} \begin{pmatrix} (a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu & -i\sqrt{2}p_\mu(\epsilon_+)^\nu (g^{\mu\nu\sigma} p_\sigma - H^{\mu\nu})\mathcal{C} \\ -i\sqrt{2}p_\mu(\epsilon_+)^*\nu [(g^{\mu\nu\sigma} p_\sigma - H^{\mu\nu})\mathcal{C}]^* & [-(a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu]^* \end{pmatrix}. \quad (41)$$

Hamiltonian (41) leads to various potential signals:

- Oscillations with unusual energy dependences can be expected (oscillation length may grow rather than shrink with energy);
- Anisotropies can arise from breakdown of rotational invariance, implying sidereal variations in observed fluxes.

Many bounds on SME parameters in the neutrino sector have been deduced by analysis of LSND, MiniBooNe and MINOS (and other) data [38]. It is also interesting to note that SME-inspired models have been proposed that reproduce current observations and may help resolve the LSND anomaly [39].

5.4. QED sector

Not surprisingly, the sharpest laboratory tests have been carried out in systems where the predominant interactions are described by QED.

Starting with the QED sector of the SME, lagrangian (33), an effective Hamiltonian can be constructed using perturbation theory for small LIV, such that $i\partial_t\chi = \hat{H}\chi$. In the

² See also the talk by Antonio De Santis at this conference.

non-relativistic approximation, by using the Foldy-Wouthuysen approach and making field redefinitions one finds for a massive fermion [40]

$$\begin{aligned} \hat{H}_{\text{pert}} = & a_\mu \gamma^0 \gamma^\mu - b_\mu \gamma_5 \gamma^0 \gamma^\mu - c_{00} m \gamma^0 - i(c_{0j} + c_{j0}) D^j + i(c_{00} D_j - c_{jk} D^k) \gamma^0 \gamma^j \\ & - d_{j0} m \gamma_5 \gamma^j + i((d_{0j} + d_{j0}) D^j \gamma_5 + i(d_{00} D_j - d_{jk} D^k) \gamma^0 \gamma_5 \gamma^j + \frac{1}{2} H_{\mu\nu} \gamma^0 \sigma^{\mu\nu}) \end{aligned} \quad (42)$$

This expression assumes fixed non-rotating axes. In the usual convention one employs a sun-centered frame with celestial equatorial coordinates, denoted by uppercase X, Y, Z, T . Passing to a frame of rotating, earth-fixed laboratory axes implies using an appropriate mapping. For instance, for the combination

$$\tilde{b}_j^e \equiv b_j^e - m d_{j0}^e - \frac{1}{2} \epsilon_{jkl} H_{kl}^e \quad (43)$$

one finds

$$\begin{aligned} \tilde{b}_1^e &= \tilde{b}_X^e \cos \chi \cos \Omega t + \tilde{b}_Y^e \cos \chi \sin \Omega t - \tilde{b}_Z^e \sin \chi \\ \tilde{b}_2^e &= -\tilde{b}_X^e \sin \Omega t + \tilde{b}_Y^e \cos \Omega t \\ \tilde{b}_3^e &= \tilde{b}_X^e \sin \chi \cos \Omega t + \tilde{b}_Y^e \sin \chi \sin \Omega t + \tilde{b}_Z^e \cos \chi \end{aligned} \quad (44)$$

Here the earth's rotation axis is chosen along Z , while the angle χ is between the $j = 3$ lab axis and the Z axis. Ω is the angular frequency corresponding to a sidereal day ($\Omega \approx 2\pi/(23\text{h } 56\text{m})$).

5.5. Photons

The most general photon kinetic term in the mSME follows from lagrangian (33):

$$\mathcal{L}_{\text{photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} + \frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} \quad (45)$$

which exhibits two LIV terms.

The k_{AF} term is CPT violating and leads to birefringence. It can give rise to vacuum Čerenkov radiation [30].

The k_F term is CPT conserving. It has 19 LIV degrees of freedom, some being parity-even, others parity-odd. One degree of freedom, denoted κ_{tr} , is rotationally invariant. Ten degrees of freedom lead to birefringence, the remaining nine (including κ_{tr}) don't. Also the k_F term can give rise to vacuum Čerenkov radiation [31].

5.6. Bounds on higher dimensional LIV operators

Much less work has been done on bounding higher dimensional operators. Laboratory experiments are concerned with low energies, thus best suited for mSME. Higher-dimension operators scale with energy, giving an a-priori advantage to astrophysical tests.

Higher-dimensional operators in photon sector have been obtained by considering the most general SME photon Lagrangian [58]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F^{\kappa\lambda} (\hat{k}_F)_{\kappa\lambda\mu\nu} F^{\mu\nu} + \frac{1}{2} \epsilon_{\kappa\lambda\mu\nu} A^\lambda (\hat{k}_{AF})^\kappa F^{\mu\nu} \quad (46)$$

where

$$(\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{d=2,4,6,\dots} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\dots\alpha_{(d-4)}} \partial_{\alpha_1} \dots \partial_{\alpha_{(d-4)}} , \quad (47)$$

$$(\hat{k}_{AF})_\kappa = \sum_{d=1,3,5,\dots} (k_{AF}^{(d)})_{\kappa}^{\alpha_1\dots\alpha_{(d-3)}} \partial_{\alpha_1} \dots \partial_{\alpha_{(d-3)}} . \quad (48)$$

Here $(k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\dots\alpha_{(d-4)}}$ and $(k_{AF}^{(d)})_{\kappa}^{\alpha_1\dots\alpha_{(d-3)}}$ are constant coefficients with mass dimension $4-d$.

By analyzing polarization changes due to birefringence in CMB radiation, various $k_{AF}^{(5)}$ coefficients have been bound to $\mathcal{O}(10^{-19} \text{ GeV}^{-1})$ and various $k_F^{(6)}$ coefficients have been bound to $\mathcal{O}(10^{-9} \text{ GeV}^{-2})$ [58]. From analyzing dispersion relations (time of flight differences) in gamma ray bursts various $k_F^{(6)}$ coefficients have been bound to $\mathcal{O}(10^{-22} \text{ GeV}^{-2})$ [58, 59].

6. Experimental tests of LIV

The sensitivity of experimental tests of Lorentz/CPT violation stems from their ability to detect anomalous energy shifts in various systems. Experiments are most effective when all energy levels are scrutinized for possible anomalous shifts. In past decade a number of new Lorentz/CPT signatures have been identified in addition to more classical tests that were known before, such as the Michelson-Morley experiment. Roughly one can distinguish two types of laboratory tests:

- Lorentz tests, which generally scrutinize sidereal time variations in energy levels;
- CPT tests, which measure differences in particle/antiparticle energy levels.

In the next subsections we discuss some examples of experiments that have been carried out recently or are under way.

6.1. Penning traps

Penning traps have been used recently in experiments with electrons and positrons. High precision measurements are done of the anomaly frequency ω_a and the cyclotron frequency ω_c of trapped particles. One can show to lowest order in the mSME parameters that [49]

$$\omega_c^{e^-} = (1 - c_{00}^e - c_{XX}^e - c_{YY}^e)\omega_c^{e,0} >, \quad (49)$$

$$\omega_a^{e^\pm} = \omega_a^{e,0} \pm 2b_Z^e + 2d_{Z0}^e m_e + 2H_{XY}^e. \quad (50)$$

Comparing the anomaly frequencies for electrons and positrons yields the bound $|\vec{b}^e| \lesssim 3 \times 10^{-25} \text{ GeV}$ [50].

6.2. Clock comparison experiments

The classic Hughes-Drever experiments amount to spectroscopic tests of isotropy of mass and space [51]. They typically examine hyperfine or Zeeman transitions, and provide many of the sharpest LIV bounds for the neutron and proton. For example, a recent Hughes-Drever-type test of Lorentz/CPT for the neutron which used a $^3\text{He}/^{129}\text{Xe}$ co-magnetometer has yielded the bound $|\vec{b}_J^n| \lesssim 10^{-33} \text{ GeV}$ for $J = X, Y$ [52]. A bound of $|\vec{b}_Z^p| \lesssim 2 \times 10^{-27} \text{ GeV}$ for $J = X, Y$ on Lorentz/CPT violation in the proton sector has been obtained by using a H maser [53]. Clock-comparison experiments conducted in space can provide access to many unmeasured coefficients for Lorentz and CPT violation. The orbital configuration of a satellite platform and the relatively large velocities attainable in a deep-space mission would permit a broad range of tests with Planck-scale sensitivity.

6.3. Hydrogen and antihydrogen

(Anti)hydrogen is the simplest (anti)atom, and it provides for some of the cleanest possible Lorentz and CPT tests involving protons or electrons [54]. The ALPHA and ATRAP experiments underway at CERN intend to make high precision spectroscopic measurements of 1S-2S transitions in H and anti-H and attain a frequency comparison at level of 10^{-18} . Inclusion of magnetic field provides leading order sensitivity to Lorentz/CPT. The ASACUSA

experiment intends to analyze ground state Zeeman hyperfine transitions which has leading order LIV corrections in the SME.³

6.4. Muon experiments

Several different types of experiments with muons have been conducted such as muonium experiments and $g - 2$ experiments. In the former, frequencies of ground-state Zeeman hyperfine transitions are measured in strong magnetic fields [55] which have yielded a bound $|\tilde{b}_J^\mu| \leq 2 \times 10^{-23}$ GeV. Analysis of relativistic $g - 2$ experiments using positive muons with large boost parameter have yielded bounds at a level of 10^{-24} GeV [56].

6.5. Spin polarized torsion pendulum

Experiments with a spin-polarized torsion pendulum at the University of Washington provide currently the sharpest bounds on Lorentz/CPT violation in electron sector. It is built out of a stack of toroidal magnets with a huge number of electron spins (8×10^{22}). The apparatus is suspended on a rotating turntable and time variations of the twisting pendulum are measured and analysed for sidereal time variations that would indicate LIV effects. Bounds have been obtained at the levels of $|\tilde{b}_J^e| \lesssim 10^{-31}$ GeV for $J = X, Y$ and $|\tilde{b}_Z^e| \lesssim 10^{-30}$ GeV [57].

6.6. The photon sector

An extensive body of work exists trying to bound the degrees of freedom of the LIV coefficients $k_F^{\alpha\beta\gamma\delta}$ and k_{AF}^μ (see (45)).

Cosmological sources with known polarization permit searching for energy-dependent polarization changes either from distant sources or from CMB. This has yielded the extremely tight bound $|(k_{AF})^\mu| \leq 10^{-42}$ GeV [41].

Regarding the components of k_F , bounds have been established on the degrees of freedom that lead to birefringence by cosmological observations. This implies that the polarization plane of photons rotates between emission and observation. Thus cosmological sources with known polarization can be used to verify model-dependent polarization changes. This way the bound $|k_F^{\alpha\beta\gamma\delta}| \leq 2 \times 10^{-32}$ has been obtained [47].

The rotationally-invariant κ_{tr} has been bound by a variety of lab experiments. Best laboratory bounds from LEP data are $\mathcal{O}(10^{-15})$ [32, 43], while the best astrophysical bound (absence of vacuum Čerenkov radiation in cosmic rays) is of $\mathcal{O}(10^{-19})$ [46].

The remaining eight (non-birefringent) coefficients have been bounded up to $\mathcal{O}(10^{-17})$ by studying sidereal effects in optical or microwave cavities [44] and up to $\mathcal{O}(10^{-12})$ by an experiment studying sidereal effects in Compton edge photons [45].

A complete updated list of currently existing bounds on the SME parameters is kept on the archives [48].

7. Conclusions

Fundamental theories may allow for Lorentz-invariance violation (LIV), typically assumed to occur at the Planck scale as a result of quantum-gravity effects. This makes LIV an attractive testing ground for search for new physics.

We described a series of testing schemes, both kinematical ones as well as effective field theories. We considered in particular the Standard Model Extension, which offers a comprehensive parametrization of Lorentz and CPT violation at low energy, allowing for systematic experimental testing. We discussed a series of current and experiments of Lorentz and CPT violating parameters.

³ See also the talk by Chloe Malbrunot in this conference.

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