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Gravity from Breaking of Local Lorentz Symmetry

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Abstract. We present a model of gravity based on spontaneous Lorentz symmetry breaking. We start from a model with spontaneously broken symmetries for a massless 2-tensor with a linear kinetic term and a nonderivative potential, which is shown to be equivalent to linearized general relativity, with the Nambu-Goldstone (NG) bosons playing the role of the gravitons. We apply a bootstrap procedure to the model based on the principle of consistent coupling to the total energy-momentum tensor. Demanding consistent application of the bootstrap to the potential term severely restricts the form of the latter. Nevertheless, suitable potentials exist that permit stable vacua. It is shown that the resulting model is equivalent, at low energy, to General Relativity in a fixed gauge.

1. Symmetry vs. Broken Symmetry

Masslessness often arises as a consequence of the existence of a symmetry. In quantum electrodynamics the masslessness of the photon is normally attributed to gauge invariance, or symmetry under local changes of phase. In quantum chromodynamics, the theory of the strong interaction, masslessness of the gluons is likewise attributed to a gauge invariance, albeit a nonlinear one. In general relativity, the masslessness of gravitons can be traced to symmetry under active diffeomorphisms: no diffeomorphism-invariant mass term exists.

In some circumstances, however, there exists an alternative reason why a field might be massless. Surprisingly, this alternative explanation involves the breaking of a symmetry rather than its existence. A general result, the Nambu-Goldstone theorem [1], states under mild assumptions that there must be a massless particle whenever a continuous global symmetry of an action is not a symmetry of the vacuum.

In this talk, based on ongoing work with Alan Kostelecký [2, 3], we show that an alternative description of gravity can be constructed from a symmetric two-tensor without the assumption of masslessness. In this picture, masslessness is a consequence of symmetry breaking rather than of exact symmetry: diffeomorphism symmetry and local Lorentz symmetry are spontaneously broken, but the graviton remains massless because it is a Nambu-Goldstone mode.

The cardinal object in the theory is a symmetric two-tensor-density, denoted by $C^{\mu\nu}$. Starting point is the Lagrange density

$$L = \frac{1}{2} C^{\mu\nu} K_{\mu\nu\alpha\beta} C^{\alpha\beta} + V(C^{\mu\nu}, \eta_{\mu\nu}). \quad (1)$$

Here, $K_{\mu\nu\alpha\beta}$ is the usual quadratic kinetic operator for a massless spin-2 field,

$$K_{\mu\nu\alpha\beta} = \frac{1}{2} [(-\eta_{\mu\nu}\eta_{\alpha\beta} + \frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha})\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta + \eta_{\alpha\beta}\partial_\mu\partial_\nu - \frac{1}{2}\eta_{\mu\alpha}\partial_\nu\partial_\beta - \frac{1}{2}\eta_{\nu\alpha}\partial_\mu\partial_\beta - \frac{1}{2}\eta_{\mu\beta}\partial_\nu\partial_\alpha - \frac{1}{2}\eta_{\nu\beta}\partial_\mu\partial_\alpha], \quad (2)$$

and V is a potential which is built out of the four scalars

$$X_1 = C^{\mu\nu}\eta_{\nu\mu}, \quad (3)$$

$$X_2 = (C \cdot \eta \cdot C \cdot \eta)_\mu^\mu, \quad (4)$$

$$X_3 = (C \cdot \eta \cdot C \cdot \eta \cdot C \cdot \eta)_\mu^\mu, \quad (5)$$

$$X_4 = (C \cdot \eta \cdot C \cdot \eta \cdot C \cdot \eta \cdot C \cdot \eta)_\mu^\mu. \quad (6)$$

We will suppose that V has a local minimum at $C^{\mu\nu} = c^{\mu\nu} \neq 0$, and that $C^{\mu\nu}$ acquires an expectation value $\langle C^{\mu\nu} \rangle = c^{\mu\nu}$. Note that this implies spontaneous breaking of Lorentz invariance. We can now decompose $C^{\mu\nu} = c^{\mu\nu} + \tilde{C}^{\mu\nu}$ where $\tilde{C}^{\mu\nu}$ are the fluctuations of $C^{\mu\nu}$.

At low energy, the values of the scalars X_n will be approximately fixed to their values x_n in the local minimum. Then, the linearized form of the potential can be taken equivalent to a sum of Lagrange multiplier terms that fix the values of the four scalars X_n to the values x_n :

$$V \rightarrow \sum_{n=1}^4 \frac{\lambda_n}{n} (X_n - x_n). \quad (7)$$

We obtain the linearized equations of motion

$$K_{\mu\nu\alpha\beta}\tilde{C}^{\alpha\beta} = -\lambda_1\eta_{\mu\nu} + \lambda_2(\eta c \eta)_{\mu\nu} + \lambda_3(\eta c \eta c \eta)_{\mu\nu} + \lambda_4(\eta c \eta c \eta c \eta)_{\mu\nu} \quad (8)$$

together with the constraints

$$\tilde{C}_\mu^\mu = 0, \quad c^{\mu\nu}\tilde{C}_{\mu\nu} = 0, \quad (c\eta c)^{\mu\nu}\tilde{C}_{\mu\nu} = 0, \quad (c\eta c \eta c)^{\mu\nu}\tilde{C}_{\mu\nu} = 0. \quad (9)$$

Noting that the left-hand side of eq. (8) equals the linear part of the Ricci tensor, it follows that the low-energy linearized dynamics of this model is equivalent to linearized general relativity in an axial-type gauge defined by conditions (9).

The four gauge conditions (9) reduce the original 10 $h_{\mu\nu}$ modes to 6 degrees of freedom. They can be expressed as the generators of the Lorentz generators $\mathcal{E}_{\mu\nu} = -\mathcal{E}_{\nu\mu}$ acting on the vacuum expectation value $c^{\mu\nu}$ of the cardinal field:

$$\tilde{C}^{\mu\nu} = \mathcal{E}^\mu{}_\alpha c^{\alpha\nu} + \mathcal{E}^\nu{}_\alpha c^{\mu\alpha}. \quad (10)$$

Imposing the equations of motion imposes masslessness as well as the Lorenz conditions:

$$\partial^2\tilde{C}_{\mu\nu} = 0, \quad \partial^\mu\tilde{C}_{\mu\nu} = 0, \quad (11)$$

reducing the number of propagating degrees of freedom to two helicities.

2. The bootstrap

In order that the cardinal model correctly describe gravity it needs to be coupled to the matter energy-momentum tensor. At the linear level, this can be done by including the term

$$\mathcal{L} \supset \tilde{C}^{\mu\nu}\tau_{M\mu\nu} \quad (12)$$

where

$$\tau_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T_{M\alpha}^\alpha \quad (13)$$

is the trace-reversed energy-momentum tensor. As a result, the equations of motion reduce to the linearized Einstein equation

$$K_{\mu\nu\alpha\beta}\tilde{C}^{\alpha\beta} \equiv R_{\mu\nu}^L = \tau_{M\mu\nu} \quad (14)$$

(where we have taken, for now, the values of the Lagrange multipliers λ_n equal to zero).

The total energy-momentum tensor consists not only of contributions of matter. There is a contribution of the gravitons themselves as well, quadratic in $\tilde{C}^{\mu\nu}$. As a consequence, the inclusion of a cubic term in (14) is required. This, in turn, implies a cubic contribution to the energy-momentum tensor, corresponding to a quartic term in the Lagrangian. This process continues indefinitely, yielding in the limit the full Einstein-Hilbert action [4]. It has been shown by Deser [5] that this “bootstrap” process can be carried out, for general relativity, in one step if one rewrites the free graviton action in first order (Palatini) form.

In order to implement this bootstrap procedure for the cardinal theory, we pass, as a first step, to the trace-reversed tensor \mathfrak{E} as

$$\mathfrak{E}^{\mu\nu} = -C^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}C^\alpha{}_\alpha. \quad (15)$$

The kinetic term in (14) can be rewritten in Palatini form as

$$\mathfrak{E}^{\mu\nu}(\Gamma^\alpha_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + \eta^{\mu\nu}(\Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu}) \quad (16)$$

where the connection coefficients $\Gamma^\alpha_{\mu\nu}$ are to be considered as independent additional variables. It can be shown that the bootstrap of the kinetic term then terminates after one step, yielding

$$\mathfrak{E}^{\mu\nu}(\Gamma^\alpha_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + (\eta^{\mu\nu} + \mathfrak{E}^{\mu\nu})(\Gamma^\alpha_{\mu\nu}\Gamma_\alpha - \Gamma^\alpha_{\beta\mu}\Gamma^\beta_{\alpha\nu}) \quad (17)$$

which is equivalent to the Einstein-Hilbert lagrangian

$$(\eta + \mathfrak{E})^{\mu\nu}R_{\mu\nu}(\Gamma). \quad (18)$$

From the latter form we conclude that $(\eta + \mathfrak{E})^{\mu\nu}$ is naturally interpreted as a curved-space metric density.

The bootstrap procedure can also be applied to the matter interaction to determine the form of the matter coupling for cardinal gravity. For this purpose, the interaction (12) is conveniently expressed in terms of the trace-reversed energy-momentum tensor $\tau_{M\mu\nu}$ for the matter. This tensor arises by variation of the Lagrange density \mathcal{L}_M for the matter fields via

$$-\frac{1}{2}\tau_{M\mu\nu} = \left. \frac{\delta\mathcal{L}_M(\eta \rightarrow \psi)}{\delta\psi^{\mu\nu}} \right|_{\psi \rightarrow \eta} \quad (19)$$

in the usual way. We can therefore write

$$\mathcal{L}_{M,\mathfrak{E}}^L = -\frac{1}{2}\mathfrak{E}^{\mu\nu}\tau_{M\mu\nu} \quad (20)$$

for the matter interaction with the cardinal field $\mathfrak{E}^{\mu\nu}$. Applying the bootstrap procedure can be shown to generate the Lagrange density

$$\mathcal{L}_{M,\mathfrak{E}} = \mathcal{L}_M|_{\eta \rightarrow \eta + \mathfrak{E}}. \quad (21)$$

This expression corresponds to the usual curved-space matter Lagrangian if we identify (as above) $(\eta + \mathfrak{E})^{\mu\nu}$ as the metric density. For example, the bootstrap procedure applied to the flat-space electromagnetic energy-momentum tensor yields

$$\mathcal{L}_{EM} = -\frac{1}{4\sqrt{|\eta + \mathfrak{E}|}}(\eta + \mathfrak{E})^{\alpha\gamma}(\eta + \mathfrak{E})^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}. \quad (22)$$

It is interesting to note at this point that the way we implemented the bootstrap procedure above is not completely unique. Instead of writing the full trace-reversed tensor $\mathfrak{G}^{\mu\nu}$ in the Palatini form (16), one could substitute its fluctuations $\tilde{\mathfrak{G}}^{\mu\nu}$ (defined analogously to $\tilde{C}^{\mu\nu}$ above) instead. While this makes no difference in the linearized expression, when applying the bootstrap to this form one ends up with a different result, yielding $(\eta + \tilde{\mathfrak{G}})^{\mu\nu}$ as the curved-space metric density. Applying this alternative procedure to the matter coupling yields a curved-space matter Lagrangian with an explicit reference to the expectation value $c^{\mu\nu}$, constituting Lorentz-violating terms that can be related to the Standard Model Extension [6, 7]. Its parameters for Lorentz violation have been subject to numerous experimental measurements [8]. For details see Ref. [3].

The most interesting application of the bootstrap is to the scalar potential V , which we now express as a function of four scalars \mathfrak{x}_i ($i = 1 \dots 4$) defined analogously to the X_i defined in Eqs. (3)–(6) but with $C^{\mu\nu}$ replaced by the trace-reversed tensor $\mathfrak{G}^{\mu\nu}$. As it turns out, for the procedure to be able to be applied to V , the latter needs to satisfy nontrivial integrability conditions that strongly restrict its functional form. For instance, it follows that the unique lowest-order (in terms of the total power of \mathfrak{C}) integrable polynomials are

$$\begin{aligned} \mathfrak{B}_0 &= 1, \\ \mathfrak{B}_1 &= \mathfrak{x}_1, \\ \mathfrak{B}_2 &= \mathfrak{x}_2 - \frac{\mathfrak{x}_1^2}{2}, \\ \mathfrak{B}_3 &= \mathfrak{x}_3 - \frac{3\mathfrak{x}_1\mathfrak{x}_2}{4} + \frac{\mathfrak{x}_1^3}{8}. \end{aligned} \tag{23}$$

More generally, it follows that any polynomial obtained as the term at order q in the series expansion of $\sqrt{|\det[1 + \mathfrak{C}\eta]|}$ is a solution \mathfrak{B}_q . Applying the bootstrap to \mathfrak{B}_q yields

$$\mathfrak{B}_q \rightarrow \sqrt{|\det[1 + \mathfrak{C}\eta]|} - \sum_{k=0}^{q-1} \mathfrak{B}_k. \tag{24}$$

We postulate that any integrable potential can be obtained as a suitable linear combination of the \mathfrak{B}_q polynomials.

Of particular interest are scalar potentials $\mathfrak{B}_{\mathfrak{C}}$ that have a Taylor expansion of the form

$$\mathfrak{B}_{\mathfrak{C}}(\{\mathfrak{x}_i\}) = \frac{1}{2} \sum_{i,j} a_{ij} (\mathfrak{x}_i - \mathfrak{x}_i) (\mathfrak{x}_j - \mathfrak{x}_j) + \mathcal{O}(\mathfrak{x}_i - \mathfrak{x}_i)^3 \tag{25}$$

with a_{ij} real constants. If we impose that a_{ij} be positive definite, $\mathfrak{B}_{\mathfrak{C}}$ has a local minimum at $\mathfrak{x}_i = \mathfrak{x}_i$ ($i = 1 \dots 4$), and it can represent a possibly stable vacuum. It can be shown that the integrability conditions impose constraints on the a_{ij} and on the higher order coefficients, but that a nontrivial solution space exists [3]. We have verified numerically that there is a non-empty subset of positive definite solutions, confirming the possibility of scalar potential with stable vacua [3].

It is reasonable to expect that, in the limit $a_{ij} \rightarrow \infty$ the integrable potentials $\mathfrak{B}_{\mathfrak{C}}$ of eq. 25 correspond to integrable potentials in the linearized limit considered above

$$\mathfrak{B}_{\mathfrak{C}}^L = \lambda_1 (\mathfrak{x}_1 - \mathfrak{x}_1) + \lambda_2 \left(\mathfrak{x}_2 - \frac{\mathfrak{x}_1^2}{2} - \mathfrak{x}_2 + \frac{\mathfrak{x}_1^2}{2} \right) + \lambda_3 \left(\mathfrak{x}_3 - \frac{3\mathfrak{x}_1\mathfrak{x}_2}{4} + \frac{\mathfrak{x}_1^3}{8} - \mathfrak{x}_3 + \frac{3\mathfrak{x}_1\mathfrak{x}_2}{4} - \frac{\mathfrak{x}_1^3}{8} \right) + \dots, \tag{26}$$

fixing the value of \mathfrak{x}_i to be \mathfrak{x}_i . We conclude that in this limit, the bootstrapped cardinal model corresponds to General Relativity in the gauge defined by the constraints $\mathfrak{x}_i = \mathfrak{x}_i$. This is also the case for a potential with finite a_{ij} , if we only consider energy scales low enough such that any fluctuations away from the minimum in the potential can be neglected.

3. Vacuum energy-momentum tensor

After applying the bootstrap procedure described above we end up with a Lagrangian density of the form

$$(\eta + \mathfrak{L}^{\mu\nu})R_{\mu\nu}(\Gamma) - \sqrt{|\eta + \mathfrak{L}|} V(\mathfrak{X}_1, \mathfrak{X}_2, \mathfrak{X}_3, \mathfrak{X}_4) + \mathcal{L}_M(C, \eta, \phi_i, \partial_\mu \phi_i) \quad (27)$$

The linearized equations of motion become:

$$K_{\mu\nu\alpha\beta}\mathfrak{L}^{\alpha\beta} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha}c^{\alpha\beta}\eta_{\beta\nu}\partial_2 + \dots)V + \tau_{\mu\nu}^{(m)}(\eta, \phi_i, \partial_\mu \phi_i) \quad (28)$$

where we defined

$$\partial_n \equiv \frac{\partial}{\partial \mathfrak{X}_n} \quad (n = 1 \dots 4). \quad (29)$$

We see from Eq. (28) that the first term on the right-hand side naturally takes the form of a (trace-reversed) energy momentum tensor. Explicitly, we can identify a “vacuum energy-momentum tensor”

$$T_{\mu\nu}^{(vac)} = \tau_{\mu\nu}^{(vac)} - \frac{1}{2}\eta_{\mu\nu}(\tau^{(vac)})^\alpha_\alpha \quad (30)$$

with

$$\tau_{\mu\nu}^{(vac)} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha}c^{\alpha\beta}\eta_{\beta\nu}\partial_2 + \dots)V. \quad (31)$$

It takes nonzero values whenever the scalar potential takes values away from the minimum.

Explicit solutions of the linearized equations of motion can be obtained with nonzero vacuum energy-momentum tensor. In the latter case, independent initial/boundary field values can be defined on maximally 4 suitably defined timelike/spacelike spacetime slices. If the matter energy-momentum tensor is known to be independently conserved (e.g., by symmetry arguments), the same has to be true for the vacuum energy-momentum tensor. In such a case, choosing $T_{\mu\nu}^{(vac)}$ to be zero at the initial value spacetime slices ensures it is zero throughout spacetime.

4. Conclusions

We showed it is possible to construct an alternative theory of gravity, the cardinal model, based on spontaneous breaking of Lorentz symmetry. The massless gravitons can be interpreted as Nambu-Goldstone modes of the spontaneously broken Lorentz symmetry. The full nonlinear form of the Lagrangian is fixed by consistent coupling of gravity to the total energy-momentum tensor, and can be constructed by a bootstrap process, starting with an initial Lagrangian for the cardinal field consisting of a quadratic kinetic term and a scalar potential. As it turns out, consistency of the bootstrap process imposes strong restrictions on the form of the scalar potential. Nevertheless, consistent potentials exist with local minima. At low energy, the Lagrangian corresponds to the Einstein-Hilbert action, with the possible presence of a nonzero “vacuum energy-momentum tensor”. At high energy, four extra massive graviton modes appear that modify the dynamics of the theory. An open problem remains the classification of all possible bootstrapped potentials and a study of their properties. Other issues that merit further study are the effect of the massive modes, in particular near singularities or at high temperatures, as well as a study of the quantization of the cardinal model.

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