Optimal Design of Geometrically Non-linear Laminated Structures

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The mechanical properties of a laminate are strongly dependent of the fiber directions and because of that the laminate should be designed to meet the specific requirements of each particular application in order to obtain the maximum advantages.

Design sensitivity analysis is very important to accurately know the effects of design variables changes on the performance of structures by calculating the search directions to find an optimum design. To evaluate these sensitivities efficiently and accurately it is important to have appropriate techniques associated to good structural models, which also must be efficient with regard to CPU time, especially when nonlinear analysis are involved.

The use of high strength composite materials and survival of structures under extreme environments and the need to obtain material savings by considering design optimizations using non-linear structural formulation is the main motivation for the present work.

The objective of this paper is to present the development and to show illustrative applications of a discrete finite element model to evaluate design sensitivities of geometric nonlinear response for laminated plate and shell type structures.

These sensitivities of response are evaluated analytically with respect to angle orientation of the fiber of each layer and the vectorial distances.

The discrete model, which has been developed and implemented, is based on a triangular flat plate element, based on Reddy's higher order displacement field.

The present triangular higher order discrete model has 24 degrees of freedom, which can be reduced to 18 when the Kirchhoff model (CPT) option is used.

After discretization, the equations of equilibrium for static nonlinear response can be written as [1] :

$$f(u,b) = \mu\, p(b) \qquad (1)$$

where f is the vector of internal forces, which is a function of displacement vector u and a design variable b, p is the vector of applied loads and μ is an amplitude parameter. It is assumed that the applied loads are not function of the displacement field. The equations of equilibrium are solved iteratively for a series of μ, using the update Lagrangian formulation in association with the arc-length method. The sensitivity of u with respect to the design variable b may be obtained by differentiating (1), yielding [1] :

$$\frac{du}{db} = -J^{-1}\left(\mu\frac{dp}{db} - \frac{\partial f}{\partial b}\right) \qquad (2)$$

where J is de Jacobian $\partial f/\partial u$, which is know as the tangential stiffness matrix.

In the proposed paper the formulation will be presented to evaluate by direct and adjoint techniques the sensitivities of limit loads, displacements, stress and strains, with respect to changes in design variables.

A hinged-free antisymmetric cylindrical panel (Fig. 1), with lay-up $[-45°/45°]$, subjected to a point load, is analysed. The geometry and material properties are :

R=2540 mm ; L=508 mm ; h=12.6 mm ; θ=0.1 rad ; E_1=3.3 GPa ; E_2=1.1 GPa ; G_{12}=0.66 GPa ; ν_{12}=0.25.

The straight edges are hinged and the curved edges are free. A model discretization with (8x8) elements is used.

The non-linear load deflection curve obtained with the present HSDT triangular element and those obtained by Madenci and Barut [2],Saigal et al [3],and Moita et al [4], are shown in Figure 2. A good agreement is found between the present solution and those from references [2] and [4]. Figure 3 shows the results of the cylindrical panel when it is optimized for the limit load and play angles and thicknesses as design variables. The structure is divide into 16 groups of elements [5]. For the second level of optimization, which gives the distribution of thickness, load displacement paths A and B, represent the intermediate and final stages of the optimization process.

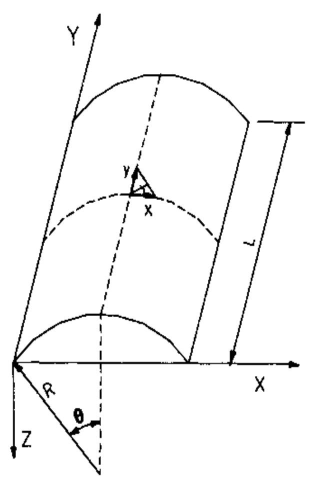

Fig. 1- Hinged-free antisymmetric cylindrical panel

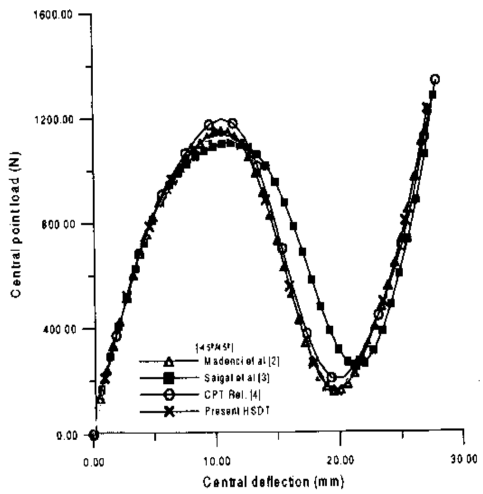

Fig. 2 – Load deflection curves under point load

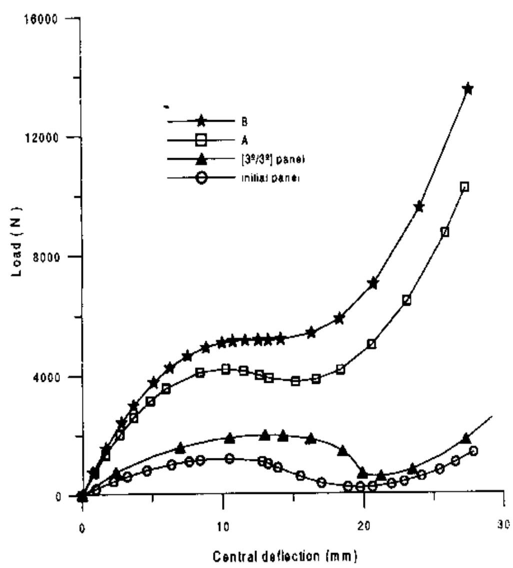

Fig. 3 – Load deflection curves for different thickness distribution and play angles.

REFERENCES

1. Haftka, R. T., "Semi-Analytical Static Nonlinear Structural Sensitivity Analysis", AIAA Journal, Vol. 31 (7), pp 1307-1312, 1993.
2. Madenci, E. and Barut, A. "A Free-Formulation-Based Flat Shell Element for Nonlinear Analysis of Thin Composite Structures", International Journal for Numerical Methods in Engineering, Vol. 37, pp 3825-3842, 1994.
3. Saigal, S., Kapania, R. K. and Yang, T. Y., "Geometrically Nonlinear Finite Element Analysis of Imperfect Laminated Shells", Journal of Composite Materials, Vol. 20, pp. 197-214, 1986.
4. Moita, J. S., Mota Soares, C. M. and Mota Soares, C. A., "Non-Linear Finite Element Analysis of Thin Composite Structures", Proc. 11th International Conference on Composite Materials, Ed. M. L. Scott, Australian Composite Structures Society, Vol. 6, pp. 808-811, 1997.
5. Moita, J. S., Infante Barbosa, J., Mota Soares, C. M. and Mota Soares, C. A., "Sensitivity Analysis and Optimal Design of Geometrically Non-Linear Laminates Plates and Shells", Computers & Structures (to appear).