An invariant for Stallings manifolds from a TQFT

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We will present our construction of a class of effectively calculable, isomorphism invariants for Stallings [1] manifolds by constructing a class of Topological Quantum Field Theories (TQFT's) [2] for these manifolds. Given a 2-dimensional oriented manifold without boundary, S, and an orientation-preserving automorphism $\varphi : S \to S$, the self-gluing of the cylinder $S \times I$, where I is the standard closed unit interval, is a 3-dimensional manifold $S_\varphi := \frac{S\times I}{\sim_\varphi}$ known as a Stallings manifold, where $\sim_\varphi$ is the relation generated by the relation $(x, 0) \sim (\varphi(x), 1)$. A fundamental feature of TQFT is the gluing together of two spaces along one or more boundary components. Our TQFT approach [3, 4] (in [4] this approach was applied in the geometric context of gerbes) describes equally well the self-gluing of a single space.

In our approach a TQFT is a monoidal functor from a topological to an algebraic category. The morphisms of the topological category are isomorphisms and gluing morphisms, where the latter are loosely described as morphisms from an oriented 3-manifold with boundary before gluing to the same manifold after gluing. On the algebraic side, the objects of the category are pairs (V, x), where V is a vector space over a field with an involution, and x is an element of V. The morphisms are linear maps which preserve the elements of the respective spaces. A TQFT functor preserves an additional structure given by two monoidal endofunctors, corresponding to change of orientation on the topological side and changing the scalar multiplication using the involution on the algebraic side.

Part of the data needed to specify a TQFT is a certain type of unitary representation of the mapping class group of S. The remaining conditions come from applying the TQFT functor to a set of topological isomorpisms and gluing morphisms, giving rise to a set of matrix equations, from which the invariant is obtained as a certain trace. We analyse the equations and present some solutions.

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