# A Technique for Reducing the PMEPR of MC-CDMA Signals

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**Abstract - In this paper we consider the use of MC-CDMA (Multi-Carrier Code Division Multiple Access) schemes with a frequency-domain orthogonal spreading. The transmitter structure is especially modified for reducing the envelope fluctuations of the transmitted signals. An analytical approach for the performance evaluation is also included which relies on the Gaussian characteristics of MC signals with a high number of subcarriers.**

**A set of performance results is presented. Our performance results show that the proposed transmitter structure is adequate for power-efficient MC-CDMA transmission since we can reduce the envelope fluctuations of the transmitted signals, while keeping essentially the same spectral efficiency of the conventional MC-CDMA transmission.**

## I. Introduction

The MC-CDMA schemes (Multicarrier Coded Division Multiple Access) [1], [2] combine a CDMA scheme with an OFDM modulation (Orthogonal Frequency Division Multiplexing) [3], so as to allow high transmission rates over severe time-dispersive channels without the need of a complex receiver implementation.

It is well-known that an important drawback of OFDM modulations is the strong envelope fluctuation and high PMEPR (Peak-to-Mean Envelope Power Ratio) of the transmitted signals. This leads to amplification difficulties, since the power amplifiers are required to have linear characteristics and/or a significant input backoff has to be adopted, so as to avoid the out-of-band radiation levels which are inherent to nonlinear distortion. For this reason, several PMEPR-reducing techniques have been proposed, in recent years, to simplify the "power amplification problem" with OFDM transmission (see [4] and the references therein). Among such techniques, those allowing a high spectral efficiency deserve a special attention. This is the case of a wide class of signal processing schemes proposed in [5], which involves a nonlinear operation in the time domain and a linear, filtering operation in the frequency domain. This class was analyzed in detail, in [6], where it was shown how to take advantage of the Gaussian-like nature of OFDM signals with a high number of subcarriers for obtaining accurate performance results in an analytical way.

Since the MC-CDMA signals with frequency domain spreading are OFDM-like multicarrier signals, they also have high envelope fluctuations. Therefore, it is important to design MC-CDMA signals with reduced envelope fluctuations. In this paper we consider the use of MC-CDMA schemes with a frequency-domain orthogonal spreading. The transmitter structure is especially modified for reducing the envelope fluctuations of the MC-CDMA signals and is based on the signal processing schemes proposed in [5], [6] for low-PMEPR, bandwidth-efficient OFDM transmission.

A very accurate analytical approach for the performance evaluation is derived. We also present exact and approximate formulas for the BER performance. For this purpose, we take advantage of the Gaussian characteristics of MC-CDMA signals with a large number of subcarriers. Our analytical approach can be regarded as a modified version of the statistical characterization of the transmitted signals proposed in [5], [6], so as to take into account the nature of MC-CDMA signals with frequency-domain spreading.

This paper is organized as follows. The MC-CDMA schemes considered in this paper are described in sec. II. Sec. III presents the transmitter structure proposed in this paper. The analytical characterization of the transmitted signals is made in sec. IV. A set o performance results is presented in sec. V and sec. VI is concerned with the conclusions of this paper.

## II. Systems Description

In this paper we consider the downlink transmission (i.e., the transmission from the BS (Base Station) to the MT (Mobile Terminal)) within MC-CDMA systems employing frequency-domain spreading (although our approach could also be employed in the uplink transmission). A constant spreading factor $K$ is assumed for all users. The frequency-domain block to be transmitted by the BS, $\{S'_k; k = 0, 1, \ldots, N-1\}$, is an interleaved version [1] of the block $\{S_k; k = 0, 1, \ldots, N-1\}$, where $N = KM$, with $K$ denoting the spreading factor and $M$ the number of data symbols for each user. The frequency-domain symbols are given by

$$S_k = \sum_{p=1}^{P} \gamma_p S_{k,p} \tag{1}$$

[1] Typically, the transmitted frequency-domain block is generated by submitting the block $\{S_k; k = 0, 1, \ldots, N-1\}$ to a rectangular interleaver with dimensions $K \times M$, i.e., the different chips associated to a different data symbol are spaced by $K$ subcarriers

where $\gamma_p$ is an appropriate weighting coefficient that accounts for the power control in the downlink (the power associated to the $p$th user is proportional to $\gamma_p^2$) and $S_{k,p} = C_{k,p}A_{\lfloor k/K \rfloor,p}$ is the $k$th chip for the $p$th user ($\lfloor x \rfloor$ denotes 'larger integer not higher that $x$'), where $\{A_{k,p}; k = 0, 1, \ldots, M-1\}$ is the block of data symbols associated to the $p$th user and $\{C_{k,p}; k = 0, 1, \ldots, N-1\}$ is the corresponding spreading sequence. An orthogonal spreading is assumed throughout this paper, with $C_{k,p}$ belonging to an QPSK constellation (Quadrature Phase Shift Keying). Without loss of generality, it is assumed that $|C_{k,p}| = 1$.

As with conventional OFDM, an appropriate cyclic extension is appended to each block transmitted by the BS. At the receiver, the cyclic extension is removed and the received samples are passed to the frequency domain, leading to the block is $\{Y_k; k = 0, 1, \ldots, N-1\}$.

It can be shown that, when the cyclic extension is longer than the overall channel impulse response, the samples $Y_k$ can be written as

$$Y_k = H_k S'_k + N_k \tag{2}$$

where $H_k$ and $N_k$ denote the channel frequency response and the noise term for the $k$th frequency, respectively.

Since the orthogonality between users is lost in frequency selective channels, an FDE (Frequency-Domain Equalizer) is required before the de-spreading operation [2]. The equalized frequency-domain samples are $\tilde{S}'_k = Y_k F_k$, with the FDE coefficients

$$F_k = \frac{SNR\, H_k^*}{1 + SNR|H_k|^2}, \tag{3}$$

where

$$SNR = \frac{E[|S_k|^2]}{E[|N_k|^2]}, \tag{4}$$

which corresponds to the minimization of the MMSE (Minimum Mean-Squared Error) in the frequency-domain samples $S'_k$. After removing the idle subcarriers, the equalized samples $\tilde{S}'_k$ are de-interleaved, leading to the block $\{\tilde{S}_k; k = 0, 1, \ldots, N-1\}$. The data symbols associated to the $p$th user can be estimated from the de-spreaded samples

$$\tilde{A}_{k,p} = \sum_{k' \in \Psi_k} \tilde{S}_{k'} C^*_{k',p}. \tag{5}$$

with $\Psi_k = \{k, k+M, \ldots, k+(K-1)M\}$ denoting the set of frequencies employed to transmit the $k$th data symbol of each user.

## III. Proposed Transmitter Structure

Since the MC-CDMA signals have large envelope fluctuations and a high PMEPR, a quasi-linear power amplification is required. To allow an efficient power amplification, it is important to reduce the PMEPR of the MC-CDMA signals to be transmitted. Fig. 1 shows the transmitted structure considered in this paper where we combine a nonlinear operation, operating on an oversampled version of the MC-CDMA block (obtained by adding $N' - N$ zeros to the original frequency-domain block (i.e., $N' - N$ idle subcarriers), followed by an IDFT operation), followed by a frequency-domain filtering operation (characterized by the set of multiplying coefficients $G_k, k = 0, 1, \ldots, N'-1$). This technique is similar to the ones proposed in [5], [6] for reducing the PMEPR of conventional OFDM signals.

even with reduced-PMEPR MC-CDMA signals, we still need a linear amplification characteristic, at least for the range of envelope fluctuations of the signals at its input. For typical amplifiers, this might mean a large back-off and, consequently, a reduced amplification efficiency. To avoid this, it is common to employ pre-distortion techniques to linearize the power amplifier [7]. This pre-distortion, which can be regarded as the inverse function of the amplification characteristic, is performed on a oversampled version of the reduced-PMEPR MC-CDMA signal.

## IV. Statistical Characterization of the Transmitted Signals

When the number of subcarriers is high ($N >> 1$) the time-domain coefficients $s'_n$ can be approximately regarded as samples of a complex Gaussian process. Throughout this paper it will be assumed that $E[S_k] = 0$ and $E[S_k S^*_{k'}] = 2\sigma_S^2 \delta_{k,k'}$ ($\delta_{k,k'} = 1$ for $k = k'$ and 0 otherwise), with $\sigma_S^2 = \frac{1}{2} E\left[|S_k|^2\right]$ ($E[\cdot]$ denotes "ensemble average"). In this case, it can be easily demonstrated that $E[s'_n] = 0$ and

$$E\left[s'_n s'^*_{n'}\right] = R_s(n-n') =$$
$$= 2\sigma^2 \frac{\operatorname{sinc}((n-n')N/N')}{\operatorname{sinc}((n-n')/N')} \exp\left(-\frac{j\pi(n-n')}{N'}\right) \tag{6}$$

($n, n' = 0, 1, \ldots, N'-1$), with $\sigma^2 = \frac{N}{(N')^2}\sigma_S^2$.

It is well-known that the output of a memoryless nonlinear device with a Gaussian input can be written as the sum of two uncorrelated components [8]: a useful one, proportional to the input, and a self-interference one, i.e.,

$$s_n^C = \alpha s'_n + d_n\ , \tag{7}$$

where $E[s'_n d^*_{n'}] = 0$ and

$$\alpha = \frac{E[s_n^C s'^*_n]}{E[|s'_n|^2]} = \frac{E[R g_C(R)]}{E[R^2]} =$$
$$= \frac{1}{2\sigma^2} \int_0^{+\infty} R g_C(R) \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right) dR. \tag{8}$$

The average power of the useful component is $S = |\alpha|^2\sigma^2$, and the average power of the self-interference component is given by $I = P_{out} - S$, where $P_{out}$ denotes the average power of the signal at the nonlinearity output, given by

$$P_{out} = \frac{1}{2} E[|g_C(R)|^2] =$$
$$= \frac{1}{2} \int_0^{+\infty} |g_C(R)|^2 \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right) dR. \tag{9}$$

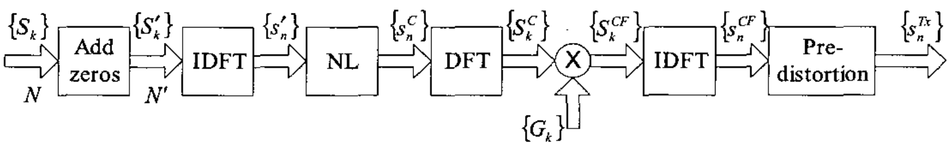


Fig. 1. Transmitter model considered in this paper (NL: nonlinear operation).

It can be shown [9] that

$$E[s_n^C s_{n'}^{C*}] = R_s^C(n-n') = \\ = \sum_{\gamma=0}^{+\infty} 2P_{2\gamma+1} \frac{(R_s(n-n'))^{\gamma+1}\,(R_s^*(n-n'))^{\gamma}}{(R_s(0))^{2\gamma+1}}, \tag{10}$$

where the coefficient $P_{2\gamma+1}$ denotes the total power associated to the IMP (Inter-Modulation Product) of order $2\gamma+1$ and can be obtained as described in [9].

Since $R_s^C(n-n') = |\alpha|^2 R_s(n-n') + E[d_n d_{n'}^*]$, it can be easily recognized that $P_1 = |\alpha|^2\sigma^2$ and

$$E[d_n d_{n'}^*] = R_d(n-n') = \\ = \sum_{\gamma=1}^{+\infty} 2P_{2\gamma+1} \frac{(R_s(n-n'))^{\gamma+1}\,(R_s^*(n-n'))^{\gamma}}{(R_s(0))^{2\gamma+1}}. \tag{11}$$

The total self-interference power is

$$I = \frac{1}{2}R_d(0) = \sum_{\gamma=1}^{+\infty} P_{2\gamma+1} = P_{out} - S. \tag{12}$$

Having in mind (7) and the signal processing chain in fig. 1, the frequency-domain block $\{S_k^{CF} = S_k^C G_k; k = 0,1,\ldots,N'-1\}$ can obviously be decomposed into useful and self-interference components:

$$S_k^{CF} = \alpha S'_k G_k + D_k G_k, \tag{13}$$

where $\{D_k; k = 0,1,\ldots,N'-1\}$ is the DFT of $\{d_n; n = 0,1,\ldots,N'-1\}$. It can be shown that $E[D_k] = 0$ and $E[D_k D_{k'}^*] = N' G_d(k)\delta_{k,k'}$ $(k,k' = 0,1,\ldots,N'-1)$, where $\{G_d(k); k = 0,1,\ldots,N'-1\}$ denotes the DFT of the block $\{R_d(n); n = 0,1,\ldots,N'-1\}$.

$E[S_k^C S_{k'}^{C*}] = 0$ for $k \neq k'$ and $E[|S_k^C|^2] = N' G_s^C(k)$, where $\{G_s^C(k) = |\alpha|^2 G_s(k) + G_d(k); k = 0,1,\ldots,N'-1\}$ denotes the DFT of $\{R_s^C(n); k = 0,1,\ldots,N'-1\}$ (given by (10)), with $\{G_s(k); k = 0,1,\ldots,N'-1\} = \text{DFT}\ \{R_s(n); n = 0,1,\ldots,N'-1\}$. Therefore, $E[S_k^{CF} S_{k'}^{CF*}] = 0$ for $k \neq k'$, and $E[|S_k^{CF}|^2] = |G_k|^2 E[|S_k^C|^2] = N'|G_k|^2 G_s^C(k)$.

From (2), the frequency-domain samples at the receiver are given by $Y_k = S_k^{Tx} H_k + N_k$, provided that the guard interval is long enough.

For an ideal Gaussian channel, the detection of the $k$th symbol transmitted by the $p$th user is based on

$$\tilde{A}_{k,p} = \sum_{k' \in \Psi_k} Y_{k'} C_{k',p}^* = \alpha\gamma_p K A_{k,p} + D_{k,p}^{eq} + N_{k,p}^{eq} \tag{14}$$

(see (5)). In (14),

$$D_{k,p}^{eq} = \sum_{k' \in \Psi_k} D_{k'} C_{k',p}^* \tag{15}$$

and

$$N_{k,p}^{eq} = \sum_{k' \in \Psi_k} N_{k',p} C_{k',p}^* \tag{16}$$

denote the equivalent self-interference and noise terms for detection purposes, respectively. It can be shown that [10] that the signal-to-self-interference ratio for detection purposes is approximately independent of $k$ and given by

$$SIR_p^{eq} \approx \frac{K}{K_U}\eta_{\gamma,p} SIR^{Tx}, \tag{17}$$

where

$$\eta_{\gamma,p} = \frac{\gamma_p^2}{\overline{\gamma^2}}, \tag{18}$$

with

$$\overline{\gamma^2} = \frac{1}{K_U}\sum_{p=1}^{K_P} \gamma_p^2, \tag{19}$$

and the signal-to-self-interference ratio of the transmitted signal is given by

$$SIR^{Tx} = \frac{\sum_k E[|\alpha S_k|^2]}{\sum_k E[|D_k|^2]}. \tag{20}$$

It can be shown that when $N' = N$ (i.e., without oversampling) then $E[|D_k|^2]$ is constant and the $SIR^{Tx}$ is simply given by [5], [6]

$$SIR^{Tx} = \frac{S}{I} \tag{21}$$

For $N' > N$ we need to use $E[|D_k|^2]$, computed analytically as described in (18) for obtaining $SIR^{Tx}$. If $N' \geq 2N$ it can be shown that the $SIR^{Tx}$ is approximately given by [10]

$$SIR^{Tx} \approx \frac{3}{2}\frac{S}{I}. \tag{22}$$

From (15), it is clear that the equivalent SIR for detection purposes increases when we decrease the number of users, for a given spreading factor $K$. This is a consequence of the samples $D_k C_{k,p}^*$ being uncorrelated, contrarily the useful components.

We can also note, from (15) (and (17)), that the equivalent SIR for detection purposes is not the same for the different users: the users with smaller attributed power (probably the ones closer to the BS) have worse $SIR_p^{eq}$, and, consequently, a larger performance degradation due to the nonlinear effects. In a practical situation, this can be compensated by increasing slightly the power attributed to these "low-power users".

Since the self-interference components $D_k$ are approximately Gaussian-distributed at the subcarrier level [5], [6], $D_k^{eq}$ is also approximately Gaussian-distributed. Therefore, the equivalent SIR levels for detection purposes, $SIR_p^{eq}$, can be employed for the BER performance evaluation.

## V. Performance Results

In this section we present some performance results concerning the transmitted structure considered in this paper. We have an MC-CDMA signal with $N = 256$ subcarriers and a spreading factor of 64. The oversampling factor is 4, which is enough for efficient PMEPR-reduction and pre-distortion operations, and $G_k = 1$ for the $N$ in-band subcarriers and 0 otherwise. We consider a perfect pre-distortion and an ideal AWGN channel.

Fig. 2 presents the analytical and simulated BER performances for a normalized clipping level $s_M/\sigma = 1.0$, when the number of users is either $K$ or $K/2$. The analytical results are obtained by using in (15) the true $SIR^{Tx}$ values, given by (18), and the approximate results are obtained by assuming that the $SIR^{Tx}$ is given by (20). Clearly our analytical approach is very accurate, even when the approximate formula is employed. Moreover, the sensitivity to nonlinear effects decreases with the system load $K_U/K$; when $K_U = K$ (i.e., for a fully loaded system), the situation is similar to conventional OFDM. It should be noted that the PMEPR of the transmitted signals, defined as in [4], is about 4.3dB, in opposition to the 8.4dB of conventional MC-CDMA signals; the nonlinear distortion effects lead to a performance degradation for BER=$10^{-2}$ of about 3dB when $K_U = K$ and 1.5dB when $K_U = K/2$. By increasing the clipping level decrease this degradation, but we increase the PMEPR of the transmitted signals. Moreover, the PMEPR reductions are achieved while keeping essentially the same spectral efficiency of conventional MC-CDMA signals.

## VI. Conclusions

The use of MC-CDMA schemes with a frequency-domain orthogonal spreading was considered in this paper. The transmitter structure was especially modified for reducing the PMEPR of the transmitted signals. An analytical approach for the performance evaluation was derived, taking advantage of the Gaussian characteristics of MC signals with a high number of subcarriers.

Our performance results show that the proposed transmitter structure is adequate for power-efficient MC-CDMA transmission since we can reduce the envelope fluctuations, while keeping essentially the same spectral efficiency of the conventional MC-CDMA transmission. It was also shown that the system load has a key influence on the robustness of MC-CDMA signals to nonlinear effects: the higher the system load the lower the robustness against nonlinear effects.

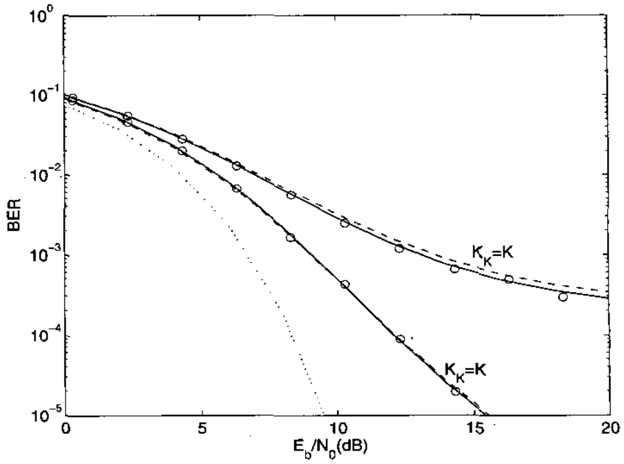


Fig. 2. Exact (solid line) and approximate (dashed line) analytical BER performances when $s_M/\sigma = 1.0$, as well as the simulated values (o). The dotted line denotes the performance of an ideal linear transmitter (i.e., when $s_M/\sigma = +\infty$), for the sake of comparisons.

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