



Decision Support

## Building sustainability composite indicators using a multi-criteria approach



António Xavier<sup>a,\*</sup>, Rui Fragoso<sup>b</sup>, Maria de Belém Costa Freitas<sup>c</sup>

<sup>a</sup> CEFAGE (Center For Advanced Studies in Management and Economics), University of Évora, Largo dos Colegiais, 7000 Évora, Portugal

<sup>b</sup> CEFAGE (Center For Advanced Studies in Management and Economics), University of Évora, Largo dos Colegiais, 7000, Évora, Portugal

<sup>c</sup> Sciences and Technology Faculty, University of Algarve, Gambelas Campus, Edif. 8 8005-139 Faro, Portugal and MED, Mediterranean Institute for Agriculture, Environment and Development

### ARTICLE INFO

JEL:

Q01

C5

Q2

Keywords:

Sustainability

Composite indicators

Multi-criteria decision analysis

Normalized entropy

### ABSTRACT

Building sustainability composite indicators is a complex process that has been addressed according to different strategies. One interesting approach is based on the compromise between the maximum aggregate solution and the most balanced solution, by considering the most displaced indicator regarding the ideal. However, some shortcomings were identified in this approach. First, several decision-making units may present an equal composite indicator, and hence the same position in the ranking, while corresponding to different sustainability situations. Second, the use of only the maximum deviation to define the most balanced solution requires a more integrated approach. Thus, this paper proposes a novel aggregation methodology for building sustainability composite indicators, where a new normalized entropy indicator for the most balanced solution is proposed and integrated with the sustainability criteria of the maximum aggregate solution and maximum deviation. The method proposed was applied to two illustrative examples from the literature and provided promising and robust results.

### 1. Introduction

One of the main purposes of natural systems is to provide goods and services, whose consumption satisfies primary human needs (Diaz-Balteiro et al., 2018). The use of these natural systems must preserve the resources for future generations, representing the sustainable development of a commitment between advancing human well-being and the biosphere's ecological limits (Moran et al., 2008).

Most studies have tried to measure sustainability using indicators (UN, 2022; UNDP, 2019). An indicator is a quantitative or qualitative measure derived from facts that can identify the relative position of a decision-making unit (DMU). When analysed at regular intervals, an indicator expresses the direction of change. Sustainability encompasses economic, environmental, and social dimensions, which implies the aggregation of different types of indicators into a composite indicator (OECD, 2008).

Composite indicators should ideally measure multidimensional concepts which cannot be captured by a single indicator (OECD, 2008). Therefore, a composite indicator is a useful tool for performance comparisons, public communication and decision support in a wide spectrum of fields (Zhou et al., 2010).

One of the main issues in composite indicators is the aggregation scheme, since their quality and reliability depend greatly on the underlying construction scheme and data aggregation (Zhou et al., 2010). The OECD (2008) presents several methods for creating composite indicators, including additive aggregation methods, geometric aggregation, and a non-compensatory multi-criteria approach. Several authors have already analysed sustainability, using different aggregation methods, as can be seen in Table 1.

It must be stressed that the study by Diaz-Balteiro and Romero (2004a), who proposed a Multi-Criteria Decision Analysis (MCDA) considering the “maximum aggregate sustainability” (an engineering solution) and the “most balanced sustainability” (an ecological solution), and identifying several intermediate solutions among them that can be used to obtain several rankings, had a considerable impact on the literature. The “most balanced sustainability” refers to an equal distribution sustainability among indicators, being related to the ideas of diversity and balance of ecological sustainability.

Despite the developments of the last 20 years, the approaches proposed after Diaz-Balteiro and Romero (2004a) - e.g. Diaz-Balteiro et al. (2011), Voces et al. (2012), Diaz-Balteiro et al. (2016a), Diaz-Balteiro et al. (2016b), Diaz-Balteiro et al. (2017a) and Diaz-Balteiro et al. (2022)

\* Corresponding author.

E-mail addresses: [amxav@sapo.pt](mailto:amxav@sapo.pt) (A. Xavier), [rfragoso@uevora.pt](mailto:rfragoso@uevora.pt) (R. Fragoso), [mbfreitas@ualg.pt](mailto:mbfreitas@ualg.pt) (M.B.C. Freitas).

<https://doi.org/10.1016/j.ejor.2025.04.024>

Received 24 February 2024; Accepted 13 April 2025

Available online 14 April 2025

0377-2217/© 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

**Table 1**  
Authorship, context, and aggregation method of different models to analyse sustainability.

Authors	Context	Aggregation method
Díaz-Balteiro and Romero (2004a)	Ranking analysis and definition of compromises between maximum aggregate sustainability and the ‘most balanced sustainability	Multi-Criteria Decision Analysis (MCDA)
Díaz-Balteiro and Romero (2004b)	Analyse the sustainability and define rankings of several forest management plans in Spain	Extended Goal Programming (EGP) based on MCDA
Gómez-Limón and Sanchez-Fernandez (2010)	Evaluation of agricultural sustainability	Weighted Sum of Indicators; Product of Weighted Indicators; Multi-Criteria Decision Analysis (MCDA)
Singh et al. (2012)	Individual analysis of 41 sustainability indicators	Different methods: Distance to Targets; Principal Components Analysis; Factor Analysis; Geometric Mean; Weighted Average; Sum of Rankings; Summation; Experts’ opinion.
Zhou et al. (2010)	Sustainability analysis	Entropy
Díaz-Balteiro et al. (2011)	Sustainability ranking of the paper industry in several EU countries	Compromise Programming (CP) based on MCDA
Voces et al. (2012)	Characterize the sustainability rankings of 17 EU countries regarding European wood manufacturing industries.	Extended Goal Programming (EGP) based on MCDA
Díaz-Balteiro et al. (2016a)	Study northwest Spain forest plantations’ sustainability.	Extended Goal Programming (EGP) based on MCDA
Díaz-Balteiro et al. (2016b)	Analysis of sustainability ranking in forest management, considering 30 plantations and 11 sustainability indicators	Extended Goal Programming (EGP) based on MCDA
Díaz-Balteiro et al. (2017a)	Identify the best management option, considering several climate change scenarios	Extended Goal Programming (EGP) based on MCDA
Xavier et al. (2018)	Analysis of agriculture sustainability, defining the rankings of considerable sets of data	Extended Goal Programming (EGP) based on MCDA improved with entropy
Wu et al. (2018)	Sustainable Society Index	Shannon entropy
Tolstykh et al. (2020)	Sustainability of industrial ecosystems	Entropy diversity
Díaz-Balteiro et al. (2022)	Circular economy, considering three different decision rules	Compromise Programming (CP) based on MCDA
Costa Freitas et al. (2022)	Build rankings of sustainable water use in Portugal	Compromise Programming (CP) based on MCDA improved with entropy
Xavier et al. (2022)	Analysis of agriculture sustainability dynamics	Compromise Programming (CP) based on MCDA improved with entropy

- have limitations:

- Several DMU may have the same value of sustainability in the “most balanced” situation, despite representing different situations;
- Use of only the maximum deviation to define the “most balanced” situation seems somewhat incomplete.

To enhance rankings, Xavier et al. (2018) and Costa Freitas et al. (2022) applied entropy in both an EGP model and CP model. However,

this method has not been generalized for composite indicator aggregation.

The present study aims to solve both problems by simultaneously integrating the following aggregation solutions: 1) maximum deviation for the most balanced ranking; 2) entropy to measure the most balanced ranking; 3) the best-aggregated ranking. This is an alternative to the composite indicators’ methodological approach of Díaz-Balteiro and Romero (2004a), which solves the problem of equal ranking position for different sustainability levels using entropy. It also addresses the use of more than two criteria in composite indicators.

This paper makes substantial contributions to the literature on composite indicators and ranking analysis. First, a novel approach generalized for all situations of ranking building is provided. Second, it uses normalized entropy to perfect the rankings and specify the most balanced situation, instead of the maximum deviation. Finally, an integrated approach combining the three solutions is suggested: 1) “maximum aggregated sustainability”; 2) “most balanced sustainability”, using normalized entropy; 3) “most balanced sustainability”, using maximum deviation.

The remainder of the paper is as follows. Section two presents the theoretical background, highlighting the major contributions and the methodological insufficiencies. Section three presents the methodological approach. Section four presents data and the empirical implementation. Section five details the results and section six discusses the main findings of this research.

## 2. Theoretical background

### 2.1. Challenges in building composite indicators

According to Singh et al. (2012), building composite indicators can pose several challenges, such as selecting data, dealing with missing data, normalizing and standardizing data, weighting methods, and aggregation methods. They can be constructed with or without weights, depending on the specific problem and available information (Atkinson, et al., 1997; Singh et al., 2012). Several steps can be considered in building composite indicators (Narula and Reddy, 2015; Salzman 2003; OECD 2008; Mazziotta and Pareto 2012), as follows: “1. defining the phenomenon to be measured; 2. selecting the individual indicators; 3. normalizing the selected indicators; 4. aggregating the normalized indicators”.

One key issue when dealing with composite indicators is the aggregation of individual indicators. The first type of aggregation methods are the additive aggregation methods, which can be divided into several subtypes. In one additive aggregation process, the value of each individual indicator is summed. However, this will imply the loss of the original information of the indicators (OECD, 2008). An alternative aggregation method involves analysing the number of indicators that exceed or fall below a specific benchmark (OECD, 2008), but there is also a loss of information. The Innovation Scoreboard, which allows a comparative analysis of innovation performance in European countries and regional European neighbours, is an example of this method (European Commission, 2001; OECD, 2008). Finally, the most used linear aggregation additive method is the sum of weighted and normalised individual indicators. This method was used by the UNDP for the Human Development Index (HDI) until 2009 (UNDP, 2010). However, this form of additive aggregation imposes restrictions on the nature of individual indicators (OECD, 2008).

Furthermore, additive aggregations may be subjacent to full compensability, where poor performance in certain indicators can be compensated by good performance in others (OECD, 2008). For the HDI, this situation occurred and countries with different development levels had a similar HDI (UNDP, 2009). Therefore, additive aggregation methods can result in biased composite indicators that do not reflect the information of the individual indicators considered for its calculus (OECD, 2008), leading to situations in which DMUs with similar

composite indicators are related to different sustainability situations. Aggregation by geometric mean has been used for the HDI since 2010 (UNDP, 2010) and has solved several of these problems.

Kwatra et al. (2020) review several studies regarding the steps required to build composite indicators, aggregation procedures and weighting methods. Before that, Dewan (2006) and Singh et al. (2012) already presented several milestones in the development of composite indicators of sustainability. Regarding aggregation, Singh et al. (2012) identified the use of several methods, such as principal components analysis, weighted average, neutralising the effect of correlation, summation, arithmetic average of the normalised indicators, regression analysis, and geometric mean.

After the review by Singh et al. (2012), we find some relevant studies proposing composite indicators of sustainability (Bravo, 2014; Jeffrey et al., 2016; Biggeri and Mauro, 2018; Hickel, 2020). One interesting approach is the Mazziotta–Pareto index (MPI) which prioritizes both high average performance and consistent performance across all indicators. However, while use of the arithmetic mean in the MPI promotes simplicity, it also introduces the drawback of compensability in aggregation. The inclusion of a ‘penalty’ component can be seen as an adjustment to address the imbalance among indicators (Greco et al., 2019).

Another alternative to compute composite indicators is the MCDA. OECD (2008) divided the MCDA methods into compensatory and non-compensatory. The former is characterized by the presence of trade-offs, where a deficiency in one criterion can be compensated for by a substantial advantage in another. The latter implies a non-compensatory relation as the criteria weights only reflect the preferences among them.

Rowley et al. (2012) examined the aggregation of sustainability indicators using various MCDA approaches, highlighting the theoretical implications of an analyst’s choices. The study provides a thorough analysis of critical aspects for normalizing, weighting, and aggregating indicators, while also presenting several MCDA approaches for sustainability analysis.

Greco et al. (2019) reviewed the literature regarding the construction of composite indicators. Due to recent developments, the authors focused their study on the following steps for building composite indicators: weighting, aggregation, and robustness. El Gibari et al. (2019) reviewed the MCDA methods for building composite indicators and highlight that the compensatory facet of an aggregation method may be: full compensatory - additive methods and compensatory MCDA approaches; partially compensatory geometric mean and partially compensatory MCDA methods; and zero compensatory - non-compensatory MCDA approaches. They classified the MCDA methods for the aggregation of composite indicators as follows: elementary methods, value and utility-based methods, outranking relation approach, data envelopment analysis (DEA) based methods and distance functions-based methods.

The focus of our study is on improving the methodology presented by Diaz-Balteiro and Romero (2004a) for ranking analysis, and the “distanced based methods”. This approach can also define compromises between the solution of the “maximum aggregate sustainability” and the solution of the “most balanced sustainability”, which is measured by the maximum deviation i.e. the solution in which the minimum normalised achievement across the considered indicators of sustainability is maximised. This methodology was applied to the management of a forest area in Spain, where a ranking of eight management alternatives was established according to four individual indicators with equal weights.

## 2.2. Distance-based methods and ranking analysis

Diaz-Balteiro and Romero (2004b) use an EGP approach with binary variables, based on Diaz-Balteiro and Romero (2004a), to analyse the sustainability and define rankings of several forest management plans in

Spain. Several targets were defined for indicators concerning minimization or maximization of the individual indicators. The weights of each individual indicator were assumed to be equal. This study provided an iterative approach to building sustainability rankings, considering the “maximum aggregate sustainability”, the “most balanced sustainability” solution and intermediate rankings.

Voces et al. (2010) analyse the European paper industry’s sustainability at the country level and define rankings using also an EGP with binary variables, but the study does not explore intermediate solutions between the aggregated sustainability and the most balanced one.

Blancas et al. (2010) analyse tourism sustainability in the coastal areas of Andalusia (Spain) using a goal programming (GP) approach, based on the positive and negative deviations, where the final indicator is aggregated by the weighted sum of the individual indicators.

Diaz-Balteiro et al. (2011) analyse the paper industry’s sustainability in 17 European countries using a binary CP approach. They consider 14 indicators whose preferential weights were derived following a “pairwise” comparison format, using AHP, maintaining only the consistent ones.

Lozano-Oyola et al. (2012), also using a GP, evaluate sustainable tourism at cultural destinations in Andalusia and propose a composite indicator based on the positive and negative deviations.

Voces et al. (2012) characterize the sustainability rankings of 17 EU countries regarding European wood manufacturing industries by using an EGP model where the indicator weights were derived following a “pairwise” comparison format based on an AHP. The main factors explaining sustainability are identified using a linear multiple regression model.

Giménez et al. (2013) analyse the sustainable management of forest plantations, in north-western Spain, using EGP with binary variables. Equal preferential weights were assigned to individual indicators, excluding relevant stakeholder opinions.

Diaz-Balteiro et al. (2016a) apply an EGP approach to study sustainability in forest plantations in northwest Spain and consider the aggregate sustainability and the most balanced one by means of minimizing the maximum deviation. Then a correlation analysis was performed to find statistically significant relationships in the case of continuous explanatory variables and a Variance Analysis (ANOVA) was implemented in variables with nominal or ordinal scales. Diaz-Balteiro et al. (2016b) present an EGP approach to rank forest plantations’ sustainability considering three solutions/orientations: “optimum average” (best aggregate, i.e. the engineering solution), “optimum balance” (minimization of the maximum deviation), and “maximum distance to the anti-ideal”. Intermediate rankings among these solutions can be obtained, and it is possible to quantify conflicts among the three orientations.

Diaz-Balteiro et al. (2017a) use an EGP approach to identify the best management option considering several climate change scenarios, where different indicators are aggregated over a one hundred year period in a forest area in Central Spain. This study innovates by presenting aggregation of the indicators over a considerable period of time.

Xavier et al. (2018) study the agricultural sustainability of Portuguese municipalities and build rankings using an EGP approach. They introduced several improvements on previous ranking building EGP approaches, namely introducing an entropy approach to solve situations of equal ranking positions. However, the entropy measure was only used to distinguish municipalities in the same ranking position, and not as a generalized alternative to the minimum maximum deviation in the most balanced solution. This approach can also solve the model simultaneously for a considerable number of municipalities, instead of an iterative approach. The individual indicator weights were also identified using an EGP model (González-Pachón and Romero, 2007; Diaz-Balteiro et al., 2009), which can identify the minority and the majority consensus. Pereira and Fragoso (2022), adapted the study by Xavier et al. (2018) to study water efficiency in several municipalities in Alto Alentejo, Portugal.

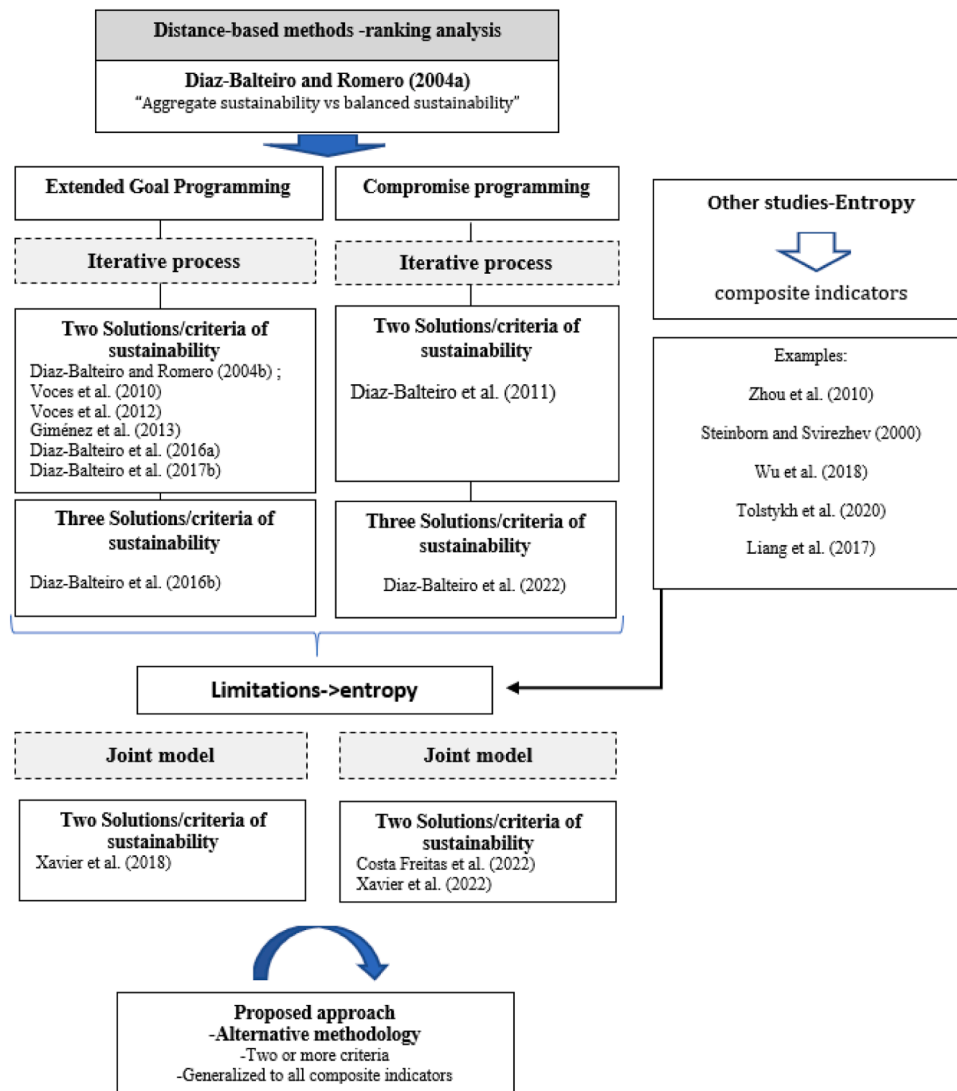


Fig. 1. Background summary of distance-based methods.

Diaz-Balteiro et al. (2022) study the circularity of the European wood industry using a CP approach. Three decision rules were included: “balanced achievement in terms of proximity to the ideal”, “balanced achievement in terms of remoteness from the anti-ideal” and “average achievement”. This paper identifies countries regarding the three different multicriteria solutions in several periods and provides a statistical analysis to explain the causality behind the rankings built. Costa Freitas et al. (2022), using CP, extended these improvements and created a full framework to analyse sustainable water use in Portugal, including a spatial analysis and a regression analysis to identify the main factors determining sustainable water use. The authors use entropy to distinguish municipalities with the same ranking position. Xavier et al. (2022) present a compromise programming approach which uses entropy to improve the agricultural sustainability ranking. The authors use an HJ-Biplot and Cluster analysis to analyse the results and establish typologies. Two periods were considered and a spatial analysis was carried out.

From the studies presented above, we can identify two approaches to define rankings and both present several limitations. The first was presented by Diaz-Balteiro and Romero (2004b) and proposes an EGP model and an iterative process in which the ranking is built unit by unit. This approach was used in several studies (e.g. Voces et al., 2010; Diaz-Balteiro et al., 2011; Voces et al., 2012; Giménez et al., 2013; Diaz-Balteiro et al., 2016a; Diaz-Balteiro et al., 2016b; Diaz-Balteiro et al.,

2022). The maximum deviation is used as the criterion of the most balanced sustainability, and sometimes its value is the same for different sustainability situations. The improvements in this research stream include the use of three criteria (Diaz-Balteiro et al., 2022; Diaz-Balteiro et al., 2016b) and the analysis of indicators over time (Diaz-Balteiro et al., 2017a), as well as statistical analysis to provide further insights into the results (Voces et al., 2012; Diaz-Balteiro et al., 2016a; Diaz-Balteiro et al., 2022).

A more recent research approach uses entropy to distinguish units that have the same ranking position but are in fact different. It also proposes a simultaneous solving of the model instead of an iterative one. This was applied to EGP (Xavier et al., 2018) and CP (Xavier et al., 2022; Costa Freitas et al., 2022) and in several studies was combined with statistical analyses.

### 2.3. Entropy and sustainability composite indicators

Entropy approaches have been used in several studies for sustainability analysis. Zhou et al. (2010) distinguish between distance-based aggregation methods and entropy aggregation. They discuss the aggregation process from an information loss perspective, related to information theory and entropy and present an entropy-based aggregation model, which can also deal with qualitative data and MCDA methods.

Steinborn and Svirezhev (2000) present an entropy-based indicator

to analyse the sustainability of agroecosystems, where the concept of entropy is taken in a physical way, instead of a statistical approach.

Wu et al. (2018) approach the Sustainable Society Index (SSI) and state that using equal weights in aggregation is likely to neglect the distinct impact of each dimension. Thus, they suggest applying ranked weights and Shannon entropy in the aggregation process. This approach may be complex because the preference scenarios would dramatically increase with the number of dimensions.

Tolstykh et al. (2020) analyse the sustainability of industrial ecosystems using an entropy model and applying the principles of complex systems. They adapt the concept of entropy diversity to the innovation of these systems. However, the study had some limitations, such as not being applicable for scenario calculations, high volatility of estimates, problems of specification selection when choosing data sources, high susceptibility to the ranking method, and parameter values. Liang et al. (2017) propose a model to measure sustainable development in Sichuan Province by using an entropy approach and the Brusselator principles. However, this index has some limitations: it does not contain all factors influencing regional sustainable development, the internal logic relationships between the indexes and a dynamic evaluation of the indexes were not studied.

Fig. 1 synthesises the papers presented in this review, many of them being based on the paper by Diaz-Balteiro and Romero (2004a).

Despite the decisive contribution to the aggregation of composite indices using MCDA approaches by Diaz-Balteiro and Romero (2004a), most of the studies presented above are not able to deal with the situation of equal ranking positions that correspond to different sustainability levels. Only Xavier et al. (2018) developed an EGP methodology to evaluate agricultural sustainability, introducing the use of entropy and the resulting information index to perfect the ranking in cases of similar composite indicators. Costa Freitas et al. (2022) also provided a CP approach to introduce the concept of normalized entropy and solve the problem of equal ranking positions corresponding to different sustainability situations. The approach proposed in this study tackles most of the limitations of previous studies and proposes a generalized approach to be implemented for composite indicator aggregation, providing an approach to complement the study by Diaz-Balteiro and Romero (2004a).

According to Diaz-Balteiro and Romero (2004a), more importance is attached to the most displaced criteria, as the value of the metric  $p$  increases to  $L_\infty$ . The dilemma between ‘aggregate’ and ‘balanced’ performance can be mitigated by joining both models (Diaz-Balteiro and Romero, 2004b). An alternative measure of the most balanced solution ( $L_\infty$ ) is the use of normalized entropy. An entropy function measures the uncertainty or randomness of a system and a maximum entropy implies equal values among indicators. Thus, entropy can be used to assess the performance of a decision-making unit (DMU) in a more nuanced way than a convex combination of performance facets: Bottleneck (the weakest link in the chain), Balanced and Maximum (best possible outcome).

A convex combination of performance facets is a weighted average of different performance measures. This approach can be useful for summarizing the overall performance of a DMU, but it can also be biased if the weights are not chosen carefully. An entropy function, on the other hand, does not require the analyst to specify weights for different performance measures and can be used to identify DMUs that are more or less balanced about their sustainable development.

### 3. The analytical framework

This section presents the proposed framework for building composite indicators, integrating simultaneously the solutions of the best aggregate, minimization of the maximum deviation and normalized entropy. Based on the suggestions of Singh et al. (2012) and Saisana and Tarantola (2002), we followed the approach of Diaz-Balteiro and Romero (2004a), which comprises three main steps: 1) Definition of preferential

**Table 2**  
Mathematical notation.

Symbol	Description
$I=\{1, \dots, n\}$	Set of individual indicators
$J=\{1, \dots, m\}$	Set of DMUs
$\lambda$	Control parameter that defines the trade-off between the best aggregate and the most balanced
$\lambda_1$ and $\lambda_2$	Control parameters that define a linear convex combination of the best aggregate, the maximum deviation and normalized entropy
$IS_i$	Sustainability composite indicator for DMU $i$
$IS_{i(p=1)}$	Sustainability composite indicator for DMU $i$ , according to metric $p=1$
$IS_{i(\infty)}$	Sustainability composite indicator for DMU $i$ , according to metric $p=\infty$
$\bar{R}_j$	Value of the normalized indicator $j$ for DMU $i$
$W_j$	Weight of indicator $j$
$p_{rj}$	Probability of the individual indicator $j$ in DMU $i$
$NE_i$	Normalized entropy of DMU $i$ .
$c$	A very small positive parameter to avoid impossible solution for the normalized entropy

weights; 2) Normalization of indicators and 3) Implementation of the MCDA approach.

To define the indicators’ weights, several techniques have been developed (González-Pachón and Romero, 2007; Diaz-Balteiro and Romero, 2008; OECD-JRC, 2008; Diaz-Balteiro et al., 2009; Nordström et al., 2009; Gómez-Limón and Sanchez-Fernandez, 2010; Nivolianitou et al., 2015; Diaz-Balteiro et al., 2016a; Xavier et al., 2016; Diaz-Balteiro et al., 2017b; Shah et al., 2019). An interesting method that can be used to compare different DMUs is the “benefit of the doubt” (BOD) (OECD, 2008; Rogge, 2018). This method is related to the Data Envelopment analysis (DEA), for constructing composite indicators, which assigns weights to individual indicators based on their relative performance (Walheer, 2024; Rogge, 2018). It is sensitive to national priorities, but has some limitations: the weights are specific to each DMU and some estimation problems might arise (OECD, 2008). In our model we propose participatory methods to assign weights to individual indicators. This approach involves incorporating the input of various stakeholders, and might ensure that the composite indicator is aligned with national policy goals.

After defining the preferential weights, it is necessary to provide the normalization of each sustainability indicator since they are measured in very different units. Several factors must be considered, including the measurement units and their robustness in the presence of outliers. (Ebert and Welsch, 2004; OECD, 2008). We used the min-max normalization method, whose values range between 0 and 1 and are normalized according to the maximum and minimum values (OECD, 2008; Diaz-Balteiro and Romero, 2004a; Kelemen et al., 2024), which is the most widely used method. Other types of normalization can also be considered, such as Standardization (or z-scores), Distance to a reference, Indicators above or below the mean and Methods for cyclical indicators (OECD, 2008; Diaz-Balteiro et al., 2018; Tamiz et al., (1998), but all of them have limitations.

To implement an MCDA approach, setting targets for composite indicators requires considering various factors. First, it is essential to define clear goals and select appropriate indicators that accurately reflect those goals. Second, setting realistic and achievable targets is crucial, ensuring they are challenging but not impossible to achieve. Several DMUs may have specific characteristics that may limit their indicators and an unrealistic target setting might penalize them. Finally, monitoring and adjustments are necessary to ensure the effectiveness of the composite indicator (Romero, 1991; OECD, 2008).

Table 2 provides an overview of the notation used to specify our mathematical formulation.

Diaz-Balteiro and Romero (2004a) used the metric  $p=1$  to define the “most sustainable” system, i.e. the engineering solution, which provides the best aggregate corresponding to the smallest deviation from the

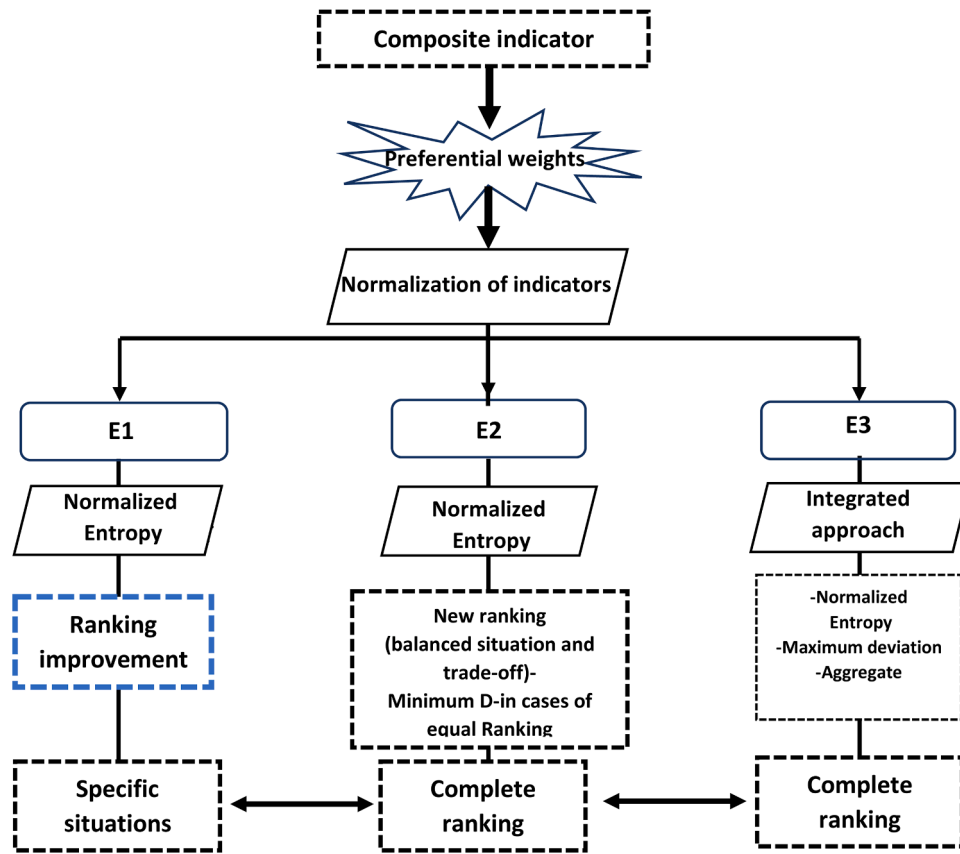


Fig. 2. The methodological lines of approach.

ideal. In this case, the sustainability indicator  $IS_i$  for DMU  $i$  may be calculated as:

$$IS_{i(p=1)} = \sum_{j=1}^m W_j \cdot \bar{R}_{ij} \quad \forall i \in \{1, 2, \dots, n\} \quad (1)$$

where  $W_j$  is the preferential weight for each  $j$  indicator;  $\bar{R}_{ij}$  is the value of the normalized indicator  $j$  for DMU  $i$ .

This approach aims to find the system with the highest overall sustainability. However, it can hide poor performance in certain areas, which might be unacceptable in the “broader context of sustainability”. However, the solution for the metric  $p=\infty$  attaches more importance to the most displaced criteria (individual indicators) and provides the “most balanced sustainability”, which maximises the following expression (e.g., Zeleny, 1974; Diaz-Balteiro and Romero, 2004a):

$$IS_i(\infty) = \left[ \min_j (W_j \cdot \bar{R}_{ij}) \right] \quad \forall i \in \{1, 2, \dots, n\} \text{ and } \forall j \in \{1, 2, \dots, m\} \quad (2)$$

The “most balanced sustainability” is the so called “ecological solution”. In this solution the values achieved by the sustainability indicators are distributed equally. This reminds the concept of ecological sustainability since the ecological systems have to be diverse and valorise each of their parts to be sustainable. Thus, the “ecological solution” is about reducing the maximum deviation among the criteria or presenting an alternative measure that quantifies the balance among indicators. This kind of solution is quite relevant when analysing sustainability, specially from an ecological point of view, and underlines the ideas of balance and equality.

Diaz-Balteiro and Romero (2004a) address all solutions from  $p=1$  to  $p=\infty$  by combining both models into a unique model, as shown in the following convex combination of sustainability measures:

$$IS_i = (1 - \lambda) \left[ \min_j (W_j \cdot \bar{R}_{ij}) \right] + \lambda \sum_{j=1}^m W_j \cdot \bar{R}_{ij} \quad \forall i \in \{1, 2, \dots, n\} \quad (3)$$

As mentioned before, one limitation of this methodology is that multiple DMUs related to different situations may have the same value in the ranking, leading to arbitrary ordering in rankings. This is particularly frequent in the most balanced solution where only the minimum maximum deviation (D) is optimized. However, the entropy concept is suitable for this type of problem and has been used to build environmental indexes (Zhou et al., 2010; Steinborn and Svirezhev, 2000; Wu et al., 2018; Liang et al., 2017).

The entropy concept was developed in the field of physics and introduced by Shannon (1948) in information science to define a unique function that measures the uncertainty of a collection of events. Let  $x$  be a random variable with possible outcome values  $x_k$  and probabilities  $p_k$ , with  $k=1, 2, \dots, K$ , subject to  $\sum_k p_k = 1$  - the entropy of  $x$  reaches a maximum when the probabilities are uniform. Therefore, Shannon (1948) defined the entropy of the distribution of probabilities,  $p = (p_1, p_2, \dots, p_k)$ , as follows:

$$H = - \sum_k p_k \ln p_k \quad (4)$$

Normalized entropy is a way to measure how much information is in a system and how important each piece of data is in reducing uncertainty (Golan, 1988; 1994 cited by Golan et al., 1996). It is calculated as follows:

$$S(\hat{p}) = \left( - \sum_k \hat{p}_k \ln \hat{p}_k \right) / \ln(K) \quad (5)$$

where  $S(\hat{p})$  is the normalized entropy indicator and  $\ln(K)$  is the entropy level of the uniform distribution with  $K$  outcomes, which represents the

**Table 3**  
Notation of management alternatives and indicators in the example of “El Pinar”.

DMUs – S management alternatives	
S <sub>1</sub>	Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory)
S <sub>2</sub>	Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint
S <sub>3</sub>	Maximize Veneer volume, with extended rotation and ending inventory constraint
S <sub>4</sub>	Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint
S <sub>5</sub>	Maximize Net Present Value (NPV), leaving the mixed stands uncut
S <sub>6</sub>	Maximize Veneer volume leaving the mixed stands uncut
S <sub>7</sub>	Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut
S <sub>8</sub>	Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory)
Ij indicators	
I <sub>1</sub>	Net Present Value (NPV) - economic indicator (Type: “more is better”)
I <sub>2</sub>	Ratio Harvest Area/Stand Growth - ecological indicator (Type: “less is better”)
I <sub>3</sub>	Average Rotation Age - forestry indicator (Type: “more is better”)
I <sub>4</sub>	Net carbon captured by the timber stand - ecological indicator (Type: “more is better”)

maximum uncertainty. A value of  $S(\hat{p}) = 0$  implies no uncertainty (i.e.,  $p_k=1$  for some  $k$  and  $p_j = 0$  for all  $j \neq k$ ). Alternatively, an  $S(\hat{p}) = 1$  implies perfect uncertainty (i.e.  $p_k=1/K$  for all  $k = 1, 2, \dots, K$ ). (see Golan et al., 1996 or Xavier et al., 2018).

The information index  $1 - S(\hat{p})$  described by Soofi (Soofi, 1992 cited by Golan et al., 1996) is an analogue measure to quantify uncertainty reduction. Xavier et al. (2018), Costa Freitas et al. (2022) and Xavier et al. (2022) have also used it to distinguish municipalities sharing an equal ranking position. In this paper, a normalized entropy indicator is proposed, as higher values of normalized entropy are indicative of a more balanced distribution within the aggregation process.

To implement the proposed method, it is essential to establish the following set of probabilities, denoted as  $pr_{ij}$ :

$$pr_{ij} = \frac{W_j \cdot \bar{R}_{ij}}{\sum_{j=1}^m (W_j \cdot \bar{R}_{ij})} \quad \forall i \in \{1, 2, \dots, n\} \text{ and } \forall j \in \{1, 2, \dots, m\} \text{ with } \sum_{j=1}^m pr_{ij} = 1 \tag{6}$$

and the normalised entropy is calculated as follows:

$$NE_i = \frac{-\sum_{j=1}^m pr_{ij} \ln(pr_{ij} + c)}{\ln(J)} \quad \forall i \in \{1, 2, \dots, n\} \tag{7}$$

where,  $NE_i$  is the normalized entropy in DMU  $i$ . To prevent situations where  $pr_{ij}=0$  and hence avoid  $\ln(0)$ , a very small positive parameter  $c$  ( $c>0$ ) was inserted.  $NE_i$  varies between 0 and 1: when  $NE_i$  is 1 the weights are distributed equally (a completely balanced distribution); when  $NE_i$  is 0 all weights are concentrated in only one indicator.

Then, the following expression may be computed and the values of the composite indicator  $IS_i$  can be obtained as follows:

$$IS_i = (1 - \lambda)NE_i + \lambda \sum_{j=1}^m W_j \cdot \bar{R}_{ij} \quad \forall i \in \{1, 2, \dots, n\} \tag{8}$$

This novel formulation incorporates the normalised entropy, enabling solutions for the composite indicator  $IS_i$  in each DMU  $i$ . However, we should emphasize that the best-aggregated solution, minimization of the maximum deviation, and normalized entropy are different measures with unique merits. Building upon the ideas of Diaz-Balteiro et al. (2016b) and Diaz-Balteiro et al. (2022), an alternative model is proposed. This model allows the trade-off between these three solutions, providing a flexible approach to address varying priorities as follows:

$$IS_i = \lambda_1 NE_i + \lambda_2 \left[ \min_j (W_j \cdot \bar{R}_{ij}) \right] + (1 - \lambda_1 - \lambda_2) \sum_{j=1}^m W_j \cdot \bar{R}_{ij} \quad \forall i \in \{1, 2, \dots, n\} \tag{9}$$

In this equation,  $\lambda_1$  and  $\lambda_2$  are control parameters, enabling the establishment of a linear convex combination of the three measures.

Thus, when  $1-\lambda_1-\lambda_2$  equals 1, the result is the best-aggregated solution; when  $\lambda_1$  equals 1 and  $\lambda_2$  equals 0, the outcome represents the most balanced/uniform distribution based on the concept of normalized entropy; and when  $\lambda_1$  equals zero and  $\lambda_2$  equals one, the solution minimizes the maximum deviation regarding the ideal. For values of the control parameters  $\lambda_1$  and  $\lambda_2$  such that  $\lambda_1 + \lambda_2 \in [0,1]$  compromises between the three solutions can be obtained.

The mathematical formulation presented in equation (9) leads us to design three methodological lines for ranking analysis, representing several innovations in relation to previous approaches. Fig. 2 presents implementation of the proposed approach.

In the first methodological line (E1), the normalized entropy indicator is used to substitute the deviation of the achievement with respect to the ideal of the most displaced indicator. This is a more comprehensive measure for the most balanced situation, and it is only implemented in cases where equal values in the ranking are obtained.

In the second methodological line (E2), the normalized entropy (NE) indicator is always used to substitute the maximum deviation regarding the ideal value,  $\min_j (W_j \cdot \bar{R}_{ij})$ , and it is computed to create an alternative composite indicator.

Finally, the third methodological line (E3) combines all solutions, valorising their trade-off and providing a unique solution, which considers the best aggregate sustainability, minimization of the maximum deviation and maximization of the normalized entropy. This model takes maximum advantage of the different measures used, never losing information, and providing a novel sustainability indicator. Despite considerable complexity, we must highlight that the three methodological lines of approach are connected.

If necessary, the results may be analysed in a complementary way. For instance, the first two methodological lines are connected as the values obtained by E2 may serve as a reference for the improved ranking obtained using the approach proposed by Diaz-Balteiro and Romero (2004a) in E1 and vice versa. In turn, E2 and E3 may serve mainly as a complementary ranking of information to guide the decision-maker.

Although our approach includes as criteria two balanced solutions (minimization of the maximum deviation and normalized entropy), the aggregation method used is partially compensatory. As mentioned before, the compensatory methods are associated with the existence of trade-offs, where a disadvantage in one criterion can be offset by a sufficiently large advantage in another, while in the non-compensatory methods no trade-off occurs. These concepts may be related to weak and strong sustainability. The strong sustainability paradigm does not allow compensations between the different criteria while the weak sustainability paradigm allows this kind of compensation (Ruiz et al., 2011).

Weak sustainability is related to the compensatory methods and is useful to guide economic decision-making. It considers the value of natural capital and the potential for technological progress to offset the loss of natural capital. Compensatory methods present several

**Table 4**  
The data used from the example of the public forest “El Pinar”.

Management Alternatives	Indicators			
	I1	I2	I3	I4
S1	1.00	0.45	0.37	0.43
S2	0.86	0.00	0.58	0.00
S3	0.33	0.17	0.99	0.31
S4	0.86	0.05	0.58	0.00
S5	0.76	1.00	0.00	0.51
S6	0.00	0.44	1.00	0.16
S7	0.59	0.73	0.33	0.87
S8	0.86	0.92	0.41	1.00
Weights $w_j$	0.25	0.25	0.25	0.25

(source: Diaz-Balteiro and Romero 2004a)

advantages to tackle sustainability (Banihabib et al., 2017) being more flexible and allowing a more holistic view of sustainability, recognizing the interconnections among different aspects. For example, a country with a high level of environmental protection might have a lower score in economic development score, but compensatory methods would still permit an interesting score overall. Sustainability is a complex issue with many different dimensions and compensatory methods can help to identify trade-offs between different aspects. Therefore, it is important not to forfeit compensatory methods and to have a model that can identify the best aggregate and the most balanced solutions. This is the case of the approach proposed in this paper.

#### 4. Data and empirical implementation

The proposed methodological approach was implemented using the data of two examples from the literature: the study by Diaz-Balteiro and Romero (2004a) and the Human Sustainable Development Index (Bravo, 2014).

The former example is related to management of the public forest “El Pinar”, covered mainly by conifers, located in northeast Spain at an average altitude of 1100 m. This example considers 8  $S$  management alternatives (DMUs) and 4  $I_j$  indicators as presented in Table 3.

The data used are available in Diaz-Balteiro and Romero (2004a) and are normalized and rounded to centesimal units (Table 4). As in the paper, equal weights ( $w_j$ ) were attributed to each indicator.

For the second example, we used a study that proposes an adaptation of the Human Development Index (HDI) to measure sustainable development (Bravo, 2014). The HDI is a metric of human development comprising three dimensions and four indicators (Hickel, 2020) and calculated as the geometric mean of life expectancy at birth, education (average years of schooling and expected years of schooling) and income (Gross National Income-GNI per capita, PPP purchasing power parity) on a natural logarithmic scale. Each of these indicators is indexed within a range defined by maximum and minimum values (Hickel, 2020; UNDP, 2019).

HDI has important limitations, one of them being the absence of an ecological sustainability indicator. This is a crucial issue, which poses a significant challenge as concerns about climate change and ecological issues increase (Hickel, 2020; UNDP, 2019). Moran et al. (2008) suggests assessing the evolution of sustainable development dimensions by using HDI as an “indicator of development and the Ecological Footprint as an indicator of human demand on the biosphere”. Neumayer et al. (2001) highlights the importance of linking HDI with sustainability, which would allow the United Nations Development Programme (UNDP) to analyse whether a country is putting the well-being of future generations at risk.

Neumayer (2012) suggests there is no substantial difference between human development and sustainable development. Several studies in the literature support this notion, indicating how sustainability and human development can mutually reinforce each other, emphasizing the

**Table 5**  
The results obtained for illustrative example 1.

Manag. Alt./λ	1	0.7	0.5	0.3	0.2	0
S1	0.563	0.422	0.328	0.234	0.187	0.093
S2	0.360	0.252	0.180	0.108	0.072	0.000
S3	0.450	0.328	0.246	0.165	0.124	0.043
S4	0.373	0.261	0.186	0.112	0.075	0.000
S5	0.568	0.397	0.284	0.170	0.114	0.000
S6	0.400	0.280	0.200	0.120	0.080	0.000
S7	0.630	0.466	0.356	0.247	0.192	0.083
S8	0.798	0.589	0.450	0.311	0.242	0.103

$S_1$  - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory);  $S_2$  - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint;  $S_3$  - Maximize Veneer volume, with extended rotation and ending inventory constraint;  $S_4$  - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint;  $S_5$  - Maximize Net Present Value (NPV), leaving the mixed stands uncut;  $S_6$  - Maximize Veneer volume leaving the mixed stands uncut;  $S_7$  - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut;  $S_8$  - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).

(Source: model results)

relationship between social progress and sustainable practices.

Many researchers have studied the ecological limitations of HDI (e.g. Desai, 1995; Ramathan, 1999; De La Vega and Urrutia, 2001; Neumayer, 2001; Morse, 2003; ; Pelenc et al., 2013), and several studies have tried to integrate environmental dimensions into HDI (Hickel, 2020). One of the most important contributions was made by Togtokh and Gaffney (2010), later enhanced by Bravo (2014), who included an index of CO<sub>2</sub> emissions besides the usual indicators, resulting in the creation of the Human Sustainable Development Index. This index is calculated using a geometric mean as follows:

$$HSDI = \sqrt[4]{I_{life} \cdot I_{education} \cdot I_{income} \cdot I_{emissions}} \tag{10}$$

where the HSDI is the Human Sustainable Development Index;  $I_{life}$  is the index of life expectancy;  $I_{education}$  is the index of education;  $I_{income}$  is the index of income; and  $I_{emissions}$  is the index related to CO<sub>2</sub> emissions.

Hickel (2020) makes a significant contribution to the field of sustainable development by proposing a modification to the Human Development Index (HDI) that accounts for ecological impact. This modification led to establishing the Sustainable Development Index (SDI). Hickel’s approach provides a new perspective on sustainable development, showing that each component of the HDI correlates with ecological impact on an exponential curve. Therefore, as life expectancy, education, and income improve, ecological impact increases exponentially. Similarly, Jin et al. (2020) constructed an indicator which considers entropy in defining the indicators’ weights, building upon the foundations of the HDI.

To implement this second illustrative example, the four indexes provided by Bravo (2014) were used. The results of HSDI do not present situations of equal position in rankings. Nevertheless, implementing an MCDA approach will provide complementary information for data analysis, which may be important as an alternative, namely when analysing the most balanced situations. Thus, as mentioned before, the reasons for testing the example provided by Bravo (2014) are twofold. First the importance of the example in the literature and second its simplicity and direct connection with the HDI.

#### 5. Results

In this section the results of applying the three proposed methodological lines to both case studies are presented. First, the illustrative example 1 of Diaz-Balteiro and Romero (2004a) is presented and then that of Bravo (2014). The last part of this section contains a robustness analysis using the MPI index.

**Table 6**  
Rankings implementation in the approach of illustrative example 1.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	S8	S8	S8	S8	S8	S8
2	S7	S7	S7	S7	S7	S1
3	S5	S1	S1	S1	S1	S7
4	S1	S5	S5	S5	S3	S3
5	S3	S3	S3	S3	S5	S2, S4, S5, S6
6	S6	S6	S6	S6	S6	
7	S4	S4	S4	S4	S4	
8	S2	S2	S2	S2	S2	

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).

(Source: model results)

5.1. Illustrative example 1: Diaz-Balteiro and Romero (2004a)

The composite indicator results using the example of Diaz-Balteiro and Romero (2004a) are presented in Table 5. For  $\lambda=1$ , the model maximizes the aggregate sustainability, while for  $\lambda=0$ , it provides the most balanced solution. Intermediate values of  $\lambda$  represent the trade-off between these two situations. Table 5 shows that for the most balanced situation ( $\lambda=0$ ), some DMUs have the same sustainability value. The intermediate rankings  $\lambda=0.7$ ,  $\lambda=0.5$ , and  $\lambda=0.3$  are identical, with only minor changes in the ranking positions of S3 and S5 at  $\lambda=0.2$  (Table 6).

Fig. 3a illustrates the differences among the four least ranked DMUs considering the normalized values and shows that the indicators are unequally distributed when considering equal balanced situations. S<sub>5</sub> has a more equal distribution of the indicators than S<sub>4</sub> and S<sub>2</sub>, which seems to have the most unbalanced distribution.

As mentioned before, an entropy approach may be implemented to solve the problem of equal values of composite indicators in the ranking, using the normalized entropy measure. Fig. 3b presents the values after this transformation and shows that the data have a similar distribution.

The results of implementing the normalized entropy according to the E1 methodological line are presented in Table 7. The improvements are

clear, since the DMUs with equal positions in the previous ranking now take different positions. Although this only occurs in the most balanced situation, it can be applied in other intermediate situations, between the best aggregate solution and the most balanced one.

The results for the second methodological line E2 are presented in Table 8. In this case, a new composite indicator was created, by substituting the maximum deviation with the normalized entropy, and no equal values for the composite indicators were found. S<sub>8</sub> is always the best DMU, followed by S<sub>7</sub>, while S<sub>2</sub> is always the worst. Some changes are identified in the ranking namely in intermediate positions. S<sub>5</sub> occupies the third place in the best aggregate solution ( $\lambda=1$ ), but for all the other  $\lambda$  presented, it is S<sub>1</sub> that takes this ranking position. The other changes in the ranking are for  $\lambda=1$ ,  $\lambda=0.7$ , and  $\lambda=0.5$ , where S<sub>3</sub> is in fifth place, while for  $\lambda=0.3$ ,  $\lambda=0.1$  and  $\lambda=0$ , it is in fourth place. In addition, for  $\lambda=1$ , S<sub>5</sub> is in third place, for  $\lambda=0.7$  and  $\lambda=0.5$  it is in fourth place, and for  $\lambda=0.3$  and  $\lambda=0.2$  and  $\lambda=0$  it is in fifth place. Compared to Diaz-Balteiro and Romero (2004a), the rankings obtained are similar, except for  $\lambda=0.3$  and  $\lambda=0$ , where there is a change in the fourth and fifth positions (Table 9).

For  $\lambda=0$ , changes regarding E1 occur in the second and third places. Although the maximum deviation from the ideal is smaller in S<sub>1</sub> than in S<sub>7</sub>, its indicators' importance has a less regular distribution, concentrated only in certain indicators. The weights for S<sub>7</sub>, show a smoother distribution of all indicators, while indicator I<sub>1</sub> in S<sub>1</sub>, concentrates the most importance. S8 continues to be the most balanced sustainable DMU and for  $\lambda=0$ , the ranking positions equal the improvements obtained with E1.

Table 10 presents some results for the methodological line E3 - the integrated approach, while Table 11 presents the resulting rankings. The first three solutions regard the best aggregate ( $\lambda_1=0$ ,  $\lambda_2=0$  and  $1-\lambda_1-\lambda_2=1$ ), the most balanced solution according to the maximum deviation ( $\lambda_1=0$ ,  $\lambda_2=1$  and  $1-\lambda_1-\lambda_2=0$ ) and the most balanced solution according to the concept of normalized entropy ( $\lambda_1=1$ ,  $\lambda_2=0$  and  $1-\lambda_1-\lambda_2=0$ ). These results are similar to the previous models. However, this methodological line focuses on the trade-offs between the three criteria and takes maximum advantage of the simultaneous analysis. All sustainability measures may be considered simultaneously, and decision-makers can attribute different weights to each sustainability criterion.

In the solution where  $\lambda_1=0.33$ ,  $\lambda_2=0.33$  and  $1-\lambda_1-\lambda_2=0.34$ , the three criteria are considered simultaneously with the same weights, providing a ranking regarding the best-aggregated solution and the most balanced sustainability: normalized entropy and maximum deviation. For  $\lambda_1=0.25$ ,  $\lambda_2=0.25$  and  $1-\lambda_1-\lambda_2=0.5$ , a ranking is obtained where equal

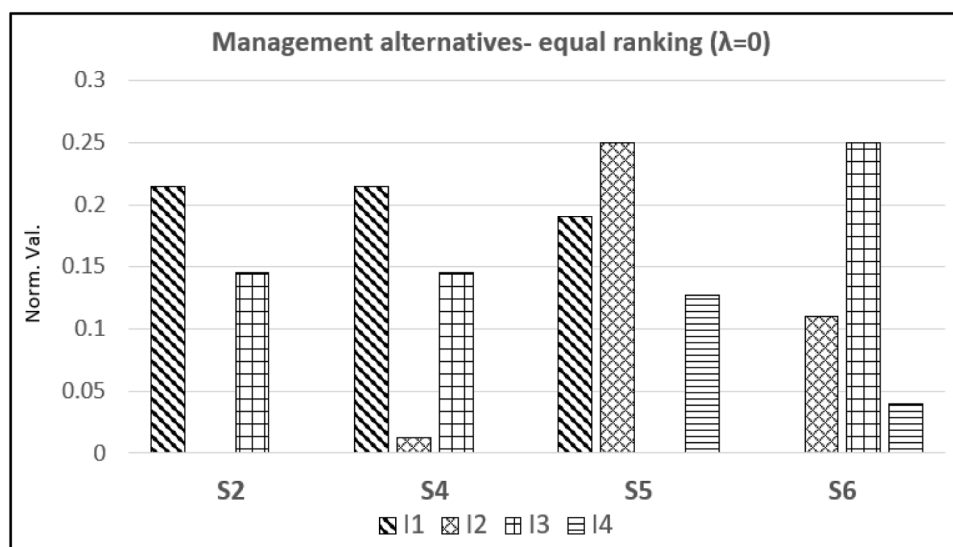


Fig. 3a. Normalized distribution of indicators – Illustrative example 1.

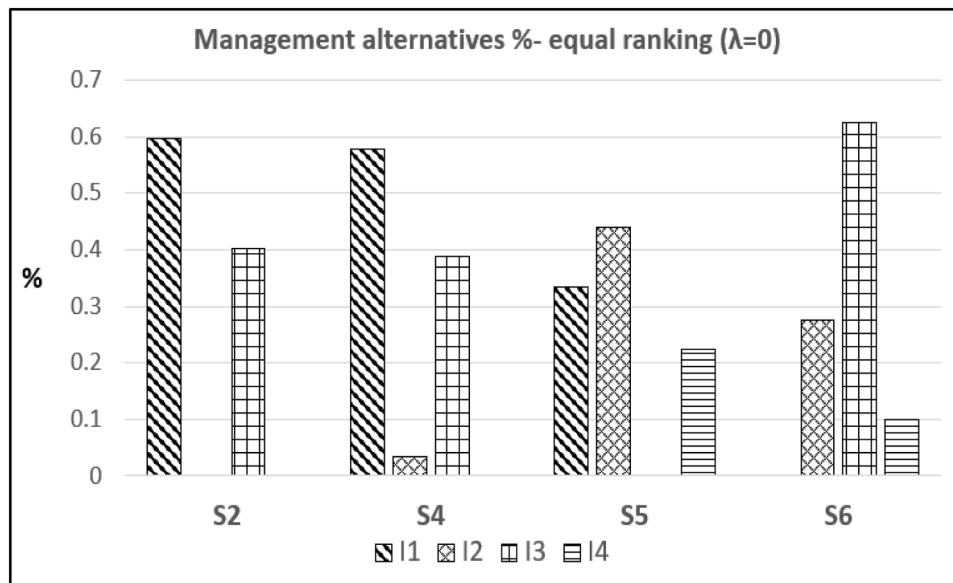


Fig. 3b. The percentage weight of the indicators regarding the total – Illustrative example 1.

Table 7

The entropy approach (E1) to improve the results for  $\lambda=0$  – Illustrative example 1.

Management alternative	Normalized Entropy	Rank
S2	0.486	4
S4	0.576	3
S5	0.767	1
S6	0.634	2

S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut.

(Source: model results)

Table 8

The composite indicator created for E2 - Illustrative example 1.

Manag. Alt./ $\lambda$	1	0.7	0.5	0.3	0.2	0
S1	0.563	0.674	0.748	0.823	0.860	0.934
S2	0.360	0.398	0.423	0.448	0.461	0.486
S3	0.450	0.567	0.645	0.724	0.763	0.841
S4	0.373	0.434	0.474	0.515	0.535	0.576
S5	0.568	0.627	0.667	0.707	0.727	0.767
S6	0.400	0.470	0.517	0.564	0.587	0.634
S7	0.630	0.729	0.795	0.862	0.895	0.961
S8	0.798	0.848	0.882	0.916	0.932	0.966

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).

(source: model results)

importance of the most balanced and most aggregated sustainability is considered. Finally, for  $\lambda_1=0.5$ ;  $\lambda_2=0.5$  and  $1-\lambda_1-\lambda_2=0$ , the model addresses the trade-off between the two balanced solutions, i.e., the

Table 9

The resulting ranking for E2 – Illustrative example 1.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	S8	S8	S8	S8	S8	S8
2	S7	S7	S7	S7	S7	S7
3	S5	S1	S1	S1	S1	S1
4	S1	S5	S5	S3	S3	S3
5	S3	S3	S3	S5	S5	S5
6	S6	S6	S6	S6	S6	S6
7	S4	S4	S4	S4	S4	S4
8	S2	S2	S2	S2	S2	S2

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).

(source: model results)

maximum deviation regarding the ideal and normalized entropy. In all these cases, the rankings remain similar except for the fourth and fifth positions in the last case.

### 5.2. Illustrative example 2: Bravo (2014)

We applied the three proposed methodological lines to all 185 countries considered in Bravo (2014). However, due to the huge amount of data, the results are presented only for the best ten ranked countries. Table 12 shows the HDSI values for the first ten ranked countries according to Bravo (2014).

The resulting ranking for the E1 methodological line application is presented in Table 13. In this case, the obtained ranking positions show some differences from Bravo (2014). For  $\lambda=1$  the ranking is the same, but for the other values of  $\lambda$  there are some differences. Norway is placed first in the ranking for  $\lambda=1$ ,  $\lambda=0.7$  and  $\lambda=0.5$ , but for  $\lambda=0.3$  and  $\lambda=0.2$  Sweden is the first ranked country. For  $\lambda=0$  (the most balanced solution according to the maximum deviation) Switzerland is first and Norway falls to eighth place, probably due to a high deviation in an indicator,

**Table 10**  
Results for integrated approach E3 - Illustrative example 1.

$\lambda_1$	1	0	0	0.5	0	0.5	0.33	0.25
$\lambda_2$	0	1	0	0.5	0.5	0	0.33	0.25
$1 - \lambda_1$	0	0	1	0	0.5	0.5	0.34	0.5
$-\lambda_2$								
S1	0.934	0.093	0.563	0.513	0.328	0.748	0.530	0.538
S2	0.486	0.000	0.360	0.243	0.180	0.423	0.283	0.302
S3	0.841	0.043	0.450	0.442	0.246	0.645	0.444	0.446
S4	0.576	0.000	0.373	0.288	0.186	0.474	0.317	0.330
S5	0.767	0.000	0.568	0.383	0.284	0.667	0.446	0.475
S6	0.634	0.000	0.400	0.317	0.200	0.517	0.345	0.359
S7	0.961	0.083	0.630	0.522	0.356	0.795	0.559	0.576
S8	0.966	0.103	0.798	0.534	0.450	0.882	0.624	0.666

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).  
(source: model results)

**Table 11**  
Ranking results for integrated approach E3 – Illustrative example 1.

$\lambda_1$	1	0	0	0.5	0	0.5	0.33	0.25
$\lambda_2$	0	1	0	0.5	0.5	0	0.33	0.25
$1 - \lambda_1 - \lambda_2$	0	0	1	0	0.5	0.5	0.34	0.5
1	S8	S8	S8	S8	S8	S8	S8	S8
2	S7	S1	S7	S7	S7	S7	S7	S7
3	S1	S7	S5	S1	S1	S1	S1	S1
4	S3	S3	S1	S3	S5	S5	S5	S5
5	S5	S5; S6; S4; S2	S3	S5	S3	S3	S3	S3
6	S6		S6	S6	S6	S6	S6	S6
7	S4		S4	S4	S4	S4	S4	S4
8	S2		S2	S2	S2	S2	S2	S2

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).  
(source: model results)

**Table 12**  
Ranking of the HSDI – Illustrative example 2.

Ranking	Country	HSDI
1	NOR	0.926
2	NZL	0.919
3	SWE	0.918
4	CHE	0.913
5	HKG	0.908
6	ISL	0.906
7	DEU	0.904
8	IRL	0.901
9	JPN	0.899
10	NLD	0.898

Source: Bravo, 2014  
NOR-Norway; CHE-Switzerland; DEU-Germany; HKG-Hong Kong, China; IRL-Ireland; ISL-Iceland; JPN-Japan; NLD-Netherlands; NZL-New Zealand; SWE-Sweden

**Table 13**  
Rankings for the line of approach E1 – Illustrative example 2.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	NOR	NOR	NOR	SWE	SWE	CHE
2	NZL	SWE	SWE	CHE	CHE	SWE
3	SWE	NZL	CHE	NOR	NOR	DNK
4	CHE	CHE	NZL	DEU	DEU	DEU
5	HKG	HKG	DEU	JPN	JPN	AUT
6	ISL	ISL	HKG	NZL	DNK	JPN
7	DEU	DEU	ISL	ISL	AUT	BEL
8	IRL	IRL	JPN	DNK	ISL	NOR
9	JPN	JPN	IRL	HKG	FRA	FRA
10	NLD	NLD	NLD	AUT	HKG	FIN

AUT-Austria; BEL-Belgium; CHE-Switzerland; DEU-Germany; DNK-Denmark; FIN-Finland; FRA-France; HKG-Hong Kong, China; IRL-Ireland; ISL-Iceland; JPN-Japan; NLD-Netherlands; NOR-Norway; NZL-New Zealand; SWE-Sweden.  
(source: model results)

**Table 14**  
Rankings for the line of approach E2 – Illustrative example 2.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	NOR	NOR	NOR	NOR	NOR	DNK
2	NZL	NZL	NZL	SWE	SWE	SWE
3	SWE	SWE	SWE	NZL	CHE	BEL
4	CHE	CHE	CHE	CHE	NZL	AUT
5	HKG	HKG	HKG	HKG	HKG	SYC
6	ISL	ISL	ISL	ISL	ISL	FIN
7	DEU	DEU	DEU	DEU	DEU	CHE
8	IRL	IRL	IRL	IRL	IRL	DEU
9	JPN	JPN	JPN	NLD	NLD	GBR
10	NLD	NLD	NLD	JPN	JPN	NLD

AUT-Austria; BEL-Belgium; CHE-Switzerland; DEU-Germany; DNK-Denmark; FIN-Finland; GBR-United Kingdom; HKG-Hong Kong, China; IRL-Ireland; ISL-Iceland; JPN-Japan; NLD-Netherlands; NOR-Norway; NZL-New Zealand; SWE-Sweden; SYC-Seychelles.  
(source: model results)

**Table 15**  
Rankings for the E3 methodological line – Illustrative example 2.

$\lambda_1$	1	0	0	0.5	0	0.5	0.33	0.25
$\lambda_2$	0	1	0	0.5	0.5	0	0.33	0.25
$1 - \lambda_1 - \lambda_2$	0	0	1	0	0.5	0.5	0.34	0.5
1	DNK	CHE	NOR	CHE	NOR	NOR	NOR	NOR
2	SWE	SWE	NZL	SWE	SWE	NZL	SWE	SWE
3	BEL	DNK	SWE	DNK	CHE	SWE	CHE	NZL
4	AUT	DEU	CHE	AUT	NZL	CHE	NZL	CHE
5	SYC	AUT	HKG	DEU	DEU	HKG	DEU	HKG
6	FIN	JPN	ISL	JPN	HKG	ISL	HKG	ISL
7	CHE	BEL	DEU	BEL	ISL	DEU	ISL	DEU
8	DEU	NOR	IRL	NOR	JPN	IRL	JPN	JPN
9	GBR	FRA	JPN	FRA	IRL	JPN	IRL	IRL
10	NLD	FIN	NLD	FIN	NLD	NLD	NLD	NLD

AUT-Austria; BEL-Belgium; CHE-Switzerland; DEU-Germany; DNK-Denmark; FIN-Finland; FRA-France; GBR-United Kingdom; HKG-Hong Kong, China; IRL-Ireland; ISL-Iceland; JPN-Japan; NLD-Netherlands; NOR-Norway; NZL-New Zealand; SWE-Sweden; SYC-Seychelles.  
(source: model results)

which is masked in the other solutions.

Table 14 presents the ranking results for the E2 methodological line. Compared to Bravo (2014), the rankings for  $\lambda=1$ ,  $\lambda=0.7$  and  $\lambda=0.5$  are equal, but several changes were identified for other values of  $\lambda$ . For  $\lambda=0$ , the ranking obtained show many different country positions.

Table 15 shows the results for the E3 methodological line. The solution for  $\lambda_1=0$ ,  $\lambda_2=0$  and  $1-\lambda_1-\lambda_2=1$ , which corresponds to the best aggregate, shows a ranking similar to Bravo (2014). For  $\lambda_1=0.25$ ,  $\lambda_2=0.25$  and  $1-\lambda_1-\lambda_2=0.5$ , equal importance is given to the best aggregate and to the most balanced solution, and the ranking produced has

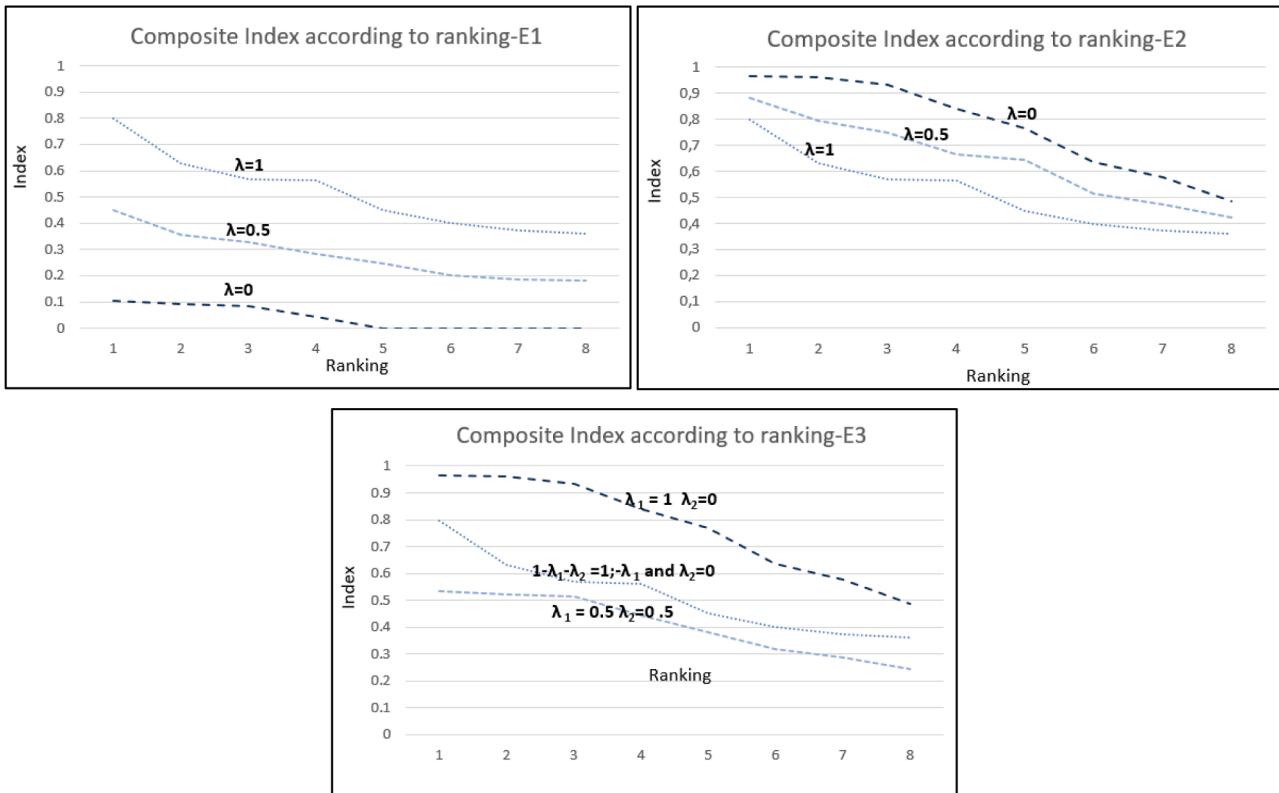


Fig. 4. The composite indicator according to the ranking for E1, E2 and E3 -example 1. (source: model results)

few differences from Bravo (2014). The solution for  $\lambda_1=0.5, \lambda_2=0.5$  and  $1-\lambda_1-\lambda_2=0$  specifies the trade-off between the two balanced sustainability solutions of maximum deviation and normalized entropy. In this case, the first three places are occupied respectively by Switzerland, Sweden, and Denmark, instead of Norway, New Zealand, and Sweden. When all the solutions are weighted more equally ( $\lambda_1=0.33, \lambda_2=0.33$  and  $1-\lambda_1-\lambda_2=0.34$ ), the three best-ranked countries are: Norway, Sweden, and Switzerland.

5.3. Robustness analysis

For the robustness analysis, the model response to different input data through a trade-off analysis was first analysed. Then, the results were tested using the Mazziotta-Pareto Index (MPI) (Mazziotta and Pareto, 2016) for the two illustrative examples. Finally, extreme situations were tested through a sensitive analysis.

For illustrative example 1, Fig. 4 shows graphically the composite indicators according to the ranking position for different values of  $\lambda$ , for the three methodological lines proposed. In E1 methodological line, the values of the composite indicators seem to be very similar after position 5 in the ranking for any value of  $\lambda$ . However, for  $\lambda=1$ , before the middle of the ranking, the values of the composite indicators are always different. For  $\lambda=0$  the curve is almost plane, indicating great proximity between the values of the composite indicators. For the E2 methodological line, where normalized entropy is used in the most balanced solution, besides a clear improvement in the values of composite indicators, their values in the ranking positions are different, especially for the solution of  $\lambda=0.5$ . The composite indicators in the case of the E3 methodological line seem to be distinct in all ranking positions for the different values of  $\lambda$ .

Fig. 5 presents the composite index trade-offs between DMUs for different values of  $\lambda$ , for the E1, E2 and E3 methodological lines.

For the three methodological lines, the trade-offs show a linear

pattern. In the E1 methodological line, the trade-offs decrease as the values move from  $\lambda=1$  to  $\lambda=0$ . However, this decrease does not imply that each DMU is becoming less sustainable. Instead, it is associated with the relative ranking positions of DMUs. For instance, there are changes in the rankings, such as that observed in the E2 methodological line between DMUs S3 and S5.

For the E3 methodological line, we present the trade-off between the solution that values the best aggregate solution ( $1-\lambda_1-\lambda_2=1$ ) and the most balanced solutions, which consider in equal terms the maximum deviation and the normalized entropy ( $\lambda_1=0.5, \lambda_2=0.5$  and  $1-\lambda_1-\lambda_2=0$ ). Thus, the trade-offs show that the positions of DMUs in the rankings are distinct for the different solutions of  $\lambda$ .

Regarding illustrative Example 2, Fig. 6 presents the composite indicators according to the ranking position for different values of  $\lambda$ , for E1, E2 and E3 methodological lines.

As in the previous illustrative example, the composite indicators in the E1 methodological line tend to have close values, especially in the most balanced solution ( $\lambda=0$ ). Composite indicators in the E2 methodological line show higher values than in the E1 methodological line. In the E3 methodological line, the values of composite indicators are also close, namely for the most balanced solutions. However, the solution corresponding to the best aggregate ( $1-\lambda_1-\lambda_2=1$ ) exhibits different composite indicators for distinct ranking positions, whose values decrease with ranking position.

Fig. 7 shows the composite index trade-offs between DMUs for different values of  $\lambda$ , for the E1, E2 and E3 methodological lines.

As mentioned before, illustrative example 2 comprises 185 DMUs. Hence, our trade-off analysis is focused only on three DMUs, including the following countries: Norway (NOR), Cuba (CUB) and Czech Republic (CZE).

For the E1 and E2 methodological lines, we considered the solutions of  $\lambda=1, \lambda=0$  and nine intermediate solutions. For the E3 methodological line, we also considered the best aggregate solution ( $1-\lambda_1-\lambda_2=1$ ) and the

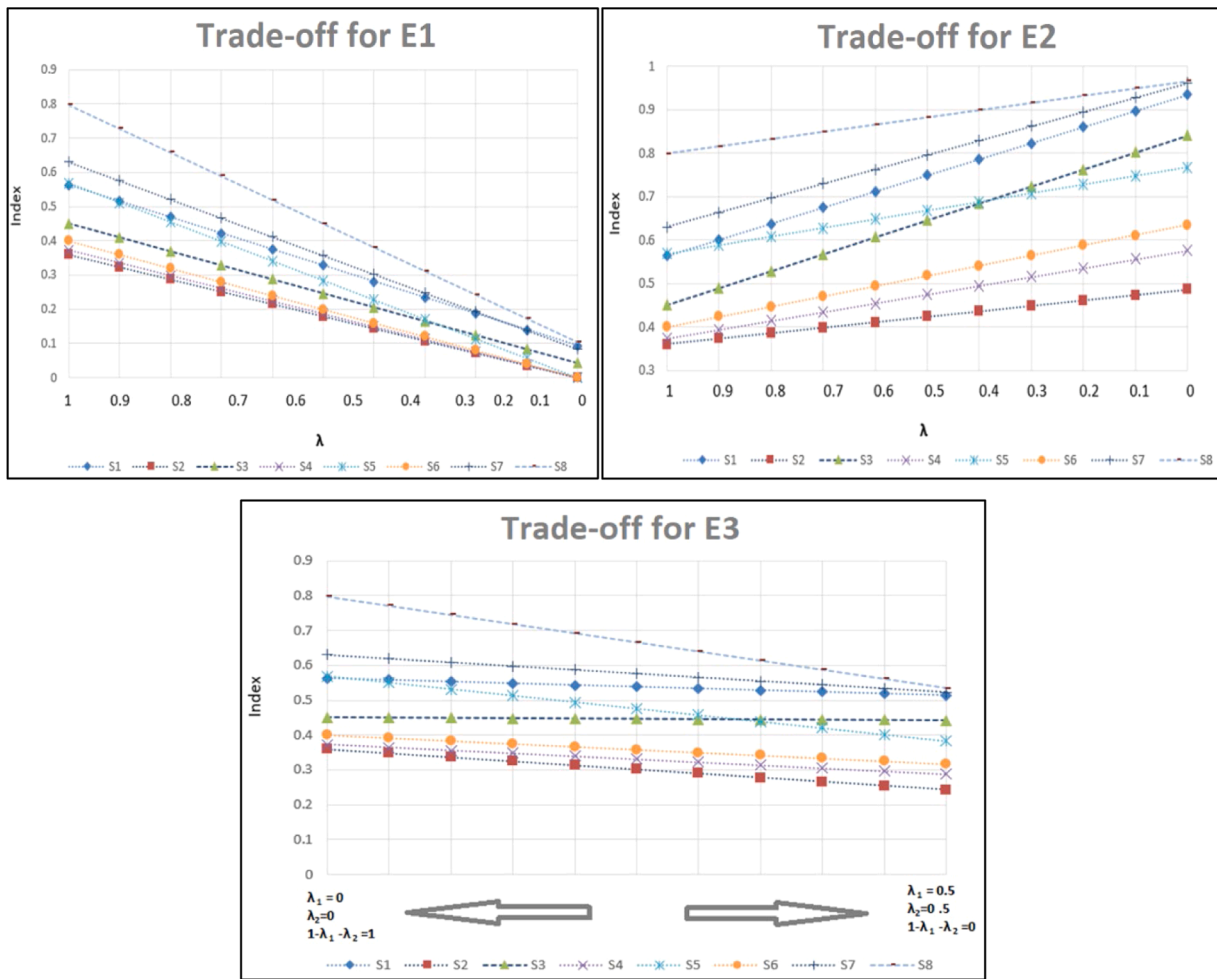


Fig. 5. The composite index trade-off between DMUs for E1, E2 and E3 -example 1. (source: model results)

most balanced solution, equally weighting the criteria of maximum deviation and normalized entropy ( $\lambda=0.5, \lambda=0.5$ ). In the three methodological lines, the trade-offs show a linear pattern. In the E1 and E2 methodological lines the trade-offs increase as the  $\lambda$  values become close to the most balanced solution ( $\lambda=0$ ). In the E3 methodological line, it is also in the most balanced solution ( $\lambda_1=0.5, \lambda_2=0.5$  and  $1-\lambda_1-\lambda_2=0$ ) that more changes in the ranking positions occur.

We tested our results using MPI for comparison. This is an approach based on a non-linear function that penalizes the DMUs with unbalanced values of the indicators and eliminates the units of measurement and the variability effects from the criteria. Table 16 presents the rankings of MPI and E1 and E2 methodological lines for illustrative example 1.

The rankings for the E1 and E2 methodological lines are identical to the MPI ranking, when  $\lambda=1$ . However, when  $\lambda=0.5$ , differences appear in the third and fourth ranking positions for both methodological lines. At  $\lambda=0$ , changes are noted in the third, fourth and fifth positions.

Table 17 shows the rankings of the MPI and E1 and E2 methodological lines for the case of illustrative example 2. In this case, the MPI ranking is different from the rankings of the E1 and E2 methodological lines. The top positioned countries in the MPI ranking are Australia (AUS), Iceland (ISL) and Ireland (IRL). However, in both methodological lines, E1 and E2, for any values of  $\lambda$ , the first ranking positions are always occupied by countries that belong to the set of the best ten countries of the MPI ranking.

We conclude our robustness analysis with a sensitivity analysis that tests extreme situations. Thus, five criteria with equal weights and twelve DMUs were simulated considering the following situations:

- DMU1- Achieves a perfect score in the first criterion, but very low in the others;
- DMU2 and DMU5 - Perfectly balanced scores (equal values in all criteria), but with different deviations;
- DMU3 - Maximum deviation equal to DMU 2 with different values in the criteria
- DMU4- A perfect balanced and perfect score of 1 in all criteria;
- DMU6 and DMU10 - Present the same minimum maximum deviation;
- DMU8 and DMU9-Equal distribution of data, but with different deviations.

The ranking of the E1 methodological line is presented in Table 18. For  $\lambda=0$ , there are two pairs of DMUs with the same composite indicator (DMU2 and DMU3, DMU6 and DMU10). However, this situation was promptly solved using an entropy approach to perfect the ranking.

Table 19 shows the ranking for the E2 methodological line based on the normalized entropy criterion. In this case, three DMUs (DMU4, DMU2 and DMU5) share the same composite indicator. That is a unique situation since it is rare to have multiple solutions with the same normalized entropy value. However, the DMUs can be distinguished and ranked using the maximum deviation, which varies for each DMU.

Finally, Table 20 presents the results of the sensitivity analysis for the E3 methodological line. Four solutions were analysed: the best aggregate ( $1-\lambda_1-\lambda_2=1$ ), the most balanced ( $\lambda_1=0.5, \lambda_2=0.5$  and  $1-\lambda_1-\lambda_2=1$ ), the balance between the best aggregate and the most balanced ( $\lambda_1=0.25, \lambda_2=0.25$  and  $1-\lambda_1-\lambda_2=0.5$ ), and the solution with equal

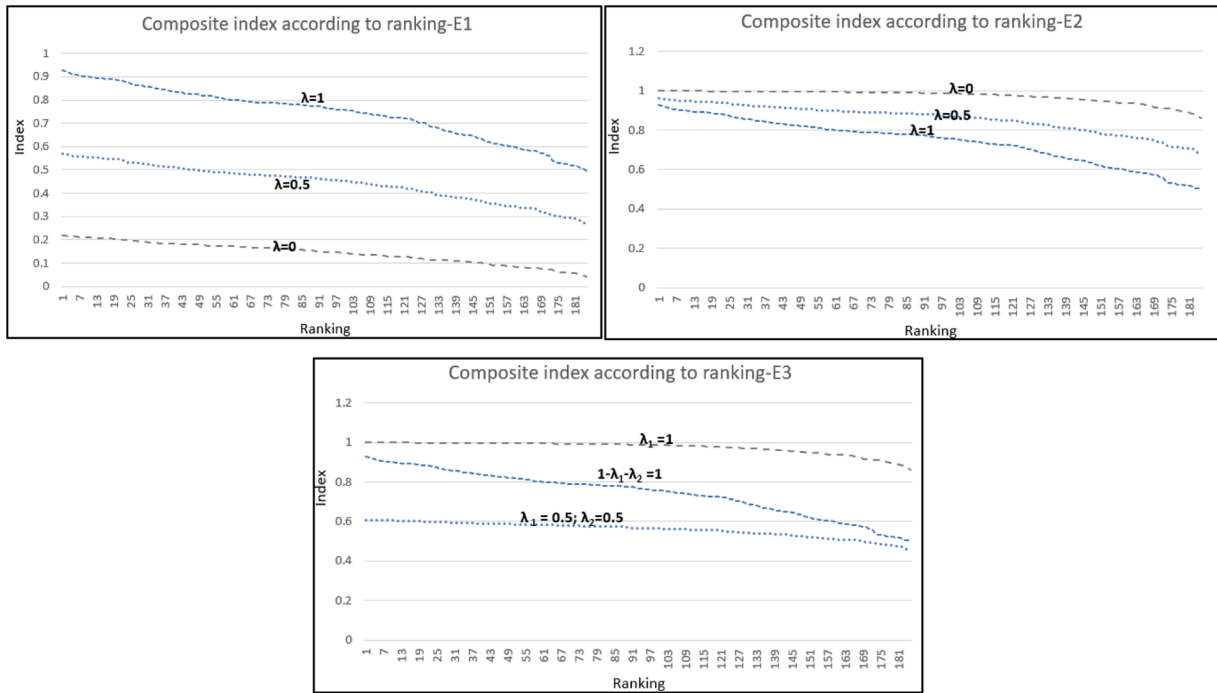


Fig. 6. The composite indicator according to the ranking for E1, E2 and E3 -example 2. (source: model results)

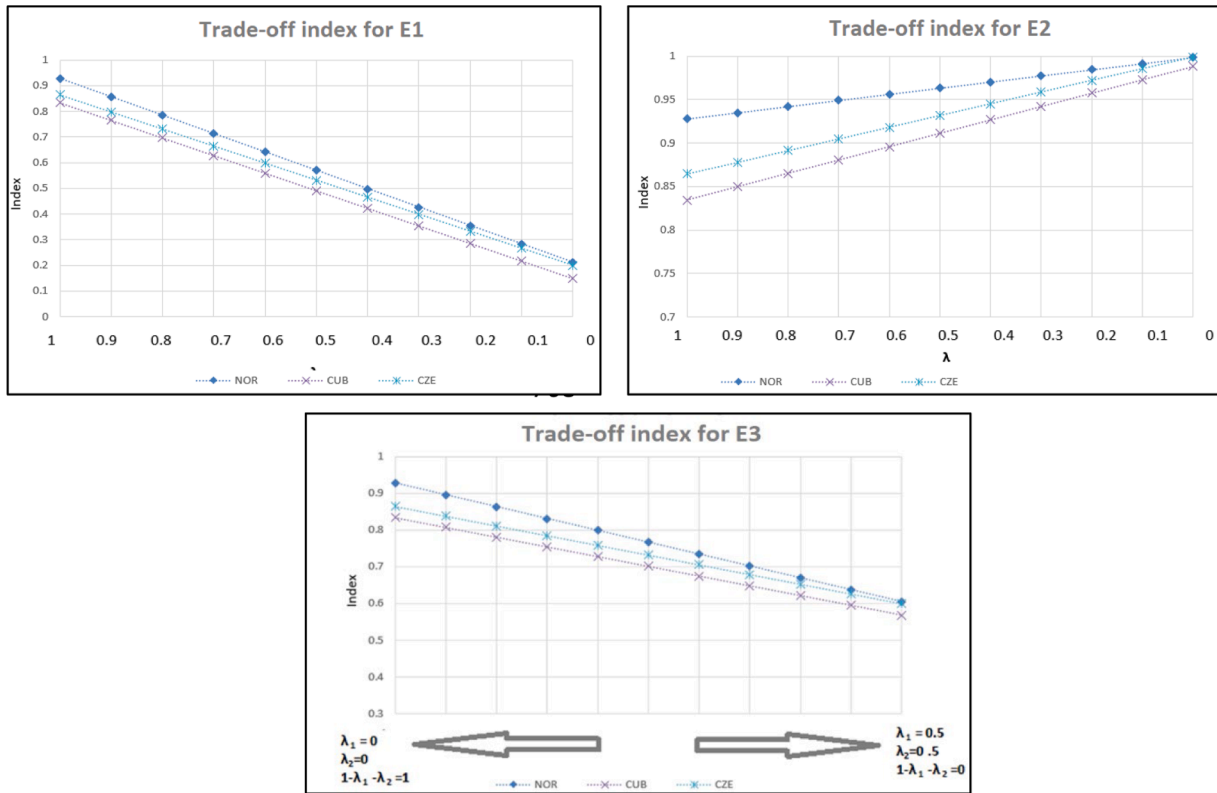


Fig. 7. The composite index trade-off between DMUs for E1, E2 and E3 - example 2. (source: model results)

criteria weights ( $\lambda_1=0.33$ ,  $\lambda_2=0.33$  and  $1-\lambda_1-\lambda_2=0.34$ ). This is an interesting set of solutions that covers the most typical scenarios, and no situations with equal ranking positions were found.

### 6. Discussion

This paper proposes a novel Multi-Criteria Decision Analysis (MCDA) approach to create a composite sustainability indicator and establish

**Table 16**  
Rankings of the MPI, E1 and E2 – Illustrative example 1.

Ranking	MPI	E1			E2		
		$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0$
1	S8	S8	S8	S8	S8	S8	S8
2	S7	S7	S7	S1	S7	S7	S7
3	S5	S5	S1	S7	S5	S1	S1
4	S1	S1	S5	S3	S1	S5	S3
5	S3	S3	S3	S2, S4, S5, S6	S3	S3	S5
6	S6	S6	S6		S6	S6	S6
7	S4	S4	S4		S4	S4	S4
8	S2	S2	S2		S2	S2	S2

S<sub>1</sub> - Maximize Net Present Value (NPV), subject to three constraints (volume control, regulation and ending inventory); S<sub>2</sub> - Maximize Net Present Value (NPV), with extended rotation and ending inventory constraint; S<sub>3</sub> - Maximize Veneer volume, with extended rotation and ending inventory constraint; S<sub>4</sub> - Maximize three objectives (NPV, veneer, volume), with extended rotation and ending inventory constraint; S<sub>5</sub> - Maximize Net Present Value (NPV), leaving the mixed stands uncut; S<sub>6</sub> - Maximize Veneer volume leaving the mixed stands uncut; S<sub>7</sub> - Maximize three objectives (NPV, veneer, volume), leaving the mixed stands uncut; S<sub>8</sub> - Maximize Net carbon, subject to three constraints (volume control, regulation, and ending inventory).

**Table 17**  
Rankings of the MPI, E1 and E2 Illustrative example 2.

Ranking	MPI	E1			E2		
		$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0.5$	$\lambda = 0$
1	AUS	NOR	NOR	CHE	NOR	NOR	DNK
2	ISL	NZL	SWE	SWE	NZL	NZL	SWE
3	IRL	SWE	CHE	DNK	SWE	SWE	BEL
4	NZL	CHE	NZL	DEU	CHE	CHE	AUT
5	CHE	HKG	DEU	AUT	HKG	HKG	SYC
6	DNK	ISL	HKG	JPN	ISL	ISL	FIN
7	BEL	DEU	ISL	BEL	DEU	DEU	CHE
8	DEU	IRL	JPN	NOR	IRL	IRL	DEU
9	SWE	JPN	IRL	FRA	JPN	JPN	GBR
10	NOR	NLD	NLD	FIN	NLD	NLD	NLD

AUS- Australia; AUT-Austria; BEL-Belgium; CHE-Switzerland; DEU-Germany; DNK-Denmark; FIN-Finland; FRA-France; HKG-Hong Kong, China; IRL-Ireland; ISL-Iceland; JPN-Japan; NLD-Netherlands; NOR-Norway; NZL-New Zealand; SWE-Sweden.

**Table 18**  
Results of the sensitivity analysis for E1.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	DMU4	DMU4	DMU4	DMU4	DMU4	DMU4
2	DMU8	DMU8	DMU8	DMU8	DMU8	DMU8
3	DMU9	DMU9	DMU9	DMU9	DMU9	DMU9
4	DMU3	DMU3	DMU3	DMU3	DMU3	<b>DMU2</b>
5	DMU2	DMU2	DMU2	DMU2	DMU2	<b>DMU3</b>
6	DMU10	DMU10	DMU10	DMU10	DMU11	DMU5
7	DMU11	DMU11	DMU11	DMU11	DMU10	DMU11
8	DMU12	DMU12	DMU6	DMU6	DMU5	<b>DMU6</b>
9	DMU6	DMU6	DMU12	DMU5	DMU6	<b>DMU10</b>
10	DMU5	DMU5	DMU5	DMU12	DMU12	DMU12
11	DMU1	DMU1	DMU1	DMU1	DMU1	DMU1
12	DMU7	DMU7	DMU7	DMU7	DMU7	DMU7

rankings. The approach consists of three complementary methodological lines: E1 – This line minimizes the maximum deviation to achieve the most balanced solution. A normalized entropy indicator is used to refine the rankings if the minimization of maximum deviation results in equal index values. E2 – In this line, normalized entropy replaces the maximum deviation to create a new composite indicator. E3 – This line provides a unique solution that combines the best aggregate solution, maximum deviation and normalized entropy. The proposed approach was tested using two examples from the literature: the study by

**Table 19**  
Results of the sensitivity analysis for E2.

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	DMU4	DMU4	DMU4	DMU4	DMU4	<b>DMU4</b>
2	DMU8	DMU8	DMU8	DMU8	DMU8	<b>DMU2</b>
3	DMU9	DMU9	DMU9	DMU9	DMU9	<b>DMU5</b>
4	DMU3	DMU3	DMU3	DMU3	DMU3	DMU8
5	DMU2	DMU2	DMU2	DMU2	DMU2	DMU9
6	DMU10	DMU10	DMU11	DMU5	DMU5	DMU3
7	DMU11	DMU11	DMU10	DMU6	DMU6	DMU6
8	DMU12	DMU6	DMU6	DMU11	DMU11	DMU11
9	DMU6	DMU12	DMU5	DMU10	DMU10	DMU10
10	DMU5	DMU5	DMU12	DMU12	DMU12	DMU12
11	DMU1	DMU1	DMU1	DMU1	DMU7	DMU7
12	DMU7	DMU7	DMU7	DMU7	DMU1	DMU1

**Table 20**  
Results of the sensitivity analysis for E3.

$\lambda_1$	0	0.5	0.33	0.25
$\lambda_2$	0	0.5	0.33	0.25
$1 - \lambda_1 - \lambda_2$	1	0	0.34	0.5

Ranking/ $\lambda$	1	0.7	0.5	0.3	0.2	0
1	DMU4	DMU4	DMU4	DMU4	DMU4	DMU4
2	DMU8	DMU8	DMU8	DMU8	DMU8	DMU8
3	DMU9	DMU9	DMU9	DMU9	DMU9	DMU9
4	DMU3	DMU2	DMU3	DMU3	DMU3	DMU3
5	DMU2	DMU3	DMU2	DMU2	DMU2	DMU2
6	DMU10	DMU5	DMU11	DMU11	DMU10	DMU10
7	DMU11	DMU6	DMU10	DMU10	DMU11	DMU11
8	DMU12	DMU11	DMU6	DMU6	DMU6	DMU6
9	DMU6	DMU10	DMU5	DMU5	DMU5	DMU5
10	DMU5	DMU12	DMU12	DMU12	DMU12	DMU12
11	DMU1	DMU7	DMU1	DMU1	DMU1	DMU1
12	DMU7	DMU1	DMU7	DMU7	DMU7	DMU7

Diaz-Balteiro and Romero (2004a) and the research by Bravo (2014), which made improvements to the Human Development Index. Additionally, a robustness analysis included a trade-off analysis, application of the Mazziotta-Pareto Index, and a sensitivity analysis based on simulated values. The results are robust and promising, demonstrating that the approach can be useful for various applications.

The E1 methodological approach, based on normalized entropy, aims to improve rankings established where composite indicators are equal. Findings from applying the E2 methodological line suggest that substituting maximum deviation with normalized entropy did not significantly change the existing rankings but did help to address certain ranking issues. Meanwhile, the E3 methodology presents an integrated approach that maximizes the benefits of all solutions, making it particularly valuable for considering various trade-offs. It is essential to consider the values of each solution and their potential interactions to accurately represent the specific objectives of the analysis. Analysts should critically evaluate and discuss the selection of solution trade-offs. Additionally, the complexity of this integrated approach must be carefully considered.

The proposed model effectively addresses various issues identified in previous approaches (e.g., Diaz-Balteiro and Romero, 2004b; Diaz-Balteiro et al., 2011; Voces et al., 2012; Diaz-Balteiro et al., 2016b). By utilizing the concept of normalized entropy instead of maximum deviation as an indicator to achieve a more balanced distribution, this model offers several advantages. Besides solving the challenges posed by equal composite indicator values and their corresponding positions in rankings, it enables the creation of a new set of indicators for different DMUs, which can serve as a reference point. It is essential to understand that the most displaced indicator's deviation and normalized entropy are complementary measures; thus, both should be employed simultaneously. Furthermore, adopting an entropy approach extends the methodology proposed by Xavier et al. (2018) to all scenarios of composite indicator aggregation, enhancing the methodology established by Diaz-Balteiro

and Romero (2004a). This new approach also allows the incorporation of additional criteria to better represent the most balanced situations, resulting in a trade-off between the best aggregated value and the new measure of entropy. Our integrated approach considers the three measures (best aggregate, maximum deviation, and normalized entropy) simultaneously, maximizing the information obtained from the various solutions.

Compared to other compromise programming approaches (Diaz-Balteiro et al., 2011; Costa Freitas et al., 2022), we observe different results. In a compromise programming approach, the best-aggregated solution is determined by summing the deviations from the ideal, while the most balanced sustainability is given by the maximum deviation. In our case, the aggregation method and the distance function employed do not weight the maximum deviation as in compromise programming.

The parameter  $\lambda$  plays a crucial role in the process of aggregating a set of indicators, making its selection a vital decision for the acceptance of composite indexes by decision-makers. When  $\lambda=1$ , models E1 and E2 yield the solution with the maximum aggregate. Conversely, when  $\lambda=0$ , these models provide solutions that ensure the most balanced achievement. For intermediate values of  $\lambda$ , compromise solutions can be reached. In the case of model E3, a more complex trade-off between  $\lambda_1$  and  $\lambda_2$  needs to be considered. With  $\lambda=1$  or  $\lambda=0$  in models E1 and E2, the results may be acceptable from both environmental and economic perspectives (Diaz-Balteiro et al., 2018). However, to reflect real sustainability, it is preferable to balance the models by setting  $\lambda$  to 0.5 in both cases. Additionally, it is essential to explore all potential solutions to identify the best trade-off for  $\lambda$ .

In the context of strong sustainability, economic and environmental capital are complementary and cannot be replaced by each other. However, many multi-criteria methods assume a certain degree of compensation between the criteria, which could lead to situations of weak sustainability. Our model can address these challenges. Although it assumes a weak sustainability context, it can also approach strong sustainability by seeking the most balanced solutions through normalized entropy and maximum deviation.

Although the proposed approach has been applied to the analysis of sustainability rankings, it may be implemented in other situations, such as human development, tourism, and stock markets, where it is necessary to critically create and analyse rankings. Therefore, this paper is a further contribution to the body of knowledge on building composite indicators using MCDA, providing a more detailed set of results.

Our study makes several innovative contributions to the literature on sustainability composite indicators, but it also has some limitations. First, our model employs an aggregation process that acts as a partial compensatory method. While the best aggregate solution allows for full compensation, the normalized entropy is a bottleneck solution, enabling us to work with no compensation between criteria. Second, the results of our model are sensitive to the values chosen for the  $\lambda$  parameter, which may lead to either the best aggregate or the most balanced solution.

In the future, we aim to enhance our model by introducing new indicators and weighting methods that facilitate compensation between criteria. We also plan to incorporate as much information as possible from the preferences of decision-makers and stakeholders. In addition, we intend to extend our application to other cases and fields to complement other composite indicators and combine our methodology with approaches such as fractional regression analysis, to identify external factors affecting sustainability.

#### CRedit authorship contribution statement

**António Xavier:** Conceptualization, Formal analysis, Methodology, Validation, Visualization, Writing – original draft, Writing – review & editing. **Rui Fragoso:** Conceptualization, Formal analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing. **Maria de Belém Costa Freitas:** Conceptualization, Formal

analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing.

#### Acknowledgements

The authors acknowledge CEFAGE (<https://doi.org/10.54499/UIDB/04007/2020>) MED (<https://doi.org/10.54499/UIDB/05183/2020>) and CHANGE (<https://doi.org/10.54499/LA/P/0121/2020>).

#### References

- Atkinson, G. D., Dubourg, R., Hamilton, K., Munasignhe, M., Pearce, D. W., & Young, C. (1997). *Measuring Sustainable Development: Macroeconomics and the Environment*. Cheltenham: Edward Elgar.
- Banihabib, M. E., Hashemi-Madani, F. S., & Forghani, A. (2017). Comparison of compensatory and non-compensatory multi criteria decision making models in water resources strategic management. *Water Resources Management*, 31, 3745–3759.
- Biggeri, M., & Mauro, V. (2018). Towards a more ‘sustainable’ human development index: Integrating the environment and freedom. *Ecological indicators*, 91, 220–231.
- Blanca, F. J., Caballero, R., González, M., Lozano-Oyola, M., & Pérez, F. (2010). Goal programming synthetic indicators: An application for sustainable tourism in Andalusian coastal counties. *Ecological Economics*, 69(11), 2158–2172.
- Bravo, G. (2014). The Human Sustainable Development Index: New calculations and a first critical analysis. *Ecological indicators*, 37, 145–150.
- Costa Freitas, M. D. B. C., Xavier, A., Fragoso, R., & Antunes, C. (2022). A composite indicator to measure sustainable water use in Portugal: A compromise programming approach. *Journal of Environmental Management*, 311, Article 114791.
- De, La, Vega, M. L., & Urrutia, A. M. (2001). HDPI: a framework for pollution-sensitive human development indicators. *Environ. Dev. Sustain.*, 3(3), 199–215.
- Dewan, H. (2006). Sustainability Index. An Economic Perspective. Accessed at <https://www.semanticscholar.org/paper/Sustainability-Index%3A-An-Economics-Perspective-Dewan/afd9ca3770eb16b2f2be5c23734ef0e8068984b3>.
- Diaz-Balteiro, L., & Romero, C. (2004a). In search of a natural systems sustainability index. *Ecological Economics*, 49, 401–405.
- Diaz-Balteiro, L., & Romero, C. (2004b). Sustainability of forest management plans: a discrete goal programming approach. *Journal of Environmental Management*, 71, 351–359.
- Diaz-Balteiro, L., & Romero, C. (2008). Making forestry decisions with multiple criteria: A review and an assessment. *Forest Ecology and Management*, 255(8), 3222–3241.
- Diaz-Balteiro, L., González-Pachón, J., & Romero, C. (2009). Forest management with multiple criteria and multiple stakeholders: An application to two public forests in Spain. *Scandinavian Journal of Forest Research*, 24(1), 87–93.
- Diaz-Balteiro, L., Voces González, R., & Romero, C. (2011). Making sustainability rankings using compromise programming. An application to European paper industry. *Silva Fennica*, 45(4), 761–773.
- Diaz-Balteiro, L., Alfranca, O., Bertomeu, M., Ezquerro, M., Giménez, J. C., González-Pachón, J., & Romero, C. (2016a). Using quantitative techniques to evaluate and explain the sustainability of forest plantations. *Canadian Journal of Forest Research*, 46(9), 1157–1166.
- Diaz-Balteiro, L., Alfranca, O., González-Pachón, J., & Romero, C. (2016b). Ranking of industrial forest plantations in terms of sustainability: A multicriteria approach. *Journal of environmental management*, 180, 123–132.
- Diaz-Balteiro, L., Alonso, R., Martínez-Jauregui, M., & Pardos, M. (2017a). Selecting the best forest management alternative by aggregating ecosystem services indicators over time: A case study in central Spain. *Ecological indicators*, 72, 322–329.
- Diaz-Balteiro, L., González-Pachón, J., & Romero, C. (2017b). Measuring systems sustainability with multi-criteria methods: A critical review. *European Journal of Operational Research*, 258(2), 607–616.
- Diaz-Balteiro, L., Belavenutti, P., Ezquerro, M., González-Pachón, J., Nobre, S. R., & Romero, C. (2018). Measuring the sustainability of a natural system by using multi-criteria distance function methods: Some critical issues. *Journal of environmental management*, 214, 197–203.
- Diaz-Balteiro, L., Romero, C., & García de Jalón, S. (2022). An analysis of the degree of circularity of the wood products industry in Europe. *Journal of Industrial Ecology*, 26(4), 1350–1363.
- Desai, M. (1995). Greening the HDI? In A. McGillivray (Ed.), *Accounting for Change* (pp. 21–36). London: The New Economics Foundation (Editor).
- Ebert, U., & Welsch, H. (2004). Meaningful environmental indices: a social choice approach. *Journal of Environmental Economics and Management*, 47, 270–283.
- El Gibari, S., Gómez, T., & Ruiz, F. (2019). Building composite indicators using multicriteria methods: a review. *Journal of Business Economics*, 89(1), 1–24.
- European Commission. (2001). *European Innovation Scoreboard*. Brussels: DG ENTR.
- Giménez, J. C., Bertomeu, M., Diaz-Balteiro, L., & Romero, C. (2013). Optimal harvest scheduling in Eucalyptus plantations under a sustainability perspective. *Forest Ecology and Management*, 29(1), 367–376.
- Golan, A., Judge, G., & Miller, D. (1996). *Maximum Entropy Econometrics: Robust Estimation with Limited Data*. New York, USA: John Wiley & Sons.
- Gómez-Limón, J. A., & Sanchez-Fernandez, G. (2010). Empirical evaluation of agricultural sustainability using composite indicators. *Ecological economics*, 69(5), 1062–1075.

- González-Pachón, J., & Romero, C. (2007). Inferring consensus weights from pairwise comparison matrices without suitable properties. *Annals of Operations Research*, 154(1), 123–132.
- Greco, S., Ishizaka, A., Tasiou, M., & Torrisi, G. (2019). On the methodological framework of composite indices: A review of the issues of weighting, aggregation, and robustness. *Social indicators research*, 141, 61–94.
- Hickel, J. (2020). The sustainable development index: Measuring the ecological efficiency of human development in the anthropocene. *Ecological Economics*, 167, Article 106331.
- Jeffrey, K., Wheatley, H., & Abdallah, S. (2016). *The Happy Planet Index 2016: A Global Index of Sustainable Wellbeing*. London: New Economics Foundation.
- Jin, H., Qian, X., Chin, T., & Zhang, H. (2020). A Global Assessment of Sustainable Development Based on Modification of the Human Development Index via the Entropy Method. *Sustainability*, 12(8), 3251.
- Kelemen, A., Szabó, Z. K., Bozókai, S., Szádóczi, Z., & Hartvig, Á. D. (2024). A sensitivity analysis of composite indicators: Min/max thresholds. *Environmental and Sustainability Indicators*, 23, Article 100453.
- Kwatra, S., Kumar, A., & Sharma, P. (2020). A critical review of studies related to construction and computation of Sustainable Development Indices. *Ecological Indicators*, 112, Article 106061.
- Liang, X., Si, D., & Zhang, X. (2017). Regional sustainable development analysis based on information entropy—Sichuan Province as an example. *International Journal of Environmental Research and Public Health*, 14(10), 1219.
- Lozano-Oyola, M., Blancas, F. J., González, M., & Caballero, R. (2012). Sustainable tourism indicators as planning tools in cultural destinations. *Ecological indicators*, 18, 659–675.
- Mazziotta, M., & Pareto, A. (2012). A non-compensatory approach for the measurement of the quality of life. In F. Maggino, & G. Nuvolati (Eds.), *Quality of life in Italy: Research and reflections* (pp. 27–40). New York: Springer.
- Mazziotta, M., & Pareto, A. (2016). On a Generalized Non-compensatory Composite Index for Measuring Socio-economic Phenomena. *Soc Indic Res*, 127, 983–1003. <https://doi.org/10.1007/s11205-015-0998-2>
- Moran, D. D., Wackernagel, M., Kitzes, J. A., Goldfinger, S. H., & Boutaud, A. (2008). Measuring sustainable development—Nation by nation. *Ecological economics*, 64(3), 470–474.
- Morse, S. (2003). Greening the United Nations' human development index? *Sustain. Dev.*, 11(4), 183–198.
- Narula, K., & Reddy, B. S. (2015). Three blind men and an elephant: the case of energy indices to measure energy security and energy sustainability. *Energy*, 80, 148–158.
- Neumayer, E. (2001). The human development index and sustainability—a constructive proposal. *Ecol. Econ*, 39(1), 101–114. Nivolianitou, Z., Synodinou, B., Manca, D., 2015. Flood disaster management with the use of AHP. *Int. J. Multicriteria Decision Making*, 5(1–2), 152–164.
- Neumayer, E. (2012). Human development and sustainability. *J. Human Dev. Capabil.*, 13(4), 561–579.
- Nivolianitou, Z., Synodinou, B., & Manca, D. (2015). Flood disaster management with the use of AHP. *International Journal of Multicriteria Decision Making*, 5(1–2), 152–164.
- Nordström, E., Romero, C., Eriksson, L., & Öhman, L. (2009). Aggregation of preferences in participatory forest planning with multiple criteria: an application to the urban forest in Lycksele, Sweden. *Canadian Journal of Forest Research*, 39(10), 1979–1992.
- OECD. (2008). Organization for Economic Co-operation and Development — JRC, Joint Research Centre. *Handbook on constructing composite indicators. Methodology and user guide*. Paris: OECD.
- Pelenc, J., Lompo, M. K., Ballet, J., & Dubois, J. L. (2013). Sustainable human development and the capability approach: integrating environment, responsibility and collective agency. *J. Human Dev. Capabil.*, 14(1), 77–94.
- Pereira, T. T., & Fragoso, R. (2022). A Governança e Eficiência no Modelo Intermunicipal de Gestão da Água no Alto Alentejo. *RPER*, 62, 123–142.
- Ramathan, B. (1999). Environment sensitive human development index: issues and alternatives. *Indian Sociol. Rev.*, 1, 193–201.
- Rogge, N. (2018). On aggregating benefit of the doubt composite indicators. In *European Journal of Operational Research*, 264 pp. 364–369.
- Romero, C. (1991). *Handbook of critical issues in goal programming*. Pergamon Press.
- Rowley, H., Peters, G., Lundie, S., & Moore, S. (2012). Aggregating sustainability indicators: Beyond the weighted sum. *Journal of Environmental Management*, 111, 24–33.
- Ruiz, F., Cabello, J. M., & Luque, M. (2011). An application of reference point techniques to the calculation of synthetic sustainability indicators. *Journal of the Operational Research Society*, 62(1), 189–197.
- Saisana, M., Tarantola, S. (2002). State-of-the-art report on current methodologies and practices for composite indicator development. Prepared for the European Commission (Report number: EUR 20408 EN).
- Salzman, J. (2003). *Methodological choices encountered in the construction of composite indices of economic and social well-being*. Ottawa: Center for the Study of Living Standards. Technical Report.
- Shah, S. A. A., Zhou, P., Walasai, G. D., & Mohsin, M. (2019). Energy security and environmental sustainability index of South Asian countries: A composite index approach. *Ecological Indicators*, 106, Article 105507.
- Shannon, C. (1948). A mathematical theory of communication. *Bell Syst. Technol. J.*, 27(3), 379–423.
- Singh, R. K., Murty, H. R., Gupta, S. K., & Dikshit, A. K. (2012). An overview of sustainability assessment methodologies. *Ecological Indicators*, 15(1), 281–299.
- Steinborn, W., & Svirezhev, Y. (2000). Entropy as an indicator of sustainability in agro-ecosystems: North Germany case study. *Ecological Modelling*, 133(3), 247–257.
- Tamiz, M., Jones, D. F., & Romero, C. (1998). Goal programming for decision making: an overview of the current state-of-the-art. *European Journal of Operational Research*, 111, 569–581.
- Togtokh, C., & Gaffney, O. (2010). *Human Sustainable Development Index, OurWorld2.0: Web-magazine of the*. United Nations University. <http://ourworld.unu.edu/en/the-2010-human-sustainable-development-index/>.
- Tolstykh, T., Shmeleva, N., Vertakova, Y., & Plotnikov, V. (2020). The entropy model for sustainability assessment in industrial ecosystems. *Inventions*, 5(4), 54.
- UN - United Nations. (2022). *Global indicator framework for the Sustainable Development Goals and targets of the 2030 Agenda for Sustainable Development*. United Nations Accessed on 15-04-2022 [https://unstats.un.org/sdgs/indicators/Global%20Indicator%20Framework%20after%202022%20refinement\\_Eng.pdf](https://unstats.un.org/sdgs/indicators/Global%20Indicator%20Framework%20after%202022%20refinement_Eng.pdf).
- UNDP - United Nations Development Programme. (2009). *Human Development Report 2009: Overcoming barriers: Human mobility and development*. New York. <https://hdr.undp.org/en/content/human-development-report-2009>.
- UNDP - United Nations Development Programme (2010). *Human Development Report 2010: The Real Wealth of Nations - Pathways to Human Development*. New York. <http://hdr.undp.org/en/content/human-development-report-2010>.
- UNDP - United Nations Development Programme. (2019). *Human Development Report 2019: Beyond income, beyond averages, beyond today: Inequalities in human development in the 21st century*. New York: UNDP-United Nations Development Programme. ISBN: 978-92-1-126439-5.
- Voces, R., Diaz-Balteiro, L., & Romero, C. (2010). In search of a European paper industry ranking in terms of sustainability by using binary goal programming. *New Developments in Multiple Objective and Goal Programming* (pp. 141–149). Berlin, Heidelberg: Springer.
- Voces, R., Diaz-Balteiro, L., & Romero, C. (2012). Characterization and explanation of the sustainability of the European wood manufacturing industries: a quantitative approach. *Expert Syst. Appl.*, 39(2012), 6618–6627.
- Walheer, B. (2024). A sequential benefit-of-the-doubt composite indicator. *European Journal of Operational Research*, 316(1), 228–239.
- Wu, S., Fu, Y., Shen, H., & Liu, F. (2018). Using ranked weights and Shannon entropy to modify regional sustainable society index. *Sustainable cities and society*, 41, 443–448.
- Xavier, A. S., Freitas, M. B. C., & Antunes, C. R. (2016). The water surfaces' ecosystem services and the opinion of different stakeholders: an approach based on goal programming. *International Journal of Management and Decision Making*, 15(2), 184–204.
- Xavier, A., Freitas, M. B. C., Fragoso, R., & Rosário, M. S. (2018). A regional composite indicator for analysing agricultural sustainability in Portugal: A goal programming approach. *Ecological Indicators*, 89, 84–100.
- Xavier, A., Costa Freitas, M. D. B., Fragoso, R., & Rosário, M. D. S. (2022). Analysing the Recent Dynamics of Agricultural Sustainability in Portugal Using a Compromise Programming Approach. *Sustainability*, 14(19), Article 12512.
- Zeleny, M. (1974). Concept of compromise solutions and the method of the displaced ideal. *Computers & Operations Research*, 1, 479–496.
- Zhou, P., Fan, L. W., & Zhou, D. Q. (2010). Data aggregation in constructing composite indicators: A perspective of information loss. *Expert Systems with Applications*, 37(1), 360–365.