

# A POST-DETECTION MAXIMUM RATIO COMBINER: EXPERIMENTAL ASSESSMENT ON HIGH DIVERSITY UNDERWATER CHANNELS

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## ABSTRACT

*Maximum Ratio Combiner (MRC) is a diversity combining technique applicable to underwater communications when the data transmitted by a single projector is received by more than one antenna/node. Post Detection MRC (PD-MRC) performs a weighted-sum over initial detected outputs of multiple nodes. PD-MRC can be applied to noise-only or ISI-only channels with the weights computed from the signal to noise ratio or from the channel impulse response of each node, respectively. In this paper it is shown that in the presence of noisy-ISI channels the weights can be computed from the detector output constellation of each node. A performance of the PD-MRC is evaluated using real data collected during RADAR'07 experiment. Results show that a gain is always attained using two nodes in the PD-MRC as compared to the node having the best MSE. Moreover, the PD-MRC gain is higher when both nodes present a similar MSE.*

## 1. INTRODUCTION

Underwater coherent communications are known to be quite unpredictable in time and space. Communication system performance and channel capacity may suffer from dramatic changes as a consequence of variations in oceanographic conditions, e.g. water-column sound speed, surface waves, and the movements of source and receivers. To overcome such problem the common approach is to implement multichannel equalizers [1] at the receiver, which usually requires intensive computing power. With spatial diversity where a source communicates with more than one receiver (nodes), placed at different locations, a performance reduction in one of the nodes does not imply a performance reduction in the other nodes. However optimally combining the distributed nodes, the overall performance of the system can be enhanced or at least maintained at the best node level.

Maximal ratio combining (MRC) is an optimal combining technique that exploits spatial diversity and is used in communications over fading channels. In MRC, two or more copies of the same information signal are

combined to minimize the Mean Squared Error (MSE) at the output. The post-detection MRC (PD-MRC) [2] weights and combines all nodes after an initial data-signal detection and synchronization. In a previous work [3], a PD-MRC was developed for ISI-only channels and it was found that the weights can be computed in a closed form expression considering that the overall IR at the detectors output is known. In this work, rather than using the IR estimates in the PD-MRC weights computation, the output constellations of initial detected signals at different nodes are used. It will be shown that the weights computed this way are optimal, in the MSE sense, for noisy-ISI channels. The proposed method finds application in underwater sensors network where the nodes presents a very low computational power, storage capacity and power consumption.

In this paper a performance comparison between the PD-MRC with weights computed with an adaptive Recursive Least Square (RLS) algorithm and with the proposed method is presented. The relevance of each of the parameters that contributes to the computation of the PD-MRC weights is clarified.

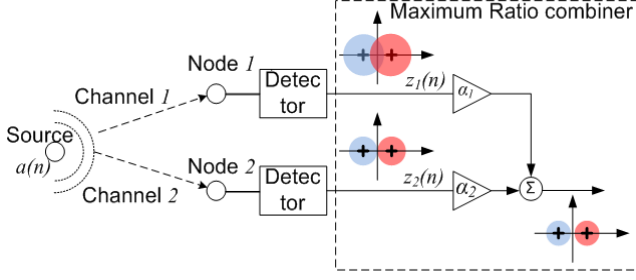
## 2. POST DETECTION MAXIMUM RATIO COMBINER

Without loss of generality, Figure 2 shows the block diagram of the PD-MRC operation with only two nodes, where a source transmits BPSK modulated signals to the nodes. The detector synchronizes and detects the transmitted sequence and its output, for node  $j$  is given by,

$$z_j(n) = a(n) * g_j(n) + u_j(n), \quad (1)$$

where  $a(n)$  denotes the transmitted sequence,  $g_j(n)$  is the overall baseband IR at the detector output and  $u_j(n)$  is the AWGN. After the initial detection the PD-MRC weighs and sums the signals with the objective of reducing the MSE at the output signal, given by

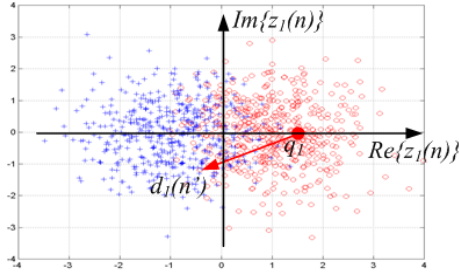
$$z(n) = \sum_j \alpha_j z_j(n). \quad (2)$$



**Figure 1: PD-MRC block diagram.**

Figure 2 shows a real detector-output constellation received at a node, acquired during RADAR'07 experiment (described in section 3). In the figure the red 'o' and the blue '+' represent the detected symbols associated with transmitted symbols '1' and '-1', respectively. At the receiver due to noise and ISI the symbols spread around two symmetric centroids  $q_j$  and  $-q_j$  that in a Rayleigh fading channel corresponds to the channel main path,  $g_j(0)$ . Also in figure 2  $d_j(n)$  represents a vector from the centroid to the symbol  $n$ , and is given by

$$d_j(n) = \frac{z_j(n)}{a(n)} - q_j. \quad (3)$$



**Figure 2: Output constellation after initial detection**

Using (3), the mean squared error (MSE) of the constellation is given by

$$MSE_j = N^{-1} q_j^{-2} \sum_n |d_j^2(n)| \quad (4)$$

where  $N$  is the total number of symbols in the constellation. From (2)-(4), the PD-MRC output MSE, considering only two nodes, is given by

$$MSE = \frac{MSE_1 |(\alpha_1 q_1)^2|}{|(\alpha_1 q_1 + \alpha_2 q_2)^2|} + \frac{MSE_2 |(\alpha_2 q_2)^2|}{|(\alpha_1 q_1 + \alpha_2 q_2)^2|} + \frac{2 \sum_n \langle \alpha_1 d_1(n), \alpha_2 d_2(n) \rangle}{N |(\alpha_1 q_1 + \alpha_2 q_2)^2|} \quad (5)$$

where it can be observed that the output MSE depends on: the input node MSEs (the first two terms) and the mean of the inner product (IP) of the vectors of the detected symbols to the centroid.

It can be shown that the MSE given by (5) is minimum if

$$\frac{\alpha_1}{\alpha_2} = \frac{MSE_2 q_2 - q_1^{-1} IP}{MSE_1 q_1 - q_2^{-1} IP}, \quad (6)$$

where

$$IP = N^{-1} \sum_n \langle d_1(n), d_2(n) \rangle \quad (7)$$

is the mean of the IP of the vectors given by (3). Equation (6) shows that there is a family of optimal solutions that results from the constant ratio of coefficients  $\alpha_1$  and  $\alpha_2$ . In the following, those coefficients are to be considered equal to the numerator and denominator of (6), respectively.

Assuming that the noise term in (1) is a zero-mean ergodic Gaussian random process and that the multipaths in  $g_j(n)$  are uncorrelated, the IP given by (7) is equal to

$$IP \approx \sum_{m \neq 0} \langle g_1(m), g_2(m) \rangle \quad (8)$$

revealing that the IP term measures the diversity between the multipath structure of the channels, excluding their main-paths. With the same assumption it can be shown that the MSE calculated using (4) is

$$MSE_j = SNR_j^{-1} + ISI_j \quad (9)$$

where the second term is the channel inter-symbolic interference (ISI), given by

$$ISI_j = \frac{\sum_m |g_j^2(m)|}{|g_j^2(0)|} \quad (10)$$

and the first is the signal to noise ratio (SNR) given by

$$SNR_j = \frac{|g_j^2(0)|}{\sigma_{u,j}^2}, \quad (11)$$

where  $\sigma_{u,j}^2$  is the noise power. Applying (8) and (9) in (6), it results that: (i) for noise-only channels

$$\frac{\alpha_1}{\alpha_2} = \frac{SNR_1 q_2}{SNR_2 q_1}; \quad (12)$$

and (ii) for ISI-only channels

$$\frac{\alpha_1}{\alpha_2} = \frac{ISI_2 q_2 - q_1^{-1} \sum_{m \neq 0} \langle g_1(m), g_2(m) \rangle}{ISI_1 q_1 - q_2^{-1} \sum_{m \neq 0} \langle g_1(m), g_2(m) \rangle}. \quad (13)$$

Equations (11) and (12) can be found in [4] and [3], respectively

### 3. EXPERIMENT DESCRIPTION

To investigate the PD-MRC performance with real data, underwater communication data collected during the RADAR'07 experiment are used. This experiment took place in the vicinity of the Setubal canyon in Portugal, from 10 to 15 July 2007. During the experiment, BPSK modulated sequences pulse shaped by a squared root raised cosine with a roll-off factor of 0.5 were transmitted from a towed source. In the following a data set with a carrier frequency of 12.5 kHz and 2000 baud is considered. The data sequence comprises one second of uniform distributed random symbols repeated 20 times. The emitted BPSK sequences were collected by two free drifting autonomous Oceanographic Acoustic Buoys (AOBs) consisting of a vertical line array of hydrophones, a preprocessing PC104+ unit and an WLAN [3]. AOB1 is composed of 8 hydrophones and AOB2 consists of 16 hydrophones. The acoustic source was towed by the research vessel NRP D.Carlos I, which

slowly cruised in the direction of the two AOBs, from an initial distance of approximately 3.5 and 5.5 kms to the AOB1 and AOB2, respectively. The nominal source depth was 60 meters. Together with the bathymetric characteristics of the site, figure 3 displays the GPS positions of the two free-drifting AOBs as well as the source ship track. The crosses indicate the locations of the three elements corresponding to the data processed in the next section. The experiment geometry is characterized by an important geological depression, the Setubal canyon that is located underneath the source with a watercolumn of 400m depth. The two AOBs were located upon the continental plateau in a slowly varying water depth region, ranging from 90m for AOB2 to 120m for AOB1.

The signals received in each AOB were initially detected and synchronized with a pTR detector [1] and further applied to the MCR developed in section 2. The pTR detector makes use of an initial channel IR estimate to deconvolve the channel multipath and reduce the MSE. However due to the range and depth variability of the transmitting and receiving devices and other environmental variabilities as surface waves, the pTR suffers a strong degradation.

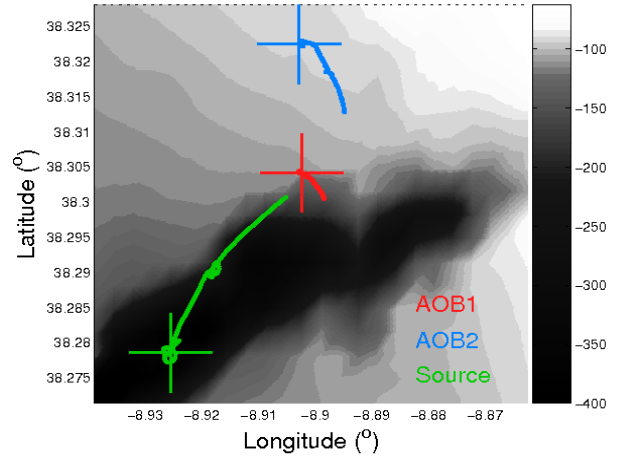
Thereafter, the two PD-MRC nodes will be named AOB1 and AOB2 and two cases will be analyzed: case “a” AOB1 and AOB2 operating with 8 and 16 hydrophones respectively; and case “b” AOB1 and AOB2 both operating with 8 hydrophones.

#### 4. PD-MRC ATTAINED RESULTS

Let PD-MRC(C) denote the PD-MRC with weights calculated from the constellation by (6), and PD-MRC(R) denote that with weights computed using an adaptive RLS algorithm.

In this section, a performance comparison between the PD-MRC(C) and PD-MRC(R), operating in non-guided mode is presented. In the PD-MRC(C), the detected signals  $z_j(n)$  is first divided into slots, each with 2000 symbols. In the first slot the PD-MRC(C) coefficients,  $a_j(0)$ , are computed with a training sequence and in the following slots the weights are computed with the estimated symbols. Except for the first slot the PD-MRC(C) technique is applied with the weights computed in the previous slot under the assumption that  $a_j(i)$  vary smoothly from one slot to the next.

Figure 4 shows the MSE and bit error rate (BER) results. It can be seen that the communications with AOB1 finds difficulties, with a MSE between -4 and 18 dB, while the communication with AOB2 obtains good results in case “a” with a MSE between -10 and -6 dB, and poor results in case “b” with a MSE between -7 and 1 dB. In both cases “a” and “b” it can be observed that PD-MRC(C) provides comparable performance to standard adaptive algorithm PD-MRC(R). Since the PD-MRC(C) only requires the computation of the coefficients 20 times, while the PD-MRC(R) 40000 times, the former becomes



**Figure 3: RADAR Experiment geometric configuration**

a preferable choice when the computation capacity is limited. Case “b” also reveals that even with 10% of BER the PD-MRC(C) technique shows a sustainable performance in decision-direct mode since the operation do not degrades over time.

The computation of the PD-MRC(C) coefficients, given by (6), depends on the MSE at the detector outputs and the IP between the two constellations. The former depends on the symbol estimates however the latter does not. With the previous reason and since the IP given by (7) is an approximation to the IP of the IRs at the detector outputs, given by (8), suggest that better results would be achieved if in (6) the IP is applied with local mean. In the following such approach will be termed PD-MRC(I) and corresponds to PD-MRC(C) with time variable coefficients due to the IP variability. Such local mean was computed using a moving average filter of 4 taps for all slots and its use is required to eliminate residual noise in the IP estimate. In the following a performance comparison is made between the PD-MRC (I) and (R) in guided mode. Figure 5 shows that in such case the MSE results become almost always better them those of the PD-MRC(R). It can be observed that the exception to such behavior happens when the AOB1 and AOB2 performances are similar, that is between the seconds: 0-2 and 12-13 seconds (as can be observed in Figure 5 for PD-MRC(I) and (R) performances and Figure 4b1 for AOB1-2 performances).

#### 5. COMMENTS AND FUTURE WORK

The work presented in this paper focused on the application of a low complexity PD-MRC to real data acquired during the RADAR’07 experiment where a source transmits data to two nodes placed at different locations. It was derived that the PD-MRC coefficients are a function of the constellations MSE at the PD outputs and of the constellations IP mean (PD-MRC(C)). Results show that the PD-MRC(C) always present a gain over the each input nodes, and higher gain is obtained when the MSE of each node is similar.

(a1)

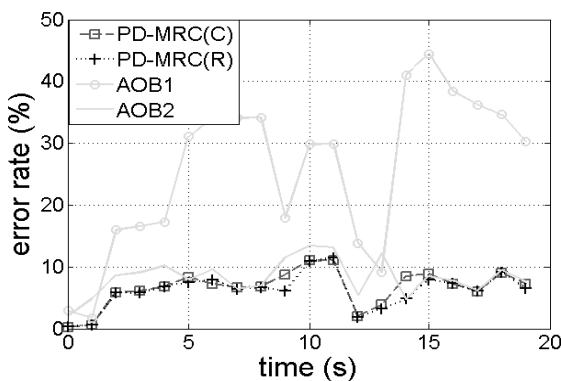
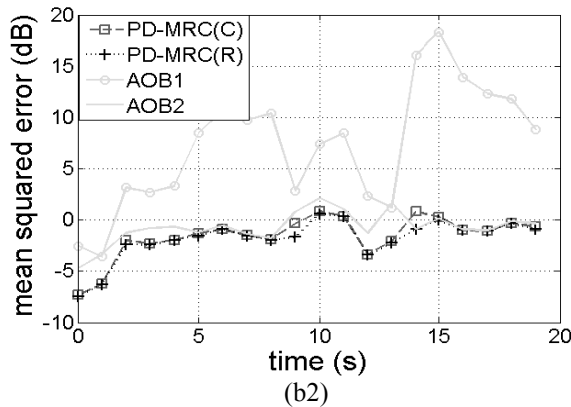
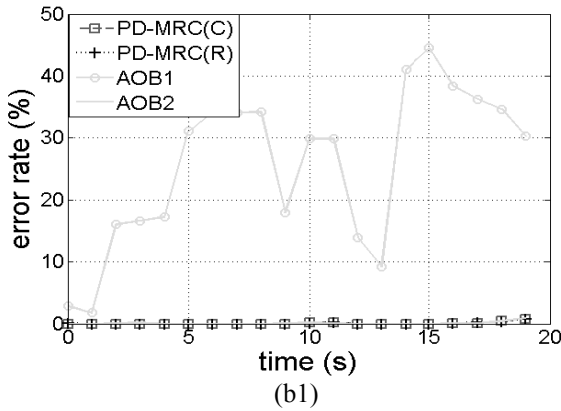
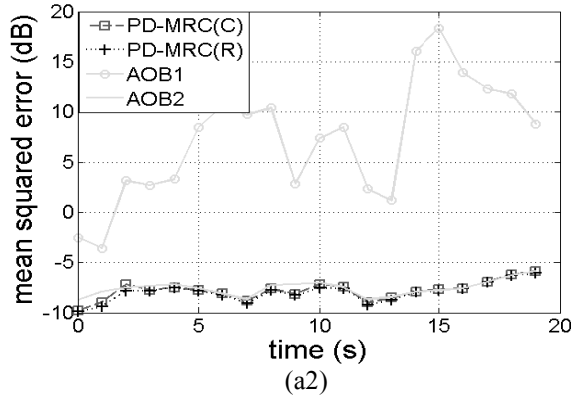


Figure 4: PD-MRC(C and R) MSE and BER results

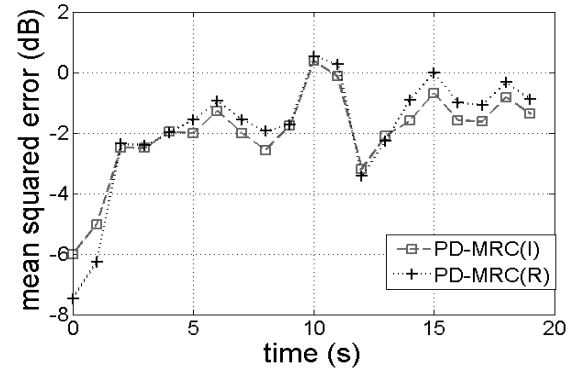


Figure 5: PD-MRC(I and R) MSE results

It was highlighted that the PD-MRC performance can be improved if instead of using the IP mean over all constellation a local IP mean is used (PD-MRC(I)). Both strategies were compared with a PD-MRC with coefficients computed using an adaptive RLS algorithm (PD-MRC(R)). Results reveal that: (i) PD-MRC(R) presents slightly better results than PD-MRC(C) however with much higher computational complexity; and (ii) PD-MRC(I) presents globally better results than PD-MRC(R). In PD-MRC(I) the moving average filter number of taps was selected empirically to be 4 for all slots. However it was observed that for optimal weights computation, different slots require a different number of taps. Future work will investigate the deep details of such behavior and a strategy to track the appropriate IP will be defined.

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