Acessing prospective early years' teachers' knowledge on fractions in order to design tasks for developing teachers' MKT*

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Fractions is perhaps one of the most complex and difficult topics pupils explore in the early years of schooling. Difficulties in learning this topic may have its genesis in the fact that fractions comprise a multifaceted construct (Kieren, 1995) or can be conceived as being grounded in the instructional approaches employed to teach fractions (Behr, Harel, Post & Lesh, 1993). Thus, students' limited understanding might be related to how their teachers understand and interpret fractions — it's thus related with teachers' knowledge and practice. Although there is a generalized agreement on teachers' role on/for students learning, most research on fractions focus on students, leaving aside teachers' role (and their knowledge on the topic). Thus, teachers' training has in certain respects been left behind. We still know little about how teachers' knowledge on fractions influences students' broader view of mathematics, and its connection and evolution within and along schooling. Aimed at conceptualize ways of improving teachers' knowledge, training and practices, it's of fundamental importance to access the areas of knowledge (here conceived as mathematical knowledge for teaching (MKT) (Ball, Thames & Phelps, 2008) in which (prospective) teachers are more deficitaries.

This paper is part of a broader research project aiming at obtaining a deeper understanding on teachers' MKT, allowing designing tasks for developing such knowledge. We aim at identify, discuss and reflect upon some mathematical critical situations identified in (prospective) teachers MKT in different mathematical topics (e.g., pictograms, subtraction), as well as concerning transversal abilities (e.g., problem solving, reasoning). Its ultimate aim is to conceptualize specific tasks for improving teachers training. Here we will focus on prospective teachers' subject matter knowledge on fractions and in particular, on the role of the whole and different kinds of whole. We will address the following research question: What critical aspects of MKT do prospective early years' teachers reveal about fractions and the role of the whole, and how can such mathematical critical situations be perceived to design tasks to develop teachers' MKT?

We opted by the MKT conceptualization because it attributes a very specific orientation to teacher's mathematical knowledge, placing emphasis on the mathematical reasoning they are immersed in during the development of tasks of teaching (Ball et al., 2008). By using

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this approach, we also aim to identify, discuss and reflect upon some mathematical critical situations, allowing to conceptualise optimal learning opportunities. Design of tasks is one of such approaches. The study in grounded in a set of tasks applied to 60 Portuguese prospective teachers focusing on their MKT on fractions and on their revealed understanding on the role of the whole and the different kinds of whole. Here we will focus only in one part of the first task: if we want to share equally 5 chocolate bars amongst 6 children's, what amount of chocolate would each children get?? We combine a qualitative methodology with an instrumental case study — and in the analysis we focus mainly on the situations where there was some evidence of gaps in knowledge, as these instances are seen as an opportunity to learn (Ribeiro & Carrillo, 2011). Such an approach intends accessing the critical aspects of (prospective) teachers' MKT, discussing not only ways of contributing to improve such MKT (were the conceptualization of tasks specifically for teachers training has a major role), but also the content of the MKT domain's and equate a theoretical refinement of the MKT conceptualization in itself.

While answering the question, the prospective teachers' revealed knowledge that was at the same level as some of the students they supposedly will be teaching. The majority of them revealed subject matter knowledge, and ways of representing the solution(s), similar to pupils in first grade —including the chosen representations. Examples of such answers are: "each child will get 5 pieces"; "each child gets 2 big squares and one small square (2/3 of 4 chocolate bars and 1/6 of other)"; "each child will get exactly 5/6 of the total amount of chocolate or 5/6 of each chocolate bar". These revealed difficulties are also our fault, as teacher educators and if such difficulties are not properly overcome, it will be impossible for teachers to prepare and implement tasks allowing their pupils to develop a broader understanding specifically, but not exclusively, on the interpretations and representations of fractions. Teacher education must start paying more attention to, and incorporate in training, the results from research (if it was already effectively incorporated, students' understanding on fractions would not be as it is). We think focusing on the specialized mathematical knowledge will be most usefull, as we assume it can be effectively taught (Hill & Ball, 2004). We will present, discuss and reflect upon the results from the prospective teachers' knowledge and the potentialities of accessing such mathematical critical situations and reasoning, for conceptualizing a specific kind of tasks for developing teachers MKT, in particular on fractions. From such results we will discuss also some aspects of the MKT conceptualization, allowing proposing some refinements.

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