Appendix V – Common sets of eigenvectors, eigenvalues and c coefficients

Examples of the four types of sets of eigenvalues, eigenvectors and c coefficients commonly found for the (8,8) dimensional demographic matrix of the stage/size structured model of a biphasic life-cycle.

1. Real and equal

$$c\lambda^t \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ \cdots \\ \\ \\ w_8 \end{bmatrix} = -1.3224 \times (-0.4918^{\,t}) \times \begin{bmatrix} 0.7067 \\ -0.0006 \\ 0.0004 \\ -0.0002 \\ 0.7065 \\ -0.0006 \\ 0.0003 \\ -0.0002 \end{bmatrix} \tag{1a}$$

$$= 2\,\mathrm{Re}(c)\left|\lambda\right|^t \cos(\theta t) \begin{bmatrix} \mathrm{Re}(w_1) \\ \mathrm{Re}(w_2) \\ \cdots \\ \mathrm{Re}(w_8) \end{bmatrix} \tag{1b}$$

$$= 2\left|\gamma\right| \cos(\rho) \left|\lambda^t\right| \cos(\theta t) \begin{bmatrix} \left|\sigma_1\right|.\cos(\omega_1) \\ \left|\sigma_2\right|.\cos(\omega_2) \\ \cdots \\ \left|\sigma_8\right|.\cos(\omega_8) \end{bmatrix} \tag{1c}$$

2. Real and symmetric

$$c\lambda^t\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ \cdots \\ \\ w_8 \end{bmatrix} = -0.0915 \times 1.4594^{\,t} \times \begin{bmatrix} -0.7511 \\ 0.0015 \\ 0.0004 \\ 0.0001 \\ 0.6602 \\ -0.0017 \\ -0.0004 \\ -0.0001 \end{bmatrix} \tag{2a}$$

$$= 2|\gamma|\cos(\rho)\left|\lambda^t\right|\cos(\theta t)\begin{bmatrix} |\sigma_1|.\cos(\omega_1) \\ \cdots \\ \\ |\sigma_5|.\cos(\pi+\omega_1) \\ \cdots \\ |\sigma_8|.\cos(\pi+\omega_4) \end{bmatrix} \tag{2b}$$

3. Complex conjugate and equal

$$(-0.005-0.336i)(-0.029+0.256i)^t \times \begin{bmatrix} 0.650+0.116i \\ -0.001-0.002i \\ -0.002+0.002i \\ 0.002+0.001i \\ 0.751+0.000i \\ -0.001-0.002i \\ -0.002+0.002i \\ 0.003+0.001i \end{bmatrix} + \tag{3}$$

$$+(-0.005+0.336i)(-0.029-0.256i)^t \times \begin{bmatrix} 0.650-0.116i \\ -0.001+0.002i \\ -0.002-0.002i \\ 0.002-0.001i \\ 0.751-0.000i \\ -0.001+0.002i \\ -0.002-0.002i \\ 0.003-0.001i \end{bmatrix}$$

4. Complex conjugate and symmetric

$$(-0.004-0.005i)(0.227+0.691i)^{t}\times\begin{bmatrix}-0.707+0.007i\\-0.002-0.010i\\0.004-0.002i\\0.001+0.002i\\0.707-0.007i\\0.002+0.010i\\-0.004+0.002i\\-0.001-0.002i\end{bmatrix}+ \tag{4}$$

$$+(-0.004+0.005i)(0.227-0.691i)^{t}\times\begin{bmatrix}-0.707-0.007i\\-0.002+0.010i\\0.004+0.002i\\0.001-0.002i\\0.707+0.007i\\0.002-0.010i\\-0.004-0.002i\\-0.001+0.002i\end{bmatrix}$$