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OPTIMAL DYNAMIC CONTROL OF LAMINATED ADAPTIVE STRUCTURES USING A HIGHER ORDER MODEL AND A GENETIC ALGORITHM

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Abstract. *This paper deals with a finite element formulation based on the classical laminated plate theory, for active control of thin plate laminated structures with integrated piezoelectric layers, acting as sensors and actuators. The control is initialized through a previous optimization of the core of the laminated structure, in order to minimize the vibration amplitude. Also the optimization of the patches position is performed to maximize the piezoelectric actuator efficiency. The genetic algorithm is used for these purposes.*
The finite element model is a single layer triangular plate/shell element with 24 degrees of freedom for the generalized displacements, and one electrical potential degree of freedom for each piezoelectric element layer, which can be surface bonded or embedded on the laminate.
To achieve a mechanism of active control of the structure dynamic response, a feedback control algorithm is used, coupling the sensor and active piezoelectric layers. To calculate the dynamic response of the laminated structures the Newmark method is considered. The model is applied in the solution of an illustrative case and the results are presented and discussed.

1. INTRODUCTION

Advanced reinforced composite structures incorporating piezoelectric sensors and actuators are increasingly becoming important due to the development of adaptive structures. These structures offer potential benefits in a wide range of engineering applications such as vibration and noise suppression, shape control and precision positioning.

In the analysis of adaptive structures integrating piezoelectric material, one of the pioneering works is due to Allik and Hughes[1] who developed a solid finite element for vibration analysis. Vibration control of composite beams with embedded or surface bonded piezoelectric material had been studied by Crawley and de Luis[2]. Tzou and Tseng[3] presented a finite element formulation for plates and shells containing integrated distributed piezoelectric sensors and actuators applied to control advanced structures. Chen et al.[4] developed a finite element based on the first order displacement field for dynamic analysis of plates. Active control is obtained through actuators potential, which is given by an amplified signal of the sensors potential.

Using a higher order shear deformation theory Samanta et al.[5] developed an eight-nodded finite element for the active vibration control of laminated plates with piezoelectric layers acting as distributed sensors and actuators. The active control capability is studied using a simple algorithm with negative velocity feedback. Lam et al.[6] and Moita et al.[7] developed finite element models based on the classical laminated theory for the active control of composite plates containing piezoelectric sensors and actuators using the Newmark method, Bathe[8], to calculate the dynamic response of laminated structures. Reviews of the modeling and the design of composite structures with adaptive capabilities are given in Franco Correia et al.[9] and Benjeddou[10].

Batra and Liang[11] used a three-dimensional linear theory of elasticity to find the optimal location of an actuator on a simple-supported rectangular laminated plate with embedded PZT layers. The optimal design is obtained by fixing the applied voltage and the size of the actuator and moving it around in order to find the maximum out-of-plane displacement. Liang et al.[12] proposed a model for the optimization of the induced-strain actuator location and configuration for active vibration control. Correia et al.[13] presented refined finite element models based on higher order displacement fields applied to the optimal design of laminated composite plates with embedded or surface bonded piezoelectric actuators and sensors.

Most of past work in the area of adaptive structures has focused on the analysis of structures with sensors and actuators, and the corresponding associate control system. Very few works have focused on the development of methodologies for the optimization of laminated structures incorporating sensors and actuators, to enhance their performance. A review of the current work in design methodologies and applications of formal optimization methodologies for adaptive structures has been carried out by Padula[14] and Frecker[15].

The finite element used in the present work is a flat three-nodded triangular element with 24 mechanical degrees of freedom and one electric degree of freedom per piezoelectric layer of the finite element and is based on the third-order shear deformation laminate theory of Reddy[16]. An integrated control is considered in order to achieve an active damping mechanism, with the amplified electrical potential of the sensors being used as electric potential input to the actuators through a negative velocity feedback control. The governing equations are solved by the Newmark method, which is a direct method for time integration. Priory to the dynamic control, to minimize the free or forced vibration amplitude through active damping, an optimal lamination sequence of the structure core, as well as an optimal placement of piezoelectric actuators patches are carried out using structural optimization methodologies based on genetic[17,18,19] and gradient-based algorithms[20,21].

2. DISPLACEMENTS AND STRAIN FIELDS.

The assumed displacement field, for the numerical higher order finite element model, is a third order expansion in the thickness coordinate for the in-plane displacements and a constant transverse displacement, conjugated with the condition that the transverse shear stresses vanish on the top and bottom faces, which is equivalent to the requirement that the corresponding strains be zero on these surfaces[16].

$$u(x,y,z)=u_0(x,y)-z\,\theta_y(x,y)+z^3\,c_1\left[\theta_y(x,y)-\frac{\partial w_0}{\partial x}\right]$$

$$v(x,y,z)=v_0(x,y)+z\,\theta_x(x,y)+z^3\,c_1\left[-\theta_x(x,y)-\frac{\partial w_0}{\partial y}\right] \tag{1}$$

$$w(x,y,z)=w_0(x,y)$$

where u_0, v_0, w_0 are displacements of a generic point in the middle plane of the laminate referred to the local axes - x,y,z directions, θ_x, θ_y are the rotations of the normal to the middle plane, about the x axis (clockwise) and y axis (anticlockwise), $\partial w_0/\partial x$, $\partial w_0/\partial y$ are the slopes of the tangents of the deformed mid-surface in x,y directions, θ_z is the rotation about the local z axis, which does not enter in the formulation in the local coordinate system, and $c_1 = 4/3h^2$.

The strains components associated with the displacements in equation (1) are conveniently represented as:

$$\begin{Bmatrix} \bar{\varepsilon}_m + z\ \bar{\varepsilon}_b + z^3\ \bar{\varepsilon}_b^* \\ \bar{\varepsilon}_s + z^2\ \bar{\varepsilon}_s^* \end{Bmatrix} = \mathcal{Z}\ \bar{\varepsilon} \tag{2}$$

where $\mathcal{Z}$ is an appropriate matrix containing powers of z.

The components of the vector $\bar{\varepsilon}_0$ are given by:

$$\bar{\varepsilon}_m = \left\{\varepsilon_{x0}\ \ \varepsilon_{x0}\ \ \varepsilon_{xy0}\right\}^T = \left\{\frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{dv_0}{\partial x}+\frac{\partial u_0}{\partial y}\right\}^T$$

$$\bar{\varepsilon}_b = \left\{k_x, k_y, k_{xy}\right\}^T = \left\{-\frac{\partial \theta_y}{\partial x}, \frac{\partial \theta_x}{\partial y}, \frac{\partial \theta_x}{\partial x}-\frac{\partial \theta_y}{\partial y}\right\}^T$$

$$\bar{\varepsilon}_b^* = \left\{k_x^*, k_y^*, k_{xy}^*\right\}^T = c_1\left\{\frac{\partial \theta_y}{\partial x}-\frac{\partial^2 w_0}{\partial x^2}, -\frac{\partial \theta_x}{\partial y}-\frac{\partial^2 w_0}{\partial y^2}, -\frac{\partial \theta_x}{\partial x}+\frac{\partial \theta_y}{\partial y}-2\frac{\partial^2 w_0}{\partial x \partial y}\right\}^T \tag{3}$$

$$\bar{\varepsilon}_s = \left\{\phi_x, \phi_y\right\}^T = \left\{-\theta_y + \frac{\partial w_0}{\partial x}, \theta_x + \frac{\partial w_0}{\partial y}\right\}^T$$

$$\bar{\boldsymbol{\varepsilon}}_s^* = \left\{\psi_x, \psi_y\right\}^T = \left\{ c_2\left(\theta_y - \frac{\partial w_0}{\partial x}\right), c_2\left(-\theta_x - \frac{\partial w_0}{\partial y}\right)\right\}^T$$

where $c_2 = 4/h^2$

5. PIEZOELECTRIC LAMINATES. CONSTITUTIVE EQUATIONS.

Assuming that a piezoelectric composite plate consists of several layers, including the piezoelectric layers, the constitutive equation for an orthotropic layer of the laminate substrate, is

$$\bar{\boldsymbol{\sigma}} = \overline{\mathbf{Q}}\,\bar{\boldsymbol{\varepsilon}} \tag{4}$$

and the constitutive equations of a deformable piezoelectric material, coupling the elastic and the electric fields are given by, Tiersten[22]

$$\bar{\boldsymbol{\sigma}} = \overline{\mathbf{Q}}\,\bar{\boldsymbol{\varepsilon}} - \bar{\mathbf{e}}\,\overline{\mathbf{E}} \tag{5}$$

$$\overline{\mathbf{D}} = \bar{\mathbf{e}}^{\mathbf{T}}\,\bar{\boldsymbol{\varepsilon}} + \bar{\mathbf{p}}\,\overline{\mathbf{E}} \tag{6}$$

where $\bar{\boldsymbol{\sigma}} = \{\sigma_x \quad \sigma_y \quad \sigma_{xy} \quad \tau_{xz} \quad \tau_{yz}\}^T$ and $\bar{\boldsymbol{\varepsilon}} = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T$ are the elastic stress and the elastic strain vectors, $\overline{Q}$ the elastic constitutive matrix, $\bar{e}$ the piezoelectric stress coefficients matrix, $\overline{E} = [\overline{E}_x \; \overline{E}_y \; \overline{E}_z]^T$ the electric field vector, $\overline{D} = [\overline{D}_x \; \overline{D}_y \; \overline{D}_z]^T$ the electric displacement vector and $\bar{p}$ the dielectric matrix, in the element local system (x,y,z) of the laminate. The $\overline{Q}_{ij}, \bar{e}_{ij}, \bar{p}_{ij}$ are functions of ply angle α for the k^{th} layer, and are given in Reddy[16].

The electric field vector is the negative gradient of the electric potential ϕ, which is assumed to be applied and varying linearly in the thickness t_k direction, i.e.

$$\overline{\mathbf{E}} = -\nabla\phi$$

$$\overline{\mathbf{E}} = \{0 \quad 0 \quad E_z\}^T \tag{7}$$

where

$$E_z = -\phi / t_k \tag{8}$$

Thus, we can define the strain vector for electro elasticity as follows:

$$\hat{\varepsilon} = \begin{Bmatrix} \bar{\boldsymbol{\varepsilon}} \\ -\overline{E} \end{Bmatrix} \tag{9}$$

6. FINITE ELEMENT FORMULATION.

The non-conforming higher order triangular finite element model has three nodes and eight degrees of freedom per node, the displacements u_{0_i}, v_{0_i}, w_{0_i}, the slopes $\partial w_0/\partial x$, $\partial w_0/\partial y$, and rotations $\theta_{xi}, \theta_{yi}, \theta_{zi}$. By assuming that c_1=0 and that sections before deformation remain plane after deformation and perpendicular to the middle surface, i.e. by neglecting the transverse shear deformation, a Kirchhoff laminated finite element model with 6 degrees of freedom per node (CPT) is also easily obtained. The introduction of fictitious stiffness coefficients $K_{\theta z}$, corresponding to rotations θ_z, to eliminate the problem of a singular stiffness matrix, for which the elements are coplanar or near coplanar, is required. The element local displacements, slopes and rotations are expressed in terms of nodal variables through shape functions $\mathbf{N}_i$ given in terms of area co-ordinates L_i, Zienkiewics[23]. The displacement field can be represented in matrix form as:

$$\mathbf{u} = \mathbf{Z}\left(\sum_{i=1}^{3} \mathbf{N}_i\, \mathbf{d}_i\right) = \mathbf{Z}\,\mathbf{N}\,\mathbf{a} \tag{10}$$

$$\mathbf{d} = \sum_{i=1}^{3} \mathbf{N}_i\, \mathbf{d}_i = \mathbf{N}\,\mathbf{a} \tag{11}$$

with

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z^3c_1 & 0 & -z+z^3c_1 & 0 \\ 0 & 1 & 0 & -z^3c_1 & 0 & +z-z^3c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$\mathbf{d}_i = \left\{ u_0 \;\; v_0 \;\; w_0 \;\; -\frac{\partial w}{\partial y} \;\; \frac{\partial w}{\partial x} \;\; \theta_x \;\; \theta_y \;\; \theta_z \right\}_i \tag{13}$$

and the strain field as follows:

$$\begin{aligned} \bar{\varepsilon}_m &= \mathbf{B}^m\,\mathbf{a} \\ \bar{\varepsilon}_b &= \mathbf{B}^b\,\mathbf{a} \\ \bar{\varepsilon}_b^* &= \mathbf{B}^{*b}\,\mathbf{a} \\ \bar{\varepsilon}_s &= \mathbf{B}^s\,\mathbf{a} \\ \bar{\varepsilon}_s^* &= \mathbf{B}^{*s}\,\mathbf{a} \end{aligned} \tag{14}$$

The electric field is given by:

$$\mathbf{E} = -\mathbf{B}^{\phi}\,\phi \tag{15}$$

where B^{ϕ} is the electric field – potential matrix, given by

$$\mathbf{B}^{\phi} = \begin{bmatrix} 1/t_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/t_{N^p} \end{bmatrix} \tag{16}$$

The dynamic equations of a laminated composite plate can be derived from the Hamilton's principle, which is given as follows:

$$\int_{t_1}^{t_2}\left\{\sum_{K=1}^{N}\left(\int_{A^e}\int_{h_{k-1}}^{h_k}\delta\hat{\boldsymbol{\varepsilon}}^T\,\hat{C}_k\,\hat{\boldsymbol{\varepsilon}}^L\,dz\,dA^e-\int_{A^e}\int_{h_{k-1}}^{h_k}\delta\dot{\mathbf{u}}^{\mathbf{T}}\,{}^t\rho_k\,\dot{\mathbf{u}}\,dz\,dA^e\right)-\left(\int_V\mathbf{f}\,\delta\mathbf{u}\,dV+\int_S\mathbf{T}\,\delta\mathbf{u}\,dS+\sum_i\mathbf{F}_i\,\delta\mathbf{u}_i+\int_S\mathbf{Q}\;\delta\phi\,dS\right)\right\}dt=0 \tag{17}$$

Entering the equations (11) to (15) into equation (16), we have

$$\int_{t_1}^{t_2}\left[\sum_{K=1}^{N}\left(\int_A\int_{h_{k-1}}^{h_k}\delta\begin{Bmatrix}a\\ \phi\end{Bmatrix}^T\begin{bmatrix}\mathbf{B}^{mec} & 0\\ 0 & \mathbf{B}^{\phi}\end{bmatrix}^T\begin{bmatrix}\overline{\mathbf{Q}} & \overline{\mathbf{e}}\\ \overline{\mathbf{e}}^T & -\overline{\mathbf{p}}\end{bmatrix}\begin{bmatrix}\mathbf{B}^{mec} & 0\\ 0 & \mathbf{B}^{\phi}\end{bmatrix}\begin{Bmatrix}a\\ \phi\end{Bmatrix}dz\,dA\;-\int_A\mathbf{N}^T\;(\sum_{k=1}^{n}\rho_k\int_{h_{k-1}}^{h_k}\mathbf{Z}^T\,dz\,)\,\mathbf{N}\;\;dA\right)+\int_V\delta\{a\}^T\,\mathbf{N}^T\,\mathbf{f}\;\,dV+\int_S\delta\{a\}^T\,\mathbf{N}^T\,\mathbf{t}\;\,dS+\delta\{a\}^T\;\mathbf{F}_c\;+\int_S\mathbf{Q}\;\;\delta\phi\,dS\right]dt=0 \tag{18}$$

To the first and second terms of first member of Eq. (18), corresponds the element stiffness and mass matrices, respectively, which are defined by:

$$\mathbf{K}=\begin{bmatrix}K_{uu} & K_{u\phi}\\ K_{\phi u} & K_{\phi\phi}\end{bmatrix}=$$

$$\int_A\begin{bmatrix}\mathbf{B}^{m^T} & \mathbf{B}^{b^T} & \mathbf{B}^{*b^T} & \mathbf{B}^{s^T} & \mathbf{B}^{*s^T} & 0\\ 0 & 0 & 0 & 0 & 0 & \mathbf{B}^{\phi}\end{bmatrix}\begin{bmatrix}\mathbf{A} & \mathbf{B} & \mathbf{D} & \mathbf{0} & \mathbf{0} & \mathbf{A}'\\ \mathbf{B} & \mathbf{C} & \mathbf{E} & \mathbf{0} & \mathbf{0} & \mathbf{B}'\\ \mathbf{D} & \mathbf{E} & \mathbf{G} & \mathbf{0} & \mathbf{0} & \mathbf{D}'\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A_s} & \mathbf{C_s} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{C_s} & \mathbf{E_s} & \mathbf{0}\\ \mathbf{A}' & \mathbf{B}' & \mathbf{D}' & \mathbf{0} & \mathbf{0} & \mathbf{A}''\end{bmatrix}\begin{bmatrix}\mathbf{B}^m & 0\\ \mathbf{B}^b & 0\\ \mathbf{B}^{*b} & 0\\ \mathbf{B}^s & 0\\ \mathbf{B}^{*s} & 0\\ 0 & \mathbf{B}^{\phi}\end{bmatrix}dA \tag{19}$$

$$\mathbf{M}=\int_A\mathbf{N}^T\;(\sum_{k=1}^{n}\rho_k\int_{h_{k-1}}^{h_k}\mathbf{Z}^T\,\mathbf{Z}\;dz\,)\,\mathbf{N}\;\;dA \tag{20}$$

in which the elements of the mechanical, piezoelectric and dielectric stiffnesses are given by:

$$\mathbf{A} = \sum_{K=1}^{N} \left[\overline{Q}_{ij} H_1\right]$$

$$\left[\mathbf{B} \quad \mathbf{D}\right] = \sum_{K=1}^{N} \left[\overline{Q}_{ij} H_2 \quad \overline{Q}_{ij} H_4\right]$$

$$\begin{bmatrix} \mathbf{C} & \mathbf{E} \\ \mathbf{E} & \mathbf{G} \end{bmatrix} = \sum_{K=1}^{N} \begin{bmatrix} \overline{Q}_{ij} H_3 & \overline{Q}_{ij} H_5 \\ \overline{Q}_{ij} H_5 & \overline{Q}_{ij} H_7 \end{bmatrix} \tag{21}$$

$$\begin{bmatrix} \mathbf{A_s} & \mathbf{C_s} \\ \mathbf{C_s} & \mathbf{E_s} \end{bmatrix} = \sum_{K=1}^{N} \begin{bmatrix} \overline{Q}_{lm} H_1 & \overline{Q}_{lm} H_3 \\ \overline{Q}_{lm} H_3 & \overline{Q}_{lm} H_5 \end{bmatrix}$$

$$\mathbf{A}' = \sum_{k=1}^{N^P} \overline{e}_{3j} \; H_1$$

$$\mathbf{B}' = \sum_{k=1}^{N^P} \overline{e}_{3j} \; H_2 \tag{22}$$

$$\mathbf{D}' = \sum_{k=1}^{N^P} \overline{e}_{3j} \; H_4$$

$$\mathbf{A}'' = \sum_{k=1}^{N^P} \overline{p}_{33} \, H_1 \tag{23}$$

$$H_n = \left(z_k^n - z_{k-1}^n\right)/n \tag{24}$$

where i, j = 1, 2, 6 and l, m = 4,5 ; n =1, 2, 3, 4, 5, 7, N is the number of layers and N^p is the number of layers or patches with piezoelectric material.

To the third term of Eq. (18), correspond the applied electric charge vector F^{ele}, and the external mechanical force vector, which is defined by:

$$\mathbf{F}_{ext}^{mec} = \int_V \mathbf{N}^\mathbf{T} \, \mathbf{f} \, dV + \int_S \mathbf{N}^\mathbf{T} \mathbf{t} \; dS + \mathbf{F}_c \tag{25}$$

where f, t, F_c are body, surface, and concentrated force vectors.

The element stiffness and mass matrices as well as external load vector are initially computed in the local coordinate system attached to the element. To solve general structures, local - global transformations are needed, Zienckiewicz[23]. After these transformations the assembled system of equations is:

$$\begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \phi \end{Bmatrix} = \begin{Bmatrix} F_{ext}^{mec}(t) \\ F^{ele}(t) \end{Bmatrix} \tag{26}$$

Assuming that piezoelectric sensors as well as actuators are bonded or embedded in the structure, the electric potential vector is subdivided in a sensor component $\phi^{(S)}$ and an actuator component $\phi^{(A)}$.

The external applied electric charge at the sensors is zero. Separating the actuator and sensor components, the system of Eq. (24) take the following form:

$$[M_{uu}]\{\ddot{q}\} + [K_{uu}]\{q\} + \left[K_{u\phi}^{(S)}\right]\left\{\phi^{(S)}\right\} = \left\{ F_{ext}^{mec}(t) - \left[K_{u\phi}^{(A)}\right] \left\{\phi^{(A)}\right\}\right\} \tag{27}$$

$$\left[K_{\phi u}^{(A)}\right]\{q\} + \left[K_{\phi\phi}^{(A)}\right] \left\{\phi^{(A)}\right\} = \left\{F^{ele}(t)\right\} \tag{28}$$

$$\left[K_{\phi u}^{(S)}\right]\{q\} + \left[K_{\phi\phi}^{(S)}\right] \left\{\phi^{(S)}\right\} = \{0\} \tag{29}$$

From the last equation, the induced sensory electric potentials $\phi^{(S)}$ are obtained as follows:

$$\left\{\phi^{(S)}\right\} = -\left[K_{\phi\phi}^{(S)}\right]^{-1} \left[K_{\phi u}^{(S)}\right]\{q\} \tag{30}$$

Thus the equation (27) takes the form:

$$[M_{uu}]\{\ddot{q}\} + \left([K_{uu}] - \left[K_{u\phi}^{(S)}\right]\left[K_{\phi\phi}^{(S)}\right]^{-1}\left[K_{\phi u}^{(S)}\right]\right)\{q\} = \left\{F_{ext}^{mec}(t) - K_{u\phi}^{(A)}\,\phi^{(A)}\right\} \tag{31}$$

6.1 Dynamic analysis

The sensor output can be obtained as follows, Reddy[24]. The charge output of each sensor, with poling in the z direction, can be expressed in terms of spatial integration of the electric displacement over its surface, taking into account that the converse piezoelectric effect is negligible. Thus we have:

$$Q^{(S)}(t) = \frac{1}{2}\left[\int_{A(z=z_{k-1})} D_z(t)\,dA + \int_{A(z=z_k)} D_z(t)\,dA\right] \tag{32}$$

From Eq. (5), the last equation can be written as follows

$$Q^{(S)}(t) = \int_A \int_{h_{k-1}}^{h_k} \frac{1}{t_k}\,\bar{e}^T\,\bar{\varepsilon}\;dz\,dA \tag{33}$$

or in the discretized form

$$Q^{(S)}(t) = (\int_A \int_{h_{k-1}}^{h_k} B^{\phi^T} \bar{e}^T B^{mec} \, dz \, dA)\{q\} = \left[K_{\phi u}^{(S)}\right]\{q\} \tag{34}$$

The current on the surface of the sensor is given by

$$I(t) = \frac{dQ^{(S)}}{dt} \tag{35}$$

When the piezoelectric sensor is used as strain rate sensor, the current can be converted into the open circuit sensor voltage output $\phi^{(S)}$ as

$$\phi^{(S)} = G_c \frac{dQ^{(S)}}{dt} \tag{36}$$

where G_c is the constant gain of the amplifier, which transforms the sensor current to voltage.

The sensor output voltage can be feed back through an amplifier to the actuator with a change of polarity. Thus, we have for the actuator voltage

$$\phi^{(A)} = -G_i G_c \frac{dQ^{(S)}}{dt} \tag{37}$$

where G_i is the gain of the amplifier to provide feedback control.

The actuator voltage written in the discretized form, is then given by

$$\phi^{(A)} = -G_i G_c \left[K_{\phi u}^{(S)}\right]\{\dot{q}\} \tag{38}$$

Use of Eq. (38) into Eq. (31) introduces an equivalent negative velocity feedback, and the motion equations become:

$$[M_{uu}]\{\ddot{q}\} - G_i G_c \left[K_{u\phi}^{(A)}\right]\left[K_{\phi u}^{(S)}\right]\{\dot{q}\} + \left(\left[K_{uu}\right] - \left[K_{u\phi}^{(S)}\right]\left[K_{\phi\phi}^{(S)}\right]^{-1}\left[K_{\phi u}^{(S)}\right]\right)\{q\} = \left\{F_{ext}^{mec}(t)\right\} \tag{39}$$

Considering Rayleigh type damping, we can write:

$$[M_{uu}]\{\ddot{q}\} + (C_R + C_A)\{\dot{q}\} + \left(\left[K_{uu}\right] - \left[K_{u\phi}^{(S)}\right]\left[K_{\phi\phi}^{(S)}\right]^{-1}\left[K_{\phi u}^{(S)}\right]\right)\{q\} = \left\{F_{ext}^{mec}(t)\right\} \tag{40}$$

with

$$C_R = \alpha M_{uu} + \beta K_{uu} \tag{41}$$

where α and β are Rayleigh's coefficients, to account for inherent structural damping, and the damping effect due to the active control is given by:

$$C_A = -G_i G_c \left[K_{u\phi}^{(A)}\right]\left[K_{\phi u}^{(S)}\right] \tag{42}$$

The solution of Eq. (40) is carried out using Newmark direct method of time integration, Bathe[8].

7. OPTIMAL DESIGN

A general structural optimization problem can be stated as:

$$\min \{\varphi(b)\} \qquad \textit{subject to}: \; b_i^l \le b_i \le b_i^u \qquad i = 1,\dots,ndv$$
$$\Psi_j(q,b) \le 0 \qquad j = 1,\dots,m \qquad (43)$$

where $\varphi(b)$ is the objective function, **b** is the vector of design variables b_i, $\Psi_j(q,b)$ are the *m* inequality behavioral constraint equations, b_i^l and b_i^u are respectively, the lower and upper limits of the design variables and *ndv* is the total number of design variables.

If the objective function and/or the constraint equations are continuous functions of the design variables, mathematical programming techniques[20,21] requiring only the computation of $\varphi(b)$, $\Psi_j(q,b)$ and their gradients, provide a general, flexible and efficient formulation for engineering design problems.

For discrete variable structural problems, a variety of methods including Genetic Algorithms[18,19] can be used. Genetic Algorithms (GAs) are directed random search techniques used to look for parameters that provide a good solution to a problem.

The inspiration for GAs came from nature and survival of the fittest. In a population, each individual has a set of characteristics that determine how well suited it is to the environment. Survival of the fittest implies that the 'fitter' individuals are more likely to survive and have a greater chance of passing their 'good' features to the next generation. In sexual reproduction, if the best features of each parent are inherited by their offspring, a new individual will be created that should have an improved probability of survival. This is the process of evolution.

In nature the 'blueprint' of individuals is contained within their DNA. The DNA can be thought of as a string of genes, with each gene or combination of genes representing a particular feature. In GA terms, a candidate solution is often referred to as a chromosome, which is a sequence of encoded numbers. This is commonly referred to as a bit string if the numbers are binary encoded. The process involved in GA optimisation works as follows:

1. Randomly generate an initial population of potential solutions.
2. Evaluate the suitability or 'fitness' of each solution.
3. Select two solutions based in favour of fitness.
4. Crossover the solutions at a random point on the string to produce two new solutions.
5. Mutate the new solutions based on a mutation probability.

Go to 2.

Selection is the procedure for choosing individuals (parents) on which to perform crossover in order to create new solutions. In tournament selection several individuals are chosen at random and the fittest becomes one of the parents. Along with mutation, crossover is the operator that creates new candidate solutions. A position is randomly chosen on the string and the two parents are 'crossed over' at this point to create two new solutions. A crossover probability (Pc) is often given which enables a chance that the parents descend into the next generation unchanged. After crossover, each bit of the string has the potential to mutate, based on a mutation probability (Pm). In binary encoding mutation involves the flipping of a bit from 0 to 1 or vice versa.

The GA used in this work was developed by Carroll[17]. This method uses a non-conventional approach to optimise, the Micro-GA[18]. Instead of using mutation to refresh the genetic information of the population, a new population is randomly generated when 95% of all the individuals bits are equal to those of the fittest. The fittest individual is included in the new population.

The main advantage of this method, in comparison with gradient-based methods, is the ability to overcome the premature convergence towards a local optimum. By other hand, a high number of objective function evaluations is usually required, which is especially relevant when the objective function evaluation is computationally expensive.

In the next section an illustrative example of the forced response with active feedback control of a composite adaptive laminated plate is presented, where firstly, it is designed for maximum stiffness being the design variables the orientation angles of the reinforcement fibers in the orthotropic layers. Secondly, the optimal location of the piezoelectric actuator discrete patches is found in order to achieve maximum piezoelectric actuator performance.

If we consider that because of manufacturing constraints, normally the fiber reinforcement directions assume discrete values in a limited set of different possible angles we can regard also this problem as a discrete design variable problem. In this case we solve this problem by using the genetic algorithm. The objective here is to maximize the elastic strain energy of the laminated composite or to minimize the elastic displacements in specific locations of the structure, being the design variables the orientation angles of the orthotropic axes in each layer. For the optimal location of the piezoelectric patches we consider that the patches can only assume the positions corresponding to the finite element mesh discretization. This is in essence a discrete variable optimization problem, which is solved by using the genetic method. In this case the objective function can be for example the maximization of the displacement in a specific point of the structure and the design variables are the discrete locations of the piezoelectric patches.

As is pointed out before, the gradient-based method is also applied for the fiber angles optimization, if we consider these variables as continuous variables. This unconstrained problem is solved by using a feasible directions non-linear interior point algorithm, developed by Herskovits[21].

8. NUMERICAL APPLICATIONS

8.1 Forced response with active feedback control of a simply-supported square plate, in free vibration and forced vibration under sinusoidal loading.

A simply-supported square ($a \times a$) laminated plate, having the initial lamination sequence of $[0^{o}/90^{o}/0^{o}]$, integrating piezoelectric actuator and sensor layers or patches made of PZT, bonded on upper and lower surfaces, is considered. The material properties of the substrate layers are $E_1 = 172.5$ GPa, $E_2 = 6.9$ GPa, $G_{12} = G_{13} = G_{23} = 3.45$ GPa, $\nu_{12} = 0.25$, ρ=1600 kg/m^3. The material and piezoelectric properties of PZT are $E_1 = E_2 = 63$ GPa, $\nu_{12} = 0.30$, $G_{12} = G_{13} = G_{23} = 24$GPa, ρ=7600 kg/m^3, $e_{31} = e_{32} = 22.86\ C/m^2$, $p_{33} = 1.5 \times 10^{-8}$ F/m. The side dimension is a = 0.18 m and the thickness of the substrate layers and PZT are 0.002 m and 0.0001 m, respectively. The plate is modeled by a (6x6) element mesh (72 triangular elements). First we search for the optimal core lamination sequence, which leads to the maximum fundamental natural frequency of the plate, by using the genetic algorithm. For the genetic problem, the design variables are chosen from a discrete set of possible ply angles defined as: $S_{dv} = \{ 0^o, \pm 15^o, \pm 30^o, \pm 45^o, \pm 60^o, \pm 75^o, 90^o \}$. The optimal lamination sequence is found to be $[45^o/-45^o/45^o]$. The same optimal design is obtained using gradient optimization.

The amplitudes and the natural frequencies of the plate in free vibrations, enabled by an applied uniform distributed transverse load *q = 10000 N/m²* and then suddenly removed, are shown in Figure 1 for both the initial and final lamination sequences. As expected, for the final design, there is a decreasing in the central amplitude and the fundamental natural frequency increases.

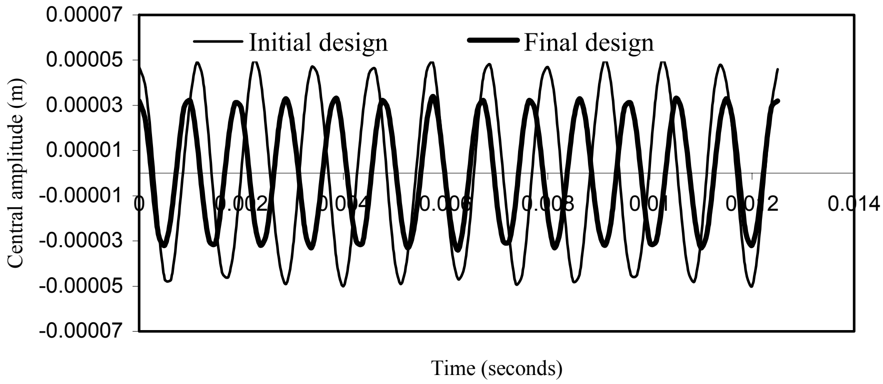

Figure 1. Central amplitudes for the initial and final lamination sequences.

Next we pretend to investigate the optimal position of the piezoelectric actuators patches, in order to maximize the control of the plate, in this case measured by the amplitude of the deflection in the center of the plate, and compare it with the control obtained by using an entire piezoelectric layer. In this application, 4 actuators patches covering 8 triangular elements are introduced. The plate is modeled by a (6x6) element mesh (72 triangular elements). Using the genetic algorithm the optimal positions of the patches are obtained as represented in Figure 2 a), i.e. the central plate elements (*Patches 1*). In Figure 2 a) and b) two different patches positions are shown, and the corresponding central line deflections, as well as those obtained with an entire actuator layer, are shown in Figure 3.

Figure 4 illustrates the responses for central amplitude of the plate, and compares these responses in three different situations. The controlled responses, obtained using a time step Δt=0.000125 s in the Newmark method, with $G_i G_c = 8000$, clearly demonstrates the damping effect of the piezoelectric actuators on the free vibration of the plate with an applied uniform distributed transverse load q = 10000 N/m^2, which was then suddenly removed.

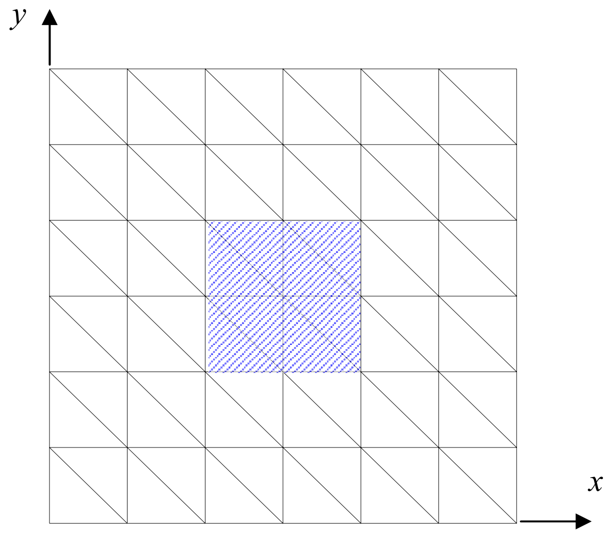

Figure 2 a). Patch locations – *Patches 1*.

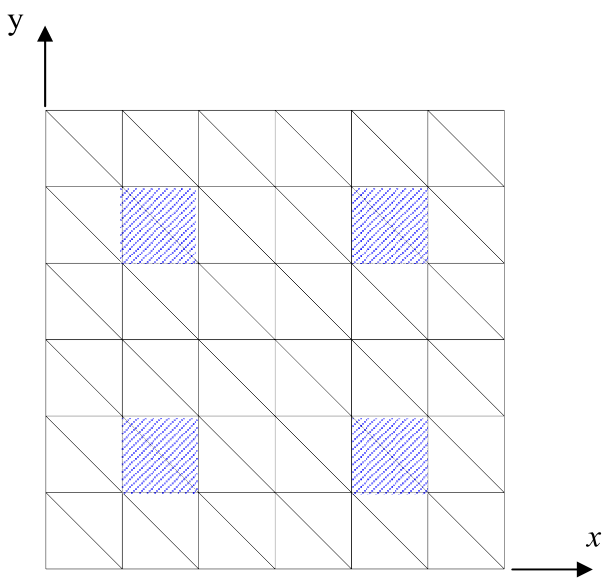

Figure 2 b). Patch locations – *Patches 2*.

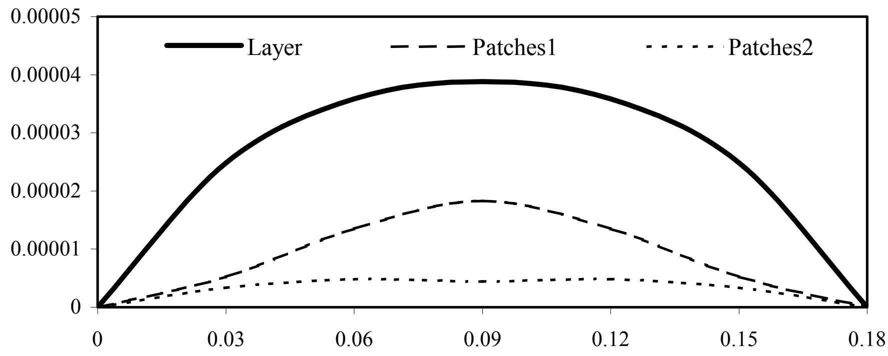

Figure 3. Central line deflections.

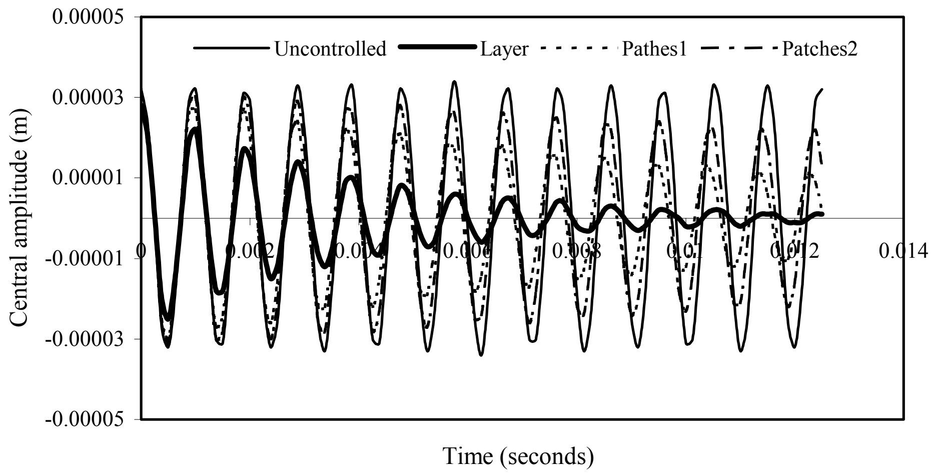

Figure 4. Uncontrolled and controlled responses on the plate central amplitude.

For an applied distributed transverse harmonic load $q(t) = q \sin 2\pi f\, t$ with a magnitude $q = 10000\,\mathrm{N}$ and frequency $f = 10\,\mathrm{Hz}$, Figure 5 shows the uncontrolled and controlled responses, where the effect of negative velocity feedback control, $G_i G_c = 3.2\times10^7$, is evident. For the same applied transverse harmonic load but now considering a frequency of $f = 1050\,\mathrm{Hz}$, which is very close to the first natural frequency, Figure 6 illustrates the uncontrolled and controlled responses for central deflection *w*. The controlled responses for gains $G_i G_c = 0.32 \times 10^5$ and $G_i G_c = 1.6 \times 10^5$, clearly demonstrates the action of the piezoelectric actuator layer, which increases the overall damping of the system.

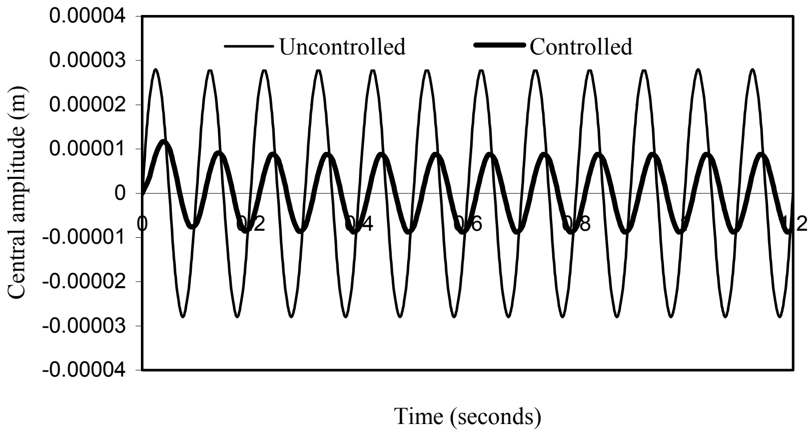

Figure 5. Uncontrolled central deflection, f=10 Hz

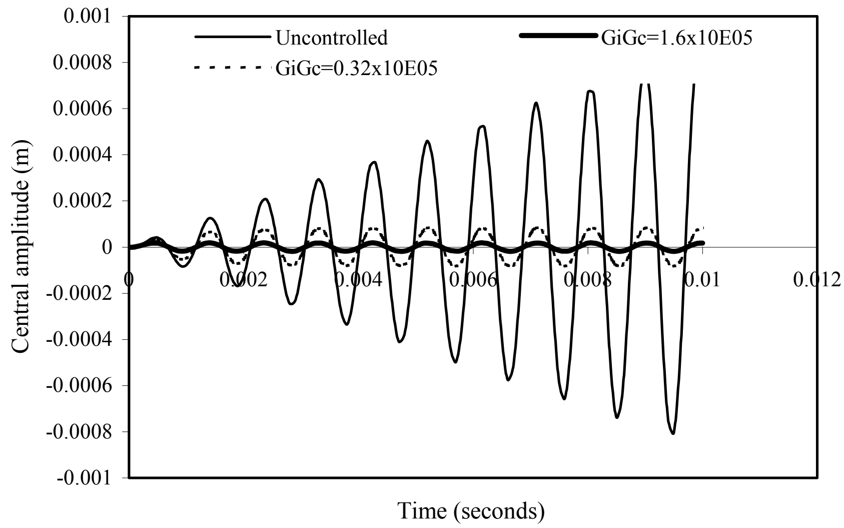

Figure 6. Effect of negative velocity feedback control on the central deflection

9. CONCLUSIONS

The active control capability of composite structures covered with piezoelectric layers or patches is investigated, using the finite element method. A finite element based on the Kirchhoff classical theory, has been developed. The present model has been validated in Moita *et al.*[25], where the solutions for deflection and sensed voltage in a bimorph beam, are compared with the solutions obtained by other authors. Here, the results obtained, show that the negative velocity feedback control algorithm used in this model is effective for an active damping control of vibration response. Also the core optimization had been performed in order to minimize the vibration amplitude and maximization of first natural frequency. For this optimization the genetic and gradient-based algorithms had been used. The patch position optimization had been also performed in order to maximize the effect a defined set of actuators. For this optimization the genetic algorithm had been used.

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