

# Triple unification of inflation, dark energy, and dark matter in two-scalar-field cosmology

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A unified description of inflation, dark energy, and dark matter is presented within a two-scalar-field cosmological model. Inflation, assumed to be of the warm type, is driven by one of the scalar fields, which, shortly after the end of the inflationary period, decouples from radiation and begins to oscillate rapidly around the minimum of its potential, thus behaving like cold dark matter; the second scalar field emerges, at recent times, as the dominant component of the universe, giving rise to a second era of accelerated expansion. For certain values of the parameters of the model, the cosmological solutions arising in this triple unification of inflation, dark energy, and dark matter are viable, reproducing the main features of the evolution of the universe.

## I. INTRODUCTION

The theory of cosmic inflation [1–5] stands now on solid observational foundations, as several of its key predictions have been confirmed by precise measurements of the cosmic microwave background radiation [6].

According to the inflationary paradigm, the universe undergoes a period of accelerated expansion in the early stages of its evolution, which is driven by a scalar field — the inflaton — slowly rolling down its potential. Such an early inflationary period not only solves the flatness, horizon, homogeneity, isotropy, and primordial monopole problems, but also provides the seeds for the formation of the observed large-scale structures of the universe.

A period of accelerated expansion is not exclusive to the early stages of evolution of the universe. In fact, cosmological observations have shown, not without surprise, that accelerated expansion is also taking place at the present time [7, 8], implying the existence of an unknown form of energy — dubbed as dark energy — which accounts for a substantial part of the total energy density of the universe [9]. Within the  $\Lambda$ CDM concordance model, this dark energy is assumed to be a cosmological constant. However, this simple explanation raises problems of its own [10], a circumstance that led to the hypothesis that dark energy could be identified with a scalar field [11], as in the inflationary paradigm.

In addition to dark energy, the concordance cosmological model also includes cold dark matter, which accounts for about one quarter of the total energy density of the universe [9]. Although the existence of dark matter has been inferred by its gravitational effects on a multiplicity of astrophysical and cosmological phenomena, it has so far eluded a direct detection and, after decades of intense experimental efforts, its physical nature remains a mystery [12]. Such circumstances led to the consideration of a wider range of dark-matter candidates, including the possibility that a scalar field, similar to those appearing

in the models of inflation and dark energy, could play the role of dark matter.

Since inflation, dark energy, and dark matter can all be identified with scalar fields, it is natural to try to unify these seemingly disparate phenomena under the same theoretical roof using these fields.

Such a unified description was proposed in Refs. [13, 14]. There, inflation was assumed to be of the usual (cold) type, followed by a post-inflationary reheating period, in which the decay of the inflaton field was required to be incomplete, leaving a remnant that behaved like cold dark matter. In order to reduce the energy density of the remnant to the level required by cosmological observations, two possibilities were considered: modification of the decay rate during the reheating process [13] or introduction of an additional period of thermal inflation, driven by a separate field, at lower energy densities [14]. In what concerns dark energy, this scenario assumed a non-zero vacuum energy for the inflaton/dark-matter field, motivated by a combination of the string landscape picture and the anthropic principle.

A new scenario for a triple unification, in which dark energy is described not by a cosmological constant, but rather by a dynamical scalar field, was soon afterwards proposed [15]. Within a two-scalar-field cosmological model inspired by supergravity, one of the fields played the role of dark energy, inducing the present accelerated expansion of the universe, while the second field played the roles of both inflaton and dark matter. Because inflation was assumed to be of the warm type [16], no distinctive post-inflationary reheating phase was required; soon after the smooth transition to the radiation-dominated era, the energy transfer from the inflaton field to the radiation bath ceased and the former began to oscillate around the minimum of the potential, thus mimicking the behavior of a cold-dark-matter fluid. However, despite its success in unifying inflation, dark energy, and dark matter within a single framework, this two-scalar-field cosmological model was not entirely satisfactory, since it accounted for just a fraction of the dark matter content of the universe.

The purpose of the present article is to provide a uni-

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fied description of inflation, dark energy, and dark matter in a more general setting, namely, within a two-scalar-field cosmological model given by the action<sup>1</sup>

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}e^{-\alpha\kappa\phi}(\nabla\xi)^2 - e^{-\beta\kappa\phi}V(\xi) \right], \quad (1)$$

where  $g$  is the determinant of the metric  $g_{\mu\nu}$ ,  $R$  is the Ricci scalar,  $\phi$  and  $\xi$  are scalar fields, and  $\alpha$  and  $\beta$  are independent dimensionless parameters<sup>2</sup>. Such an action, with a non-standard kinetic term and an exponential potential, arises in a great variety of gravity theories, such as the Jordan-Brans-Dicke theory, Kaluza-Klein theories,  $f(R)$ -gravity, and string theories (see Refs. [17, 18] for a derivation of the above action in the context of these theories). More recently, it has been shown that this action also arises in the context of hybrid metric-Palatini theories of gravity [19, 20].

For an appropriate choice of the potential  $V(\xi)$ , the two-scalar-field cosmological model given by action (1) allows for a unified description of inflation, dark energy, and dark matter, in which the scalar field  $\xi$  plays the roles of both inflaton and dark matter, while the scalar field  $\phi$  plays the role of dark energy.

The simplest potential providing such a triple unification has the form

$$V(\xi) = V_a + \frac{1}{2}m^2\xi^2, \quad (2)$$

where  $V_a$  and  $m$  are constants, related, respectively, to the energy density of dark energy and to the  $\phi$ -dependent mass of the scalar field  $\xi$ , defined as

$$M_\xi^2(\phi) = m^2 e^{-\beta\kappa\phi}. \quad (3)$$

While emphasizing that a triple unification such as the one proposed in this article could be achieved by any potential whose expansion around its minimum has the form  $A + B\xi^2 + \dots$ , for definiteness we will use the potential given by Eq. (2).

In what follows, let us briefly outline the key aspects of the triple unification proposed in this article.

Inflation is assumed to be of the warm type. Energy is continuously transferred from the inflaton field  $\xi$  (and

also from the dark-energy field  $\phi$ ) to a radiation bath, thereby ensuring that the energy density of the latter is substantial — albeit sub-dominant — throughout the inflationary expansion and that a smooth transition to a radiation-dominated era takes place without the need for a distinctive post-inflationary reheating phase.

The dissipation coefficients, mediating the energy transfer from the scalar fields to the radiation bath, have a generic dependence on the temperature, namely,  $\Gamma \propto T^p$  ( $p$  constant), and, immediately after the end of the inflationary period, are exponentially suppressed, becoming negligible shortly afterwards.

Shortly after the end of the inflationary period, the inflaton  $\xi$  decouples from radiation and begins to oscillate rapidly around the minimum of its potential, thus behaving on average like a pressureless nonrelativistic fluid, i.e., like cold dark matter.

Due to the non-standard kinetic term and the exponential factor in the potential, the energy density of cold dark matter depends explicitly on the scalar field  $\phi$ , implying that, in general, this quantity does not evolve exactly as ordinary baryonic matter.

After a radiation-dominated era, encompassing the primordial nucleosynthesis period, cold dark matter, together with ordinary baryonic matter, dominates the dynamics of the universe, giving rise to a matter-dominated era, long enough to allow for structure formation.

At recent times, the scalar field  $\phi$  finally emerges as the dominant component of the universe, giving rise to a second era of accelerated expansion, thus behaving like dark energy.

These key aspects of the proposed triple unification will be detailed in the body of the article.

To conclude this introductory section, let us point out that unified descriptions of inflation, dark energy, and dark matter have been proposed in several other contexts [21–27].

This article is organized as follows. The evolution equations for the two-scalar-field cosmological model are presented in the next section. For clarity, the cosmic evolution is divided into two stages, the first corresponding to the inflationary period and the transition to the radiation-dominated era (Sect. II A) and the second encompassing the radiation-, matter-, and dark-energy-dominated eras (Sect. II B). The continuity of the different physical quantities at the transition between the first and second stages of evolution is analyzed in Sect. II C. Numerical solutions are presented in Sect. III, which is divided into three subsections. In the first, we analyze the case  $\alpha = \beta$ , while the second is devoted to the case  $\alpha \neq \beta$ . Dissipative effects during inflation are analyzed in the last subsection. Finally, in Sect. IV, we present our conclusions.

<sup>1</sup> Throughout this article we will adopt the natural system of units and use the notation  $\kappa \equiv \sqrt{8\pi G} = \sqrt{8\pi}/m_P$ , where  $G$  is the gravitational constant and  $m_P = 1.22 \times 10^{19}$  GeV is the Planck mass.

<sup>2</sup> The triple unifications of inflation, dark energy, and dark matter proposed in Refs. [13, 14] and Ref. [15] correspond to models given by action (1) with, respectively,  $\alpha = \beta = 0$ ,  $\phi = 0$ , and  $V(\xi) = V_0 + \frac{1}{2}M^2\xi^2$ , where  $V_0$  and  $M$  are arbitrary constants, and  $\alpha = 0$ ,  $\beta = -\sqrt{2}$ , and  $V(\xi) = A_1[1 - 2A_2 \exp(-\sqrt{2}\kappa\xi) + A_3 \exp(-2\sqrt{2}\kappa\xi)]$ , where the constants  $A_i$  are related to fundamental quantities of the Salam-Sezgin six-dimensional supergravity theory.

## II. TWO-SCALAR-FIELD COSMOLOGICAL MODEL

Our analysis of the cosmic evolution is divided into two stages: the first corresponds to the inflationary period and the transition to the radiation-dominated era, while the second encompasses the radiation-, matter-, and dark-energy-dominated eras.

### A. First stage of evolution: the inflationary era

We assume inflation to be of the warm type (for reviews, see Refs. [28, 29]). In this inflationary paradigm, a continuous transfer of energy from the inflaton field to radiation ensures that the energy density of the latter remains substantial — albeit sub-dominant — throughout the inflationary era. This energy transfer also guarantees that the transition to a radiation-dominated era takes place in a smooth manner. It contrasts with the usual (cold) inflationary paradigm, in which radiation is severely diluted during inflation, a circumstance that makes a distinctive post-inflationary reheating process necessary in order to recover the standard cosmic evolution.

In our model, radiation is described by a perfect fluid with an equation-of-state parameter  $w_R = p_R/\rho_R = 1/3$ , where  $p_R$  and  $\rho_R$  are the pressure and the energy density of the fluid, respectively.

The energy density of radiation is sustained, during the inflationary period, by a continuous transfer of energy from the scalar fields  $\xi$  and  $\phi$ , which is accomplished by the introduction of dissipative terms with coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  into the equations of motion. This energy transfer prevents the radiation bath from being diluted, keeping the expanding universe “warm”.

To implement warm inflation, it would suffice to have a continuous and significative transfer of energy from the inflaton field  $\xi$  to the radiation bath, in which case one could simply set the dissipation coefficient  $\Gamma_\phi$  to zero and ignore any energy exchange between the dark-energy field  $\phi$  and radiation. However, as it follows from action (1), a direct transfer of energy between the two scalar fields also takes place (for nonvanishing  $\alpha$  and/or  $\beta$ ). Therefore, it seems natural to also allow for a direct energy transfer from the scalar field  $\phi$  to the radiation bath, mediated by a non-zero  $\Gamma_\phi$ , although we shall emphasize that this is not essential for the implementation of the warm-inflation scenario.

Inflation comes to an end when, due to an increase of dissipative effects, the energy density of the radiation bath smoothly takes over and begins to dominate the evolution of the universe. At this point, the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  are exponentially suppressed and, consequently, the radiation bath decouples from the scalar fields  $\xi$  and  $\phi$  and begins to evolve in the usual manner.

Let us now present the equations governing the cosmic

evolution during the inflationary period and the transition to a radiation-dominated era.

We assume a flat Friedman-Robertson-Walker universe<sup>3</sup>, given by the metric

$$ds^2 = -dt^2 + a^2(t)d\Sigma^2, \quad (4)$$

where  $a(t)$  is the scale factor and  $d\Sigma^2$  is the metric of the three-dimensional Euclidean space.

The equations of motion for the scalar fields  $\xi(t)$  and  $\phi(t)$  and for the energy density of radiation  $\rho_R(t)$  are then

$$\ddot{\xi} + 3\frac{\dot{a}}{a}\dot{\xi} - \alpha\kappa\dot{\phi}\dot{\xi} + \frac{\partial V}{\partial \xi}e^{(\alpha-\beta)\kappa\phi} = -\Gamma_\xi\dot{\xi}e^{\alpha\kappa\phi}, \quad (5)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\alpha\kappa}{2}\dot{\xi}^2e^{-\alpha\kappa\phi} - \beta\kappa V e^{-\beta\kappa\phi} = -\Gamma_\phi\dot{\phi}, \quad (6)$$

$$\dot{\rho}_R + 4\frac{\dot{a}}{a}\rho_R = \Gamma_\xi\dot{\xi}^2 + \Gamma_\phi\dot{\phi}^2, \quad (7)$$

while the Einstein equations for the scale factor  $a(t)$  are given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\xi}^2}{2}e^{-\alpha\kappa\phi} + V e^{-\beta\kappa\phi} + \rho_R\right), \quad (8)$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{3}\left(\dot{\phi}^2 + \dot{\xi}^2e^{-\alpha\kappa\phi} - V e^{-\beta\kappa\phi} + \rho_R\right), \quad (9)$$

where an overdot denotes a derivative with respect to time  $t$  and the potential  $V$  is given by Eq. (2).

Note that the above evolution equations differ from the usual ones in warm inflationary models in that they contain extra terms arising due to the presence, in action (1), of a non-standard kinetic term for the field  $\xi$ .

Instead of the comoving time  $t$ , let us use a new variable  $u$ , related to the redshift  $z$ ,

$$u = -\ln\left(\frac{a_0}{a}\right) = -\ln(1+z), \quad (10)$$

where  $a_0 \equiv a(u_0)$  denotes the value of the scale factor at the present time  $u_0 = 0$ .

With this change of variables, the above equations for  $\xi$ ,  $\phi$ , and  $\rho_R$  become

$$\xi_{uu} = -\left\{\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}}{a}\Gamma_\xi e^{\alpha\kappa\phi}\right]\xi_u - \alpha\kappa\left(\frac{\dot{a}}{a}\right)^2\phi_u\xi_u + m^2\xi e^{(\alpha-\beta)\kappa\phi}\right\}\left(\frac{\dot{a}}{a}\right)^{-2}, \quad (11)$$

$$\phi_{uu} = -\left\{\left[\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{\dot{a}}{a}\Gamma_\phi\right]\phi_u + \frac{\alpha\kappa}{2}\left(\frac{\dot{a}}{a}\right)^2\xi_u^2e^{-\alpha\kappa\phi} - \beta\kappa\left(V_a + \frac{1}{2}m^2\xi^2\right)e^{-\beta\kappa\phi}\right\}\left(\frac{\dot{a}}{a}\right)^{-2}, \quad (12)$$

$$\rho_{Ru} = -4\rho_R + \frac{\dot{a}}{a}(\Gamma_\xi\xi_u^2 + \Gamma_\phi\phi_u^2), \quad (13)$$

<sup>3</sup> Since the current cosmological measurements constrain the present-time value of the curvature density parameter  $\Omega_k$  to be very small [9], a spatially flat universe can be assumed without much loss of generality.

where the subscript  $u$  denotes a derivative with respect to  $u$ ;  $\dot{a}/a$  and  $\ddot{a}/a$  are functions of  $u$ ,  $\xi$ ,  $\xi_u$ ,  $\phi$ , and  $\phi_u$ , given by

$$\left(\frac{\dot{a}}{a}\right)^2 = 2\kappa^2 \frac{(V_a + \frac{1}{2}m^2\xi^2)e^{-\beta\kappa\phi} + \rho_R}{6 - \kappa^2\phi_u^2 - \kappa^2\xi_u^2 e^{-\alpha\kappa\phi}} \quad (14)$$

and

$$\begin{aligned} \frac{\ddot{a}}{a} = \frac{\kappa^2}{3} & \left\{ 2\kappa^2 \left[ \left( V_a + \frac{1}{2}m^2\xi^2 \right) e^{-\beta\kappa\phi} + \rho_R \right] \right. \\ & \times \frac{\phi_u^2 + \xi_u^2 e^{-\alpha\kappa\phi}}{\kappa^2\phi_u^2 + \kappa^2\xi_u^2 e^{-\alpha\kappa\phi} - 6} \\ & \left. + \left( V_a + \frac{1}{2}m^2\xi^2 \right) e^{-\beta\kappa\phi} - \rho_R \right\}. \end{aligned} \quad (15)$$

In order to solve the above system of equations, one has to specify the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$ .

Over the years, a variety of forms has been adopted for these coefficients, from the simplest, based on general phenomenological considerations, to the more elaborate ones, derived from microscopic quantum field theory. In general, the dissipation coefficients  $\Gamma$  appearing in the literature are functions of the temperature  $T$  and/or the inflaton field  $\xi$ , as, for instance,  $\Gamma \propto T^3/\xi^2$  [25, 30],  $\Gamma \propto T$  [31–33], or  $\Gamma \propto T^{-1}$  [34].

In this article, we will not be concerned with the specific microscopic models used to derive the dissipation coefficients. We will adopt instead a model-independent approach, assuming that, during inflation, these coefficients have a generic dependence on the temperature of the radiation bath, namely,  $\Gamma \propto T^p$ . We also assume that, immediately after the end of the inflationary period, the dissipation coefficients are exponentially suppressed, becoming negligible soon afterwards. In short, we assume the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  to be given by

$$\Gamma_{\xi,\phi} = f_{\xi,\phi} \times \begin{cases} T^p, & T \geq T_E, \\ T^p \exp \left[ 1 - \left( \frac{T_E}{T} \right)^q \right], & T \leq T_E, \end{cases} \quad (16)$$

where  $T_E$  is the temperature of the radiation bath at the end of the inflationary period,  $f_\xi$  and  $f_\phi$  are positive constants with dimension  $(\text{mass})^{1-p}$  encoding the details of the microscopic models used to derive the dissipation coefficients, and  $q > 0$  and  $p$  are parameters determining the temperature dependence of these coefficients.

The suppression of the dissipation coefficients immediately after the inflationary period is of paramount importance in our unification proposal in order to guarantee that the inflaton field  $\xi$  survives and that it has enough energy to mimic the behavior of cold dark matter in a way consistent with cosmological observations (see Sect. IIB below for details). A microscopic model in which this suppression is achieved naturally has been proposed recently [32], in the context of the warm little

inflaton scenario [31]. More specifically, the dissipation coefficient mediating the energy transfer from the inflaton field to radiation, which initially is proportional to  $T$ , becomes exponentially suppressed when the temperature drops below a certain threshold value (roughly coinciding with the end of the inflationary period), thus leading to a stable inflaton remnant. Such suppression of the dissipative effects below a threshold temperature is also present in another warm-inflation model [34] in which the dissipation coefficient during inflation is proportional to  $T^{-1}$ .

As will be shown in Sect. IIIC, for dissipation coefficients proportional to  $T^p$ , with  $p > 2$ , suppression of the dissipation coefficients below a threshold temperature occurs naturally, as a result of the background dynamics, making it unnecessary, for such values of  $p$ , to introduce explicitly the exponential factor in Eq. (16).

For our base scenario we will choose  $p = 1$ , corresponding to dissipation coefficients linearly dependent on the temperature, and  $q = 2$  (see Sect. IIIA); other values of the parameters  $p$  and  $q$  will be considered in Sect. IIIC.

Finally, let us recall that the temperature  $T$  of the radiation bath is related to its energy density by

$$\rho_R = \frac{\pi^2}{30} g_* T^4, \quad (17)$$

where  $g_*$  denotes the effective number of relativistic degrees of freedom at temperature  $T$ . Assuming the standard model of particle physics and taking into account that, at the relevant temperatures, all the degrees of freedom of this model are relativistic and in thermal equilibrium,  $g_*$  takes the value 106.75.

Solving Eqs. (11)–(15) allows us to determine the density parameters for radiation and for the scalar fields  $\xi$  and  $\phi$ ,

$$\Omega_R = \frac{\rho_R}{\rho_c} = \frac{\kappa^2}{3} \rho_R \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (18)$$

$$\Omega_\xi = \frac{\rho_\xi}{\rho_c} = \frac{\kappa^2}{6} \left[ \xi_u^2 e^{-\alpha\kappa\phi} + m^2 e^{-\beta\kappa\phi} \xi^2 \left( \frac{\dot{a}}{a} \right)^{-2} \right], \quad (19)$$

$$\Omega_\phi = \frac{\rho_\phi}{\rho_c} = \frac{\kappa^2}{3} \left[ \frac{\phi_u^2}{2} + V_a e^{-\beta\kappa\phi} \left( \frac{\dot{a}}{a} \right)^{-2} \right], \quad (20)$$

as well as the effective equation-of-state parameter,

$$w_{\text{eff}} = \frac{1}{3} \left( \Omega_R + 3\Omega_\phi \frac{p_\phi}{\rho_\phi} + 3\Omega_\xi \frac{p_\xi}{\rho_\xi} \right), \quad (21)$$

where  $\rho_c = (3/\kappa^2)(\dot{a}/a)^2$  is the critical density and the energy density and pressure of the scalar fields  $\xi$  and  $\phi$  are given by, respectively,

$$\rho_\xi = \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \xi_u^2 e^{-\alpha\kappa\phi} + \frac{1}{2} m^2 e^{-\beta\kappa\phi} \xi^2, \quad (22)$$

$$p_\xi = \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \xi_u^2 e^{-\alpha\kappa\phi} - \frac{1}{2} m^2 e^{-\beta\kappa\phi} \xi^2, \quad (23)$$

and

$$\rho_\phi = \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \phi_u^2 + V_a e^{-\beta\kappa\phi}, \quad (24)$$

$$p_\phi = \frac{1}{2} \left( \frac{\dot{a}}{a} \right)^2 \phi_u^2 - V_a e^{-\beta\kappa\phi}. \quad (25)$$

In this article we will be focused on the background dynamics of the two-scalar-field cosmological model and its capability to reproduce the main stages of evolution of the universe, leaving the analysis of the primordial spectrum of density perturbations and its agreement with cosmological observations to future work. Nevertheless, we would like to emphasize here that warm-inflation models have been shown to be consistent with cosmic microwave background (CMB) data for a large variety of potentials and dissipation coefficients. In particular, predictions of the tensor-to-scalar ratio and the spectral tilt of the primordial spectrum were shown to agree with cosmological data for models with a quartic potential and dissipation coefficients proportional to  $T$  and  $T^3$  [35–39], and also in the case of a quadratic potential and dissipation coefficient proportional to  $T^{-1}$  [34]. The former models favor mostly the weak dissipative regime, while the latter is consistent with strong dissipation.

We now turn to the description of the second stage of evolution, which encompasses the radiation-, matter-, and dark-energy-dominated eras.

### B. Second stage of evolution: the radiation-, matter-, and dark-energy-dominated eras

As mentioned above, at the end of the inflationary period, the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  are exponentially suppressed and, soon afterwards, become negligible, allowing us to set them exactly to zero. This marks the end of the first stage of evolution.

During the second stage of evolution, in the absence of dissipation, radiation decouples from the scalar fields  $\xi$  and  $\phi$  and Eq. (13) yields the solution

$$\rho_R = \rho_{R0} e^{-4u}, \quad (26)$$

where  $\rho_{R0} \equiv \rho_R(u_0)$  denotes the energy density of radiation at the present time  $u_0 = 0$ .

For its part, the scalar field  $\xi$  begins to oscillate rapidly around its minimum, behaving like a nonrelativistic dark-matter fluid with equation of state  $\langle p_\xi \rangle = 0$  [40], where the brackets  $\langle \dots \rangle$  denote the average over an oscillation<sup>4</sup>.

Let us derive an expression for the energy density of the dark-matter fluid in terms of  $u$  and  $\phi(u)$ . To that

end, we multiply Eq. (11) by  $\xi_u$  and use the definition of  $\rho_\xi$  given by Eq. (22) to obtain

$$\rho_{\xi u} + 3 \left( \frac{\dot{a}}{a} \right)^2 \xi_u^2 e^{-\alpha\kappa\phi} - \frac{\alpha\kappa}{2} \left( \frac{\dot{a}}{a} \right)^2 \xi_u^2 \phi_u e^{-\alpha\kappa\phi} + \frac{\beta\kappa}{2} m^2 \xi^2 \phi_u e^{-\beta\kappa\phi} = 0. \quad (27)$$

Averaging over an oscillation period and taking into account that  $\langle p_\xi \rangle = 0$  implies  $\langle \xi^2 \rangle = \rho_\xi m^{-2} e^{\beta\kappa\phi}$  and  $\langle \xi_u^2 \rangle = \rho_\xi (\dot{a}/a)^{-2} e^{\alpha\kappa\phi}$ , the above equation can be written as

$$\rho_{\xi u} + 3\rho_\xi - \frac{(\alpha - \beta)\kappa}{2} \rho_\xi \phi_u = 0, \quad (28)$$

yielding the solution

$$\rho_\xi = C e^{-3u} e^{\frac{(\alpha - \beta)\kappa}{2} \phi}, \quad (29)$$

where  $C$  is a constant whose value is fixed by current cosmological measurements [see Eq. (35) below].

As expected, the energy density of dark matter is proportional to  $e^{-3u}$  (or, in terms of the scale factor, proportional to  $a^{-3}$ ), due to the fact that the potential  $V(\xi)$  was chosen to be quadratic. But it also depends directly on the scalar field  $\phi$ , through an exponential factor, as a consequence of both the non-standard kinetic term of the scalar field  $\xi$  and the exponential potential [see action (1)]. As will be seen in Sect. III, such dependence of  $\rho_\xi$  on the dark-energy field  $\phi$  has implications on the cosmic evolution, leading to a non-simultaneous peaking of the energy densities of dark matter and ordinary baryonic matter.

Now, taking into account the above expressions for the energy densities of radiation and dark matter, the evolution Eqs. (11)–(15) can be considerably simplified, yielding

$$\phi_{uu} = - \left\{ \left[ \frac{\ddot{a}}{a} + 2 \left( \frac{\dot{a}}{a} \right)^2 \right] \phi_u - \beta\kappa V_a e^{-\beta\kappa\phi} + \frac{(\alpha - \beta)\kappa C}{2} e^{\frac{(\alpha - \beta)\kappa}{2} \phi} e^{-3u} \right\} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (30)$$

with

$$\left( \frac{\dot{a}}{a} \right)^2 = 2\kappa^2 \left[ V_a e^{-\beta\kappa\phi} + \left( \rho_{\text{BM0}} + C e^{\frac{(\alpha - \beta)\kappa}{2} \phi} \right) e^{-3u} + \rho_{R0} e^{-4u} \right] (6 - \kappa^2 \phi_u^2)^{-1}, \quad (31)$$

and

$$\frac{\ddot{a}}{a} = \frac{\kappa^2}{6} \left\{ 4\kappa^2 \left[ V_a e^{-\beta\kappa\phi} + \left( \rho_{\text{BM0}} + C e^{\frac{(\alpha - \beta)\kappa}{2} \phi} \right) e^{-3u} + \rho_{R0} e^{-4u} \right] \phi_u^2 (\kappa^2 \phi_u^2 - 6)^{-1} + 2V_a e^{-\beta\kappa\phi} - \left( \rho_{\text{BM0}} + C e^{\frac{(\alpha - \beta)\kappa}{2} \phi} \right) e^{-3u} - 2\rho_{R0} e^{-4u} \right\}. \quad (32)$$

<sup>4</sup> These oscillations take place if the mass of the scalar field  $\xi$ , given by Eq. (3), is much bigger than the Hubble parameter,  $M_\xi \gg H \equiv \dot{a}/a$ , a condition that can be easily satisfied by choosing a large enough value for the constant  $m$ .

In the above equations, we have introduced ordinary baryonic matter, described as a perfect fluid with pressure  $p_{\text{BM}} = 0$  and energy density

$$\rho_{\text{BM}} = \rho_{\text{BM}0} e^{-3u}, \quad (33)$$

where, as usual, the subscript 0 indicates present-time values.

Agreement with current cosmological measurements [9] requires  $\rho_{\text{R}0} = 9.02 \times 10^{-128} m_{\text{P}}^4$  and  $\rho_{\text{BM}0} = 8.19 \times 10^{-125} m_{\text{P}}^4$ , as well as

$$\frac{1}{2} \left[ \left( \frac{\dot{a}}{a} \right)^2 \phi_u^2 \right]_{u=u_0} + V_a e^{-\beta \kappa \phi_0} = \rho_{\text{DE}0}, \quad (34)$$

$$C e^{\frac{(\alpha-\beta)\kappa}{2} \phi_0} = \rho_{\text{DM}0}, \quad (35)$$

where the present-time energy densities of dark energy and dark matter are  $\rho_{\text{DE}0} = 1.13 \times 10^{-123} m_{\text{P}}^4$  and  $\rho_{\text{DM}0} = 4.25 \times 10^{-124} m_{\text{P}}^4$ , respectively<sup>5</sup>.

As will be shown in Sect. III, for a choice of  $V_a$  and  $C$  satisfying the above conditions, Eqs. (30)–(32) describe a radiation-dominated era, encompassing the primordial nucleosynthesis period, followed by an era dominated by the scalar field  $\xi$  (dark matter) and ordinary baryonic matter, lasting long enough for structure formation to occur, and, finally, a dark-energy-dominated era, induced by the scalar field  $\phi$ , during which the universe undergoes accelerated expansion. The requirement that the transition from the radiation- to the matter-dominated era does not occur too early in the cosmic history and, consequently, does not conflict with primordial nucleosynthesis, as well as the requirement that the expansion of the universe is accelerating at the present time, imposes constraints on the parameters  $\alpha$  and  $\beta$ , namely,  $|\alpha - \beta| \lesssim 1$  and  $|\beta| \lesssim 3/2$ .

During the second stage of evolution, the density parameter for the scalar field  $\phi$  is given by Eq. (20), while the density parameters for radiation, baryonic matter, and the scalar field  $\xi$ , are given by, respectively,

$$\Omega_{\text{R}} = \frac{\rho_{\text{R}}}{\rho_c} = \frac{\kappa^2}{3} \rho_{\text{R}0} e^{-4u} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (36)$$

$$\Omega_{\text{BM}} = \frac{\rho_{\text{BM}}}{\rho_c} = \frac{\kappa^2}{3} \rho_{\text{BM}0} e^{-3u} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (37)$$

$$\Omega_{\xi} = \frac{\rho_{\xi}}{\rho_c} = \frac{\kappa^2}{3} C e^{\frac{(\alpha-\beta)\kappa}{2} \phi} e^{-3u} \left( \frac{\dot{a}}{a} \right)^{-2}. \quad (38)$$

The effective equation-of-state parameter, during this stage of evolution, is

$$w_{\text{eff}} = \frac{1}{3} \left[ 1 - \Omega_{\text{BM}} - \Omega_{\xi} - \Omega_{\phi} \left( 1 - 3 \frac{p_{\phi}}{\rho_{\phi}} \right) \right], \quad (39)$$

where  $\rho_{\phi}$  and  $p_{\phi}$  are given by Eqs. (24) and (25), respectively.

A detailed analysis of this stage of evolution for the case  $\alpha = 2/\sqrt{6}$  and arbitrary  $\beta$  can be found in Ref. [41], where a unified description of dark matter and dark energy was proposed within the generalized hybrid metric-Palatini theory of gravity.

### C. Transition between the first and second stages of evolution

As we have just seen, the first stage of evolution, corresponding to the inflationary era, is described by Eqs. (11)–(15), while the second stage of evolution, encompassing the radiation-, matter-, and dark-energy-dominated eras, is described by Eqs. (30)–(32).

The transition from the first to the second stage of evolution occurs shortly after the end of inflation, at the beginning of the radiation-dominated era, when the dissipation coefficients  $\Gamma_{\xi}$  and  $\Gamma_{\phi}$ , given by Eq. (16), are exponentially suppressed and become negligible. The moment at which the dissipation coefficients are set exactly to zero marks the end of the first stage of evolution and the beginning of the second. In what follows, this transition moment will be denoted by  $u = u_*$ .

At the transition between the first and second stages of evolution the different physical quantities should be continuous.

For the scalar field  $\phi$  this is achieved by simply requiring the initial value of the second stage to be equal to the final value of the first stage.

For the energy density of the scalar field  $\xi$ , continuity at the transition requires

$$\rho_{\xi}(u_*) = C e^{-3u_*} e^{\frac{(\alpha-\beta)\kappa}{2} \phi_*}, \quad (40)$$

where  $\phi_* \equiv \phi(u_*)$  and  $\rho_{\xi}(u_*)$  denotes the energy density of the  $\xi$  field during the first stage of evolution, given by Eq. (22), evaluated at  $u = u_*$ . Since the constant  $C$  is fixed by Eq. (35), satisfying the above continuity condition amounts to fix the value of  $u_*$  or, equivalently, the duration of the first stage of evolution,

$$\begin{aligned} \Delta^{[\text{I}]} u &\equiv u_* - u_i \\ &= -\frac{1}{3} \ln \frac{\rho_{\xi}(u_*)}{\rho_{\text{DM}0}} - \frac{(\alpha - \beta)\kappa}{6} [\phi_0 - \phi_*] - u_i, \end{aligned} \quad (41)$$

where  $u_i$  denotes the value of  $u$  at the beginning of the first stage of evolution.

Finally, for the energy density of radiation, continuity at the transition requires that

$$\rho_{\text{R}}(u_*) = \rho_{\text{R}0} e^{-4u_*}, \quad (42)$$

where  $\rho_{\text{R}}(u_*)$  denotes the energy density of radiation during the first stage of evolution [i.e., the solution of Eq. (13)] evaluated at  $u = u_*$ . Because  $\rho_{\text{R}0}$  is fixed by current cosmological measurements (see the discussion in

<sup>5</sup> Note that these values for  $\rho_{\text{R}0}$ ,  $\rho_{\text{BM}0}$ ,  $\rho_{\text{DM}0}$ , and  $\rho_{\text{DE}0}$  correspond to a Hubble constant  $H_0 \equiv (\dot{a}/a)_0 = 1.17 \times 10^{-61} m_{\text{P}}$  or, in more familiar units,  $H_0 = 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

Sect. II B), the above continuity condition leads, in general, to a value of  $u_*$  different from the one determined from Eq. (40). To avoid this and, at the same time, to maintain adherence to the convention  $u_0 = 0$ , the value of the variable  $u$  at the beginning of the first stage of evolution,  $u_i$ , has to be shifted by an appropriate amount, i.e., by an amount ensuring that  $u_*$ , determined from Eq. (40), also satisfies Eq. (42). This procedure fixes the duration of the second stage of evolution to be

$$\Delta^{[\text{II}]}u \equiv u_0 - u_* = \frac{1}{4} \ln \frac{\rho_R(u_*)}{\rho_{R0}}. \quad (43)$$

Now, we can proceed to the numerical analysis of the equations of our two-scalar-field cosmological model.

### III. NUMERICAL SOLUTIONS

Let us solve numerically Eqs. (11)–(15) and Eqs. (30)–(32), corresponding to the first and second stages of evolution, respectively, and present a unified description of inflation, dark energy, and dark matter within the two-scalar-field cosmological model given by action (1).

For appropriate choices of the initial values for the variables  $\xi$ ,  $\xi_u$ ,  $\phi$ ,  $\phi_u$ , and  $\rho_R$  and the values of the constants  $\alpha$ ,  $\beta$ ,  $V_a$ ,  $m$ ,  $C$ ,  $f_\xi$ ,  $f_\phi$ ,  $p$ , and  $q$  our model allows for a cosmic evolution consistent with current cosmological observations.

Note that, because of the symmetries of action (1), we can assume  $\alpha \geq 0$  without loss of generality. The solutions corresponding to negative values of  $\alpha$  can be obtained from the solutions with positive values using the transformation  $\alpha \rightarrow -\alpha$ ,  $\beta \rightarrow -\beta$ , and  $\phi \rightarrow -\phi$ .

For clarity of presentation, this section is divided into three subsections, where we analyze the case  $\alpha = \beta$ , the case  $\alpha \neq \beta$ , and the dependence of the dissipative effects on the parameters  $p$  and  $q$ .

#### A. Case $\alpha = \beta$

We first analyze the case  $\alpha = \beta = 1$ , choosing representative values for the initial conditions, namely  $\xi(u_i) = 0.52 m_P$ ,  $\phi(u_i) = 10^{-3} m_P$ ,  $\xi_u(u_i) = 10^{-2} m_P$ ,  $\phi_u(u_i) = 10^{-5} m_P$ ,  $\rho_R(u_i) = 0.25 \times 10^{-12} m_P^4$ , and for the parameters<sup>6</sup>, namely,  $V_a = 7.64 \times 10^{-123} m_P^4$ ,  $m = 10^{-5} m_P$ ,  $f_\xi = f_\phi = 2$ ,  $p = 1$ ,  $q = 2$ . We call this case the base scenario.

Initially, the scalar field  $\xi$  (the inflaton) slowly rolls down its quadratic potential, leading to an accelerated

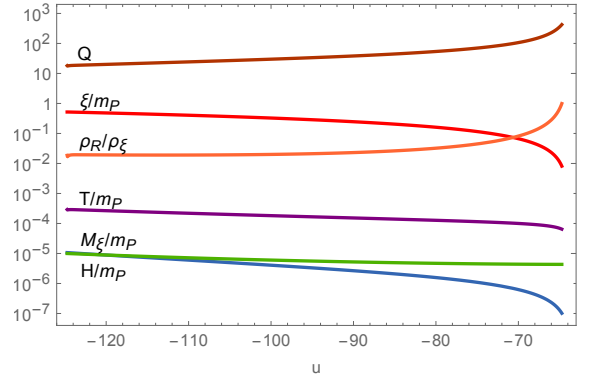


FIG. 1: Evolution of the inflaton field  $\xi$ , its mass  $M_\xi$ , the dissipation ratio  $Q$ , the temperature  $T$  of the radiation bath, the Hubble parameter  $H$ , and the ratio  $\rho_R/\rho_\xi$  during the inflationary period, which extends from  $u \approx -124.7$  to  $u \approx -64.7$  (60  $e$ -folds of expansion). The energy scale of inflation is  $E_{\text{inf}} \approx 2.3 \times 10^{16}$  GeV.

expansion of the universe (60  $e$ -folds). The energy scale of inflation, defined as

$$E_{\text{inf}} = \left[ e^{-\beta \kappa \phi(u_i)} V[\xi(u_i)] \right]^{1/4}, \quad (44)$$

is  $2.3 \times 10^{16}$  GeV. Because of the dependence on the scalar field  $\phi$ , the inflaton mass  $M_\xi$  decreases from  $10^{-5} m_P$  to  $4.3 \times 10^{-6} m_P$  during this stage of cosmic evolution. Dissipative effects guarantee that energy is continuously transferred from both fields  $\xi$  and  $\phi$  to the radiation bath, preventing it from being diluted away by accelerated expansion. Throughout the inflationary period, the dissipation ratios, defined as<sup>7</sup>

$$Q_{\xi,\phi} = \frac{\Gamma_{\xi,\phi}}{3H}, \quad (45)$$

remain larger than unity (strong dissipative regime) and the temperature of the radiation bath decreases only slightly (from  $3.5 \times 10^{15}$  GeV to  $7.9 \times 10^{14}$  GeV). The evolution of the relevant quantities during the inflationary period is shown in Fig. 1.

At a certain point of the evolution, radiation emerges as the dominant component of the universe and the inflationary period comes to an end (see Fig. 2). At that moment, when the temperature of the radiation bath is  $T_E \approx 7.9 \times 10^{14}$  GeV, the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  begin to decrease exponentially, implying that soon afterwards the dissipation ratio  $Q$  becomes negligible (see Fig. 3). At  $u = u_* \approx -63.7$ , the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  are set exactly to zero and the dynamics of the cosmic evolution becomes governed by Eqs. (30)–(32). This marks the beginning of the second stage of evolution.

<sup>6</sup> The parameters  $f_\xi$  and  $f_\phi$ , which determine the amplitude of the energy transfer between the scalar fields and the radiation bath, are, in general, different. Here, for simplicity, they are assumed to be equal.

<sup>7</sup> Since we are considering  $\Gamma_\phi = \Gamma_\xi$ , in what follows, for simplicity, both dissipation ratios  $Q_\phi$  and  $Q_\xi$  will be denoted by  $Q$ .

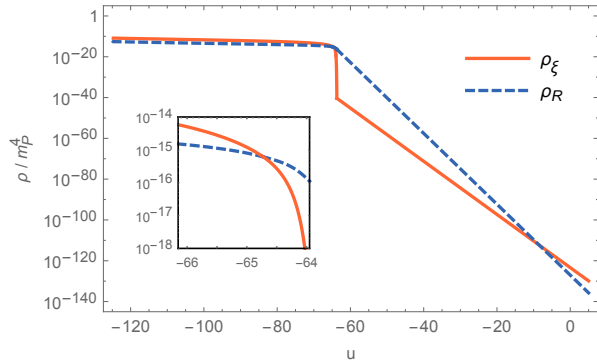


FIG. 2: Evolution of the energy densities of the scalar field  $\xi$  and of radiation. During the inflationary period, due to dissipative effects, the latter remains almost constant. At  $u \approx -64.7$ , radiation emerges as the dominant component of the universe and the inflationary period comes to an end (inset plot). During the radiation-dominated era, the scalar field  $\xi$  behaves like a pressureless nonrelativistic fluid (dark matter), becoming dominant, together with ordinary baryonic matter, at  $u \approx -8.6$ .

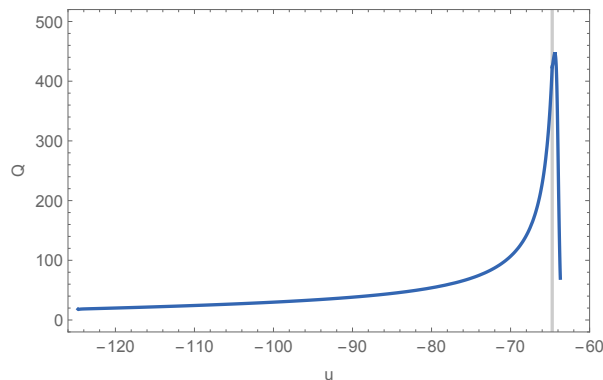


FIG. 3: Evolution of the dissipation ratio  $Q$ . Strong dissipation,  $Q \gg 1$ , is maintained throughout inflation. At the end of the inflationary period ( $u \approx -64.7$ , vertical grey line), the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  are exponentially suppressed, implying that soon afterwards  $Q$  becomes negligible. At  $u = u_* \approx -63.7$ , the dissipation coefficients are set exactly to zero, marking the end of the first stage of evolution.

In the scenario we have been considering, the initial conditions and the values of the parameters are such that a strong dissipative regime ( $Q > 1$ ) is maintained throughout inflation. However, as will be shown below in Sect. III C, a weak dissipative regime can also be obtained for other choices of initial conditions and values of the parameters.

The value of the energy density of the field  $\xi$  at the transition between the first and second stages of evolution,  $\rho_\xi(u_*)$ , is of crucial importance (see Fig. 2). It cannot be too large, otherwise the radiation-dominated era will be too short, conflicting with primordial nucleosynthesis, but it cannot be too small, otherwise the matter-dominated era will not be long enough for structure for-

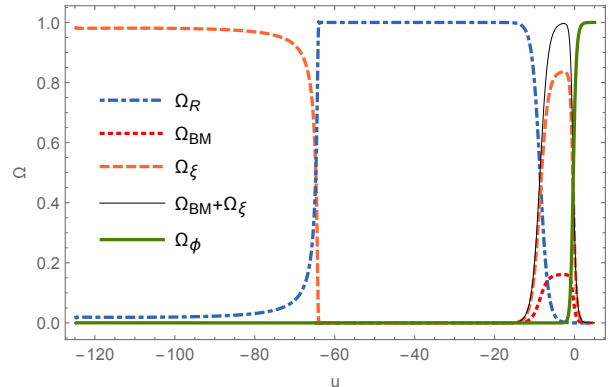


FIG. 4: Evolution of the density parameters for radiation, baryonic matter, and the scalar fields  $\xi$  (inflaton and dark matter) and  $\phi$  (dark energy). The successive inflationary, radiation, matter, and dark energy eras are clearly delimited. At the present time,  $u_0 = 0$ , the density parameters are  $\Omega_\phi(u_0) \approx 0.69$ ,  $\Omega_\xi(u_0) \approx 0.26$ ,  $\Omega_{\text{BM}}(u_0) \approx 0.05$ , and  $\Omega_{\text{R}}(u_0) \approx 5.5 \times 10^{-5}$ , in agreement with cosmological measurements.

mation to take place or, worse, such an era may not even occur. As discussed above [see Eq. (41)], in order to guarantee an adequate value of  $\rho_\xi(u_*)$ , the duration of the first stage of evolution should be chosen carefully; in the example we have been considering (base scenario),  $\Delta^{[\text{I}]}u \approx 61.0$ , implying  $\rho_\xi(u_*) \approx 4.2 \times 10^{-41} m_{\text{P}}^4$ , which allows for radiation- and matter-dominated eras with durations consistent with current cosmological observations.

Let us open here a parenthesis to briefly comment on a recent proposal of unification of inflation and dark matter [32], in which inflation is also assumed to be of the warm type. There, the potential of the inflaton field, in addition to a quadratic term, also has a quartic one, which dominates for large field values, implying that this scalar field behaves, during most of the radiation-dominated era, including the primordial nucleosynthesis period, as dark radiation. It begins to behave like dark matter just before the transition between the radiation- and the matter-dominated eras. This contrasts with the situation in our model, in which the scalar field  $\xi$  behaves like dark matter from the very beginning of the radiation-dominated era [see Eq. (29) and Fig. 2].

Returning to our base scenario, we point out that, as discussed in Sect. II C, continuity of the energy density of radiation at the transition between the first and second stages of evolution fixes the duration of the second stage to be  $\Delta^{[\text{II}]}u \approx 63.7$ ; at the transition,  $\rho_{\text{R}}(u_*) \approx 4.2 \times 10^{-17} m_{\text{P}}^4$ .

During the second stage of evolution, the energy densities  $\rho_{\text{R}}$  and  $\rho_\xi$  evolve according to Eqs. (26) and (29), leading successively to radiation- and dark-matter-dominated eras. Meanwhile, the scalar field  $\phi$  plays no significant role in the dynamics of the universe, only becoming dominant at recent times ( $u \approx -0.3$ ) when it induces a period of accelerated expansion (see Fig. 4).



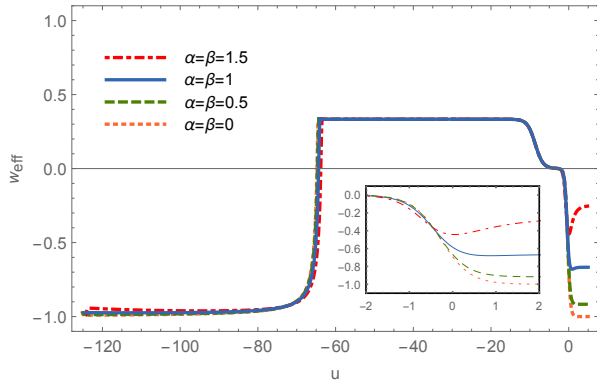


FIG. 5: Evolution of the effective equation-of-state parameter  $w_{\text{eff}}$ , clearly showing the inflationary, radiation, matter, and dark-energy eras (the solid blue line corresponds to the base scenario,  $\alpha = \beta = 1$ ). The value of  $w_{\text{eff}}$  for  $u \rightarrow +\infty$  depends on the parameter  $\beta$  (but not on  $\alpha$ ), such that, for  $|\beta| < \sqrt{2}$ , the universe enters a period of everlasting accelerated expansion. For  $\beta = 3/2$  this accelerated expansion is only temporary, since asymptotically  $w_{\text{eff}}$  approaches the value  $-1/4$ .

For our choice of the initial conditions and parameters (and  $C \approx 4.25 \times 10^{-124} m_{\text{P}}^4$ ) the conditions given by Eqs. (34) and (35) are satisfied, implying that the density parameters at the present time  $u_0 = 0$  become  $\Omega_\phi(u_0) \approx 0.69$ ,  $\Omega_\xi(u_0) \approx 0.26$ ,  $\Omega_{\text{BM}}(u_0) \approx 0.05$ , and  $\Omega_{\text{R}}(u_0) \approx 5.5 \times 10^{-5}$ , in agreement with cosmological measurements [9].

In the future (i.e., for  $u > 0$ ), the density parameters for radiation  $\Omega_{\text{R}}$ , baryonic matter,  $\Omega_{\text{BM}}$ , and dark matter  $\Omega_\xi$  become negligible in comparison with the density parameter for dark energy  $\Omega_\phi$ , implying that the effective equation-of-state parameter  $w_{\text{eff}}$  tends to the value  $-1 + \beta^2/3$  (see Fig. 5).

Indeed, for negligible  $\Omega_{\text{R}}$ ,  $\Omega_{\text{BM}}$ , and  $\Omega_\xi$ , Eq. (30) simplifies considerably, becoming

$$\phi_{uu} = -\frac{1}{2\kappa}(\kappa\phi_u - \beta)(6 - \kappa^2\phi_u^2). \quad (46)$$

This differential equation admits an analytical solution, which, in the limit  $u \rightarrow +\infty$  and for  $\beta^2 < 6$ , becomes  $\phi_u = \beta/\kappa$ . Now, inserting this asymptotic solution, in Eq. (31), one obtains  $V_a e^{-\beta\kappa\phi} = (6 - \beta^2)H^2/(2\kappa^2)$ . Finally, substituting the above expressions for  $\phi$  and  $\phi_u$  into Eq. (39), one obtains  $w_{\text{eff}} = -1 + \beta^2/3$  for  $u \rightarrow +\infty$ .

The asymptotic behavior of the effective equation-of-state parameter implies that, for  $|\beta| < \sqrt{2}$ , the universe enters a period of everlasting accelerated expansion. For values of  $|\beta|$  slightly above  $\sqrt{2}$ , this accelerated expansion still takes place, but does not last forever. For instance, in the case  $\beta = 3/2$ , shown in Fig. 5, accelerated expansion occurs at the present time ( $w_{\text{eff}} < -1/3$ ) and then ceases as  $w_{\text{eff}}$  tends to  $-1/4$ .

In summary, in the example we have been considering (base scenario), inflation, driven by the scalar field  $\xi$ , begins at  $u \approx -124.7$  and extends for 60  $e$ -folds, till

$u \approx -64.7$ . During this period, a radiation bath with temperature of about  $10^{15}$  GeV is sustained by a continuous and copious energy transfer from the scalar fields  $\xi$  and  $\phi$ . At the end of inflation, the dissipation coefficients are exponentially suppressed; as a consequence, radiation decouples from the scalar fields  $\xi$  and  $\phi$  and begins to evolve in the usual manner, dominating the dynamics of the universe till  $u \approx -8.6$ . In the absence of dissipative effects, the scalar field  $\xi$  oscillates around its minimum, behaving like a pressureless nonrelativistic fluid (dark matter), and, together with ordinary baryonic matter, becomes dominant at  $u \approx -8.6$ . The scalar field  $\phi$  (dark energy), having played no significant role in the dynamics of the universe during the preceding eras, finally becomes dominant at  $u \approx -0.3$ , giving rise to an everlasting period of accelerated expansion of the universe.

Because we are only interested in models that support accelerated expansion at the present time, we restrict our analysis to the cases  $\beta \lesssim 3/2$ . Furthermore, as pointed out above, we can assume  $\alpha \geq 0$  without loss of generality. Our numerical simulations show that, for values of  $\alpha = \beta$  lying in this interval, the cosmic evolution is quite similar to the base scenario, making it unnecessary to present here a detailed analysis. We just refer the reader to the base scenario and also to Fig. 5, where, the cosmic evolution is outlined for three more cases, namely  $\alpha = \beta = 0, 1/2$  and  $3/2$ .

## B. Case $\alpha \neq \beta$

Let us start by considering a varying  $\beta$  for fixed  $\alpha$  (say  $\alpha = 1$ , as in the base scenario considered in the previous subsection). The evolution of the density parameters for the scalar fields  $\xi$  and  $\phi$ , as well as for radiation and baryonic matter, are shown in Figs. 6 and 7 for the cases  $\beta = 1/2$  and  $\beta = 0$ , respectively, while the evolution of the effective equation-of-state parameter is shown in Fig. 8 for  $\beta = 0, 1/2, 1$  and  $3/2$ .

A first change in the cosmic evolution, as compared with the case  $\alpha = \beta$ , is related to the duration of the matter-dominated era. The more  $\beta$  differs from  $\alpha$ , the earlier the transition from a radiation to a matter-dominated universe takes place and the longer the duration of the latter. This effect is quite mild for  $|\alpha - \beta| \lesssim 1$ , having no implications on the viability of the cosmological solutions (see Fig. 6 and the inset of Fig. 8). However, for  $|\alpha - \beta| \gtrsim 1$ , the effect becomes so strong that it begins to conflict with primordial nucleosynthesis. For instance, in the case  $\alpha = 1$  and  $\beta = 0$ , the transition from the radiation- to the matter-dominated era takes place already at  $u \approx -15.2$  (see Fig. 7), quite near to the primordial nucleosynthesis value  $u \approx -18$ . If one further increases the value of  $|\alpha - \beta|$ , conflict with primordial nucleosynthesis can only be avoided by dropping the requirement that the present-time density parameter  $\Omega_\xi(u_0)$  must be equal to the observational value  $\Omega_{\text{DMO}}$ .

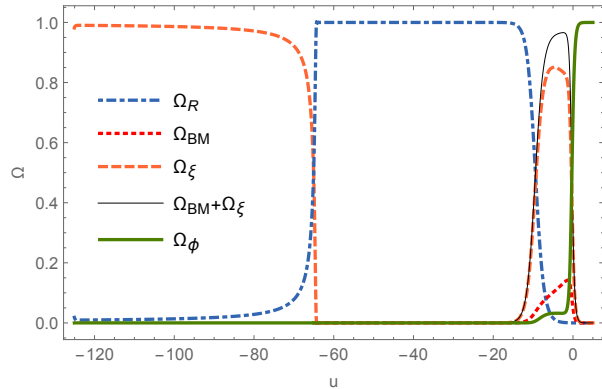


FIG. 6: Evolution of the density parameters for the case  $\alpha = 1$  and  $\beta = 1/2$ . The transition from the radiation- to the matter-dominated era takes place at  $u \approx -9.4$ , a little earlier than in the base scenario. During the matter-dominated era, the density parameter of dark energy is a non-negligible fraction (about 3%) of the density parameter of matter (dark plus baryonic). The density parameter of dark matter reaches its maximum value at  $u \approx -4.8$ , while for baryonic matter this peaking occurs later, at  $u \approx -1.4$ .

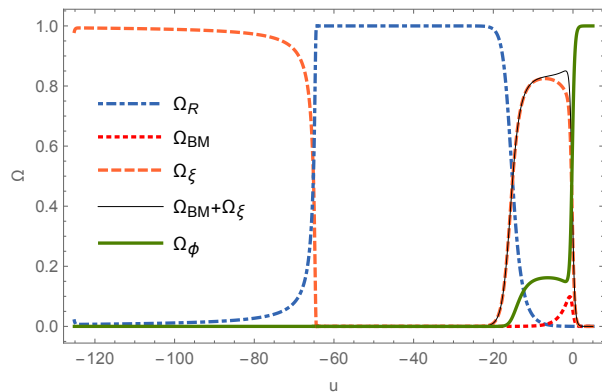


FIG. 7: Evolution of the density parameters for the case  $\alpha = 1$  and  $\beta = 0$ . The transition from the radiation- to the matter-dominated era takes place at  $u \approx -15.2$ , much earlier than in the base scenario. During the matter-dominated era, the density parameter of dark energy is a significant fraction (about 20%) of the density parameter of matter (dark plus baryonic). The density parameter of dark matter reaches its maximum value at  $u \approx -7$ , while for baryonic matter this peaking occurs much later, at  $u \approx -0.8$ .

In other words, one must accept that the scalar field  $\xi$  accounts only for part of the dark-matter content of the universe; the rest must be introduced by hand, together with ordinary baryonic matter. In fact, this was the situation reported in Ref. [15], where, for  $\alpha = 0$  and  $\beta = -\sqrt{2}$ , the scalar field contributed only about 6% to the total matter content of the universe.

Another change in the cosmic evolution concerns the behavior of the scalar field  $\phi$  (dark energy). For  $\alpha \neq \beta$ , it starts to influence the dynamics of the universe much earlier, at the beginning of the matter-dominated era, and

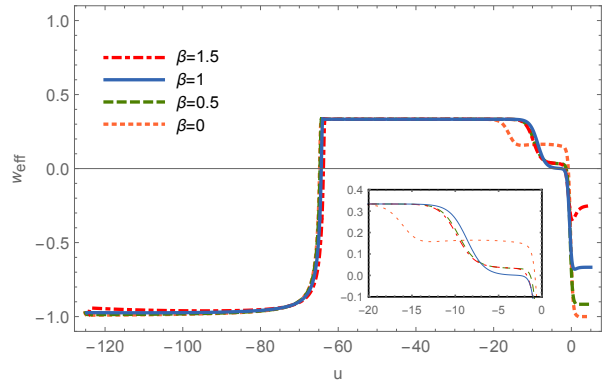


FIG. 8: Evolution of the effective equation-of-state parameter  $w_{\text{eff}}$  for  $\alpha = 1$  and different values of  $\beta$  (the solid blue line corresponds to the base scenario,  $\beta = 1$ ). During the matter-dominated era, the value of  $w_{\text{eff}}$  is approximately zero for  $\beta = 1/2$  and  $3/2$ , similarly to the base scenario (inset plot). However, for  $\beta = 0$ , the situation changes drastically, with the value of  $w_{\text{eff}}$  departing significantly from zero, due to a greater influence of dark energy during this era. For  $\beta = 0, 1/2$ , and  $1$ , the universe initiates, at recent times, a period of everlasting accelerated expansion. For  $\beta = 3/2$  accelerated expansion takes place at the present time  $u_0 = 0$ , but does not last forever, as  $w_{\text{eff}}$  asymptotically approaches the value  $-1/4$ .

its energy density is a non-negligible fraction of the total energy density throughout the matter-dominated era. The higher  $|\alpha - \beta|$ , the greater the fraction of dark energy during this era. For instance, in the case  $\alpha = 1$  and  $\beta = 1/2$ , shown in Fig. 6, the density parameter of dark energy is about 3% of the density parameter of matter (dark plus baryonic), while in the case  $\alpha = 1$  and  $\beta = 0$ , shown in Fig. 7, this percentage increases to 20%. This behavior of dark energy implies that, during the matter-dominated era, the value of the effective equation-of-state parameter  $w_{\text{eff}}$  differs from zero (see Fig. 8). Note, however, that for the cases  $\beta = 1/2$  and  $3/2$  the dark-energy fraction of the total energy density during the matter-dominated era is much smaller than in the case  $\beta = 0$ , meaning that the value of  $w_{\text{eff}}$  in these cases remains close to zero during the matter-dominated era.

A third change occurring in the cosmic evolution is related to the peaking of the energy densities of dark and baryonic matter. In the case  $\alpha = \beta$ , the energy density of dark energy, given by Eq. (29), evolves exactly as the energy density of ordinary baryonic matter (i.e., as  $e^{-3u}$  or, in terms of the scale factor, as  $a^{-3}$ ), meaning that the ratio between  $\rho_\xi$  and  $\rho_{\text{BM}}$  is constant throughout time and, consequently, the peaking of these two quantities occurs simultaneously. For  $\alpha \neq \beta$  the situation is quite different. The ratio  $\rho_\xi/\rho_{\text{BM}}$  depends directly on the behavior of the scalar field  $\phi$ , namely,  $\rho_\xi/\rho_{\text{BM}} \propto \exp[(\alpha - \beta)\kappa\phi/2]$ . This implies that the peaking of the energy densities of dark matter and ordinary baryonic matter does not occur at the same time. For instance, in the case  $\alpha = 1$  and  $\beta = 0$ , shown in Fig. 7, the density parameter of dark matter reaches its maximum value at  $u \approx -7$ , while for baryonic

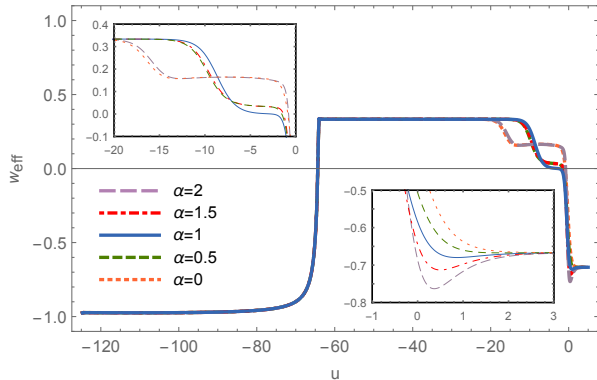


FIG. 9: Evolution of the effective equation-of-state parameter  $w_{\text{eff}}$  for  $\beta = 1$  and different values of  $\alpha$  (the solid blue line corresponds to the base scenario,  $\alpha = 1$ ). At recent times, the universe experiences a second period of accelerated expansion; as clearly shown in the bottom right inset plot, independently of the value of parameter  $\alpha$ ,  $w_{\text{eff}}$  asymptotically approaches  $-2/3$ . Due to the earlier influence of dark energy, during the matter-dominated era,  $w_{\text{eff}}$  differs significantly from zero in the cases  $\alpha = 0$  and  $\alpha = 2$  and mildly in the cases  $\alpha = 1/2$  and  $\alpha = 3/2$  (see upper left inset plot).

matter this peaking occurs much later, at  $u \approx -0.8$ .

As seen above, agreement with current cosmological data requires  $|\alpha - \beta| \lesssim 1$ , in order to guarantee that the radiation-dominated era lasts long enough to encompass primordial nucleosynthesis, and  $|\beta| \lesssim 3/2$ , in order to guarantee accelerated expansion at the present time ( $|\beta| < \sqrt{2}$  to guarantee that this accelerated expansion lasts forever). For such values of the parameters  $\alpha$  and  $\beta$ , the two-scalar-field cosmological model given by action (1) allows for a triple unification of inflation, dark energy, and dark matter which is, at least qualitatively, consistent with observations.

The above conclusions are confirmed by the numerical solutions obtained for other values of the parameters  $\alpha$  and  $\beta$ . For completeness, the evolution of the effective equation-of-state parameter for the case  $\beta = 1$  and varying  $\alpha$  is presented in Fig. 9.

### C. Dissipative effects

We conclude the present section with an analysis of the dissipative effects during both the inflationary period and the transition to the radiation-dominated era.

So far we have considered the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  to depend linearly on the temperature, i.e., we have chosen  $p = 1$  in Eq. (16). However, as already referred to in Sect. II A, in the context of warm inflation several other possibilities have been considered, from the simplest, based on general phenomenological considerations, to the more elaborate ones, derived from microscopic quantum field theory. Accordingly, we can extend our analysis to include other values of  $p$ , corresponding,

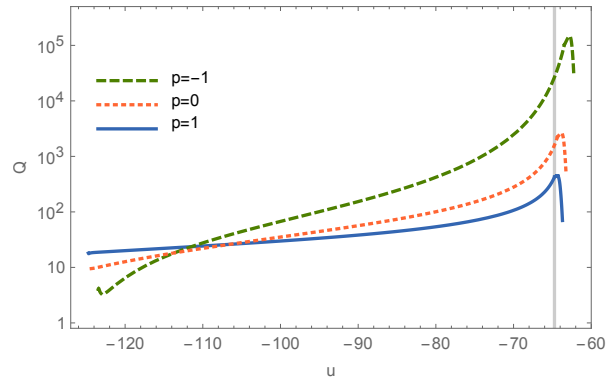


FIG. 10: Evolution of the dissipation ratio  $Q$  for  $q = 2$  and  $p = -1, 0, 1$ , corresponding to dissipation coefficients inversely proportional to the temperature, constant, and proportional to the temperature, respectively.

in particular, to dissipation coefficients constant throughout the inflationary period ( $p = 0$ ) and dissipation coefficients inversely proportional to the temperature ( $p < 0$ ).

Our numerical simulations show that, for these values of  $p$ , the cosmic evolution proceeds in a similar way to the case  $p = 1$ . In Fig. 10 the evolution of the dissipation ratio  $Q$  is shown for  $p = -1, 0, 1$ . For comparison purposes, we choose the duration of the inflationary period to be the same in all three cases, which in turns requires the choice  $f_\xi = f_\phi = 2$  for  $p = 1$ ,  $f_\xi = f_\phi = 3.1 \times 10^{-4} m_{\text{P}}$  for  $p = 0$ , and  $f_\xi = f_\phi = 3.8 \times 10^{-8} m_{\text{P}}^2$  for  $p = -1$  (the initial conditions and the values of the other parameters are the same as those of the base scenario).

In warm inflation, the dissipative effects — and the consequent energy transfer from the inflaton to the radiation bath — play an essential role, but they should vanish soon after the end of the inflationary period, allowing cosmic evolution to further proceed in the usual way. In our cosmological model, this is achieved by assuming that, immediately after the end of inflation, the dissipation coefficients are suppressed by an exponential term parameterized by  $q$  [see Eq. (16)]. In all the cases considered so far, we have chosen  $q = 2$ . However, the suppression of dissipative effects after the inflationary period can be chosen to proceed slower or faster. This is illustrated in Fig. 11, where the dissipation ratio  $Q$  is shown for different values of  $q$ , namely  $q = 1, 2, 3$ , with  $p = 1$  and  $f_\xi = f_\phi = 2$ .

Let us note that for  $p > 2$ , the dissipation coefficients  $\Gamma_\xi$  and  $\Gamma_\phi$  are suppressed naturally after inflation, with no need for an explicit exponential suppression term in Eq. (16). This can be seen as follows.

After the end of the inflationary period ( $u > u_{\text{E}}$ ), cosmic evolution is dominated by radiation. Let  $\tilde{u} > u_{\text{E}}$  be the value of  $u$  above which the dissipative terms in Eq. (13) are much smaller than the energy density of radiation, i.e.,  $(\dot{a}/a)(\Gamma_\xi \xi_u^2 + \Gamma_\phi \phi_u^2) \ll \rho_{\text{R}}$ . Then, for  $u > \tilde{u}$ , the energy density of radiation evolves approximately as  $\rho_{\text{R}}(u) \simeq \tilde{\rho}_{\text{R}} e^{-4(u-\tilde{u})}$ , where  $\tilde{\rho}_{\text{R}} \equiv \rho_{\text{R}}(\tilde{u})$ . Inspection of the

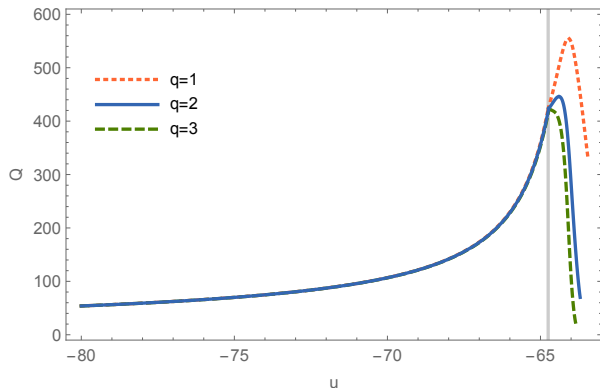


FIG. 11: Evolution of the dissipation ratio  $Q$  for  $p = 1$  and  $q = 1, 2, 3$ . The higher the value of  $q$ , the faster the dissipation ratio  $Q$  is suppressed after the inflationary period.

numerical solutions of Eq. (14) reveals that, for  $u > \tilde{u}$ , the Hubble parameter is given, in a good approximation, by  $H \simeq \kappa(\rho_R/3)^{1/2}$ . Then, from Eqs. (16) and (45) one obtains an approximate analytical expression for the dissipation ratio as a function of  $u$ , valid for  $u > \tilde{u}$ ,

$$Q \simeq A_1 \exp \left[ 1 - (p-2)(u - \tilde{u}) - A_2 e^{q(u-\tilde{u})} \right], \quad (47)$$

where  $A_1$  and  $A_2$  are given by

$$A_1 = \frac{f_{\xi, \phi}}{\sqrt{3}\kappa} \left( \frac{30}{\pi^2 g_*} \right)^{\frac{p}{4}} \tilde{\rho}_R^{\frac{p-2}{4}}, \quad (48)$$

$$A_2 = \left( \frac{\rho_{RE}}{\tilde{\rho}_R} \right)^{\frac{q}{4}}, \quad (49)$$

and  $\rho_{RE} \equiv \rho_R(u_E)$  denotes the energy density of radiation at the end of the inflationary period. For  $p \leq 2$ , the dissipation ratio  $Q$  does not decrease if  $q = 0$ , but for  $p > 2$  the parameter  $q$  can be set to zero and suppression nevertheless takes place (see Fig. 12). This result is in agreement with Ref. [25], where, in the context of a quintessential inflationary model, the dissipation coefficient, assumed to depend both on the temperature and the inflaton field as  $\Gamma \propto T^p \xi^c$ , is shown to be a decreasing function of the number of  $e$ -folds for  $p > 2$ .

So far, the cases considered correspond to a strong dissipative regime, for which  $Q > 1$ . However, it is of relevance to show that a weak dissipative regime can also be obtained in our two-scalar-field cosmological model for other choices of the initial conditions and the values of the parameters. Indeed, as already mentioned above, the confrontation of warm-inflation predictions with the latest CMB data [34–39] has shown a preference for the weak dissipation regime for certain combinations of the inflaton's potential and the dissipation coefficients (for instance, for a quartic potential and a dissipation coefficient proportional to  $T$  or  $T^3$ ).

From Eq. (45), it follows that a smaller value of the dissipation ratio at the onset of the first stage of evolution,  $Q_\xi(u = u_i)$ , can be obtained by decreasing  $f_\xi$

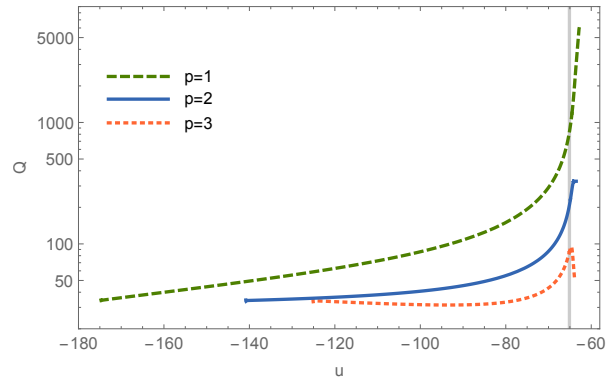


FIG. 12: Evolution of the dissipation ratio  $Q$  for  $q = 0$  and  $p = 1, 2, 3$ . For  $p > 2$ , as a result of the background dynamics, suppression takes place even in the absence of the exponential factor in Eq. (16).

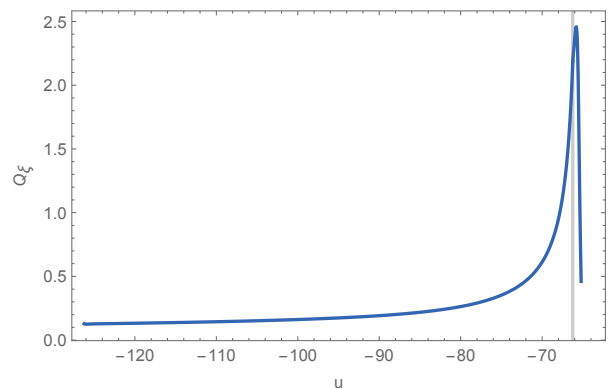


FIG. 13: Evolution of the dissipation ratio  $Q_\xi$ . Weak dissipation is maintained throughout most of the inflationary period. Towards its end, the dissipation ratio sharply increases, reaching values above unity, which facilitates the transition to the radiation-dominated era.

and/or increasing  $\xi(u_i)$  and  $\rho_R(u_i)$ . Starting from the base scenario, considered in detail in Sect. III A, it is then straightforward to find a set of initial conditions and values of the parameters corresponding to weak dissipation, as, for instance,  $\xi(u_i) = 3.19 m_P$ ,  $\phi(u_i) = 10^{-3} m_P$ ,  $\xi_u(u_i) = 10^{-2} m_P$ ,  $\phi_u(u_i) = 10^{-5} m_P$ ,  $\rho_R(u_i) = 2.2 \times 10^{-12} m_P^4$ ,  $V_a = 2.69 \times 10^{-123} m_P^4$ ,  $m = 10^{-5} m_P$ ,  $f_\xi = 0.05$ ,  $f_\phi = 25$ ,  $p = 1$ , and  $q = 2$ . The evolution of the dissipation ratio  $Q_\xi$  for this case is shown in Fig. 13 (the effective equation-of-state parameter and the density parameters evolve similarly as in the base scenario, making it unnecessary to present them here).

The primordial spectrum of density perturbations of our unification model and its agreement with observational data, for the quadratic potential, given by Eq. (2), and the dissipation coefficients, given by Eq. (16), both in the strong and weak dissipative regimes, will be explored in future work.

#### IV. CONCLUSIONS

In this article we have presented a unified description of inflation, dark energy, and dark matter in a two-scalar-field cosmological model with a non-standard kinetic term and an exponential potential. Such models arise in a great variety of gravity theories, such as the Jordan-Brans-Dicke theory, Kaluza-Klein theories,  $f(R)$ -gravity, string theories, and hybrid metric-Palatini theories of gravity.

In the proposed triple unification, one of the scalar fields plays the role of inflaton and dark matter and the other plays the role of dark energy. More specifically, inflation, assumed to be of the warm type, is driven by the scalar field  $\xi$ , which, shortly after the end of the inflationary period, decouples from radiation and begins to oscillate rapidly around the minimum of its potential, thus behaving like a cold-dark-matter fluid; the second scalar field  $\phi$  emerges, at recent times, as the dominant component of the universe and gives rise to an era of accelerated expansion. Thus, seemingly disparate phenomena like inflation, dark energy, and dark matter are unified under the same theoretical roof using scalar fields.

The two-scalar-field cosmological model given by action (1) contains two dimensionless parameters,  $\alpha$  and  $\beta$  (for  $\alpha = 0$ , the kinetic term for the scalar field  $\xi$  becomes canonical; for  $\beta = 0$ , the direct coupling in the potential between the two scalar fields  $\xi$  and  $\phi$  disappears). These parameters could, in principle, be chosen freely; however, as detailed in Sect. IIIB, the requirement that the transition from the radiation- to the matter-dominated era does not occur too early in the cosmic history and, consequently, does not conflict with primordial nucleosynthesis, as well as the requirement that the expansion of the universe is accelerating at the present time, imposes constraints on the parameters  $\alpha$  and  $\beta$ , namely,  $|\alpha - \beta| \lesssim 1$  and  $|\beta| \lesssim 3/2$ .

For such values of  $\alpha$  and  $\beta$ , the picture emerging in our unified description of inflation, dark energy, and dark matter is consistent with the standard cosmological model. Indeed, the inflationary period is followed by a radiation-dominated era that encompasses the primordial nucleosynthesis epoch; the matter-dominated era lasts long enough for structure formation to occur; the transition to a dark-energy-dominated universe takes place in a recent past; and the density parameters for dark matter and dark energy, as well as for radiation and ordinary baryonic matter, evaluated at the present time, are in agreement with current cosmological observations (see Fig. 4 for the case  $\alpha = \beta = 1$ , Fig. 6 for the case  $\alpha = 1$ ,  $\beta = 1/2$ , and Fig. 7 for the case  $\alpha = 1$ ,  $\beta = 0$ ).

Of crucial importance for consistency with the standard cosmological model is the value of the energy density of the scalar field  $\xi$  at the moment when this field begins to oscillate rapidly around its minimum, changing its behavior from an inflaton field to a nonrelativistic dark-matter fluid (i.e., at the transition from the first to second stage of cosmic evolution). If this energy density

is too large, the radiation-dominated era is too short, conflicting with primordial nucleosynthesis; if it is too small, the matter-dominated era is not long enough for structure formation to take place (see Fig. 2). In our model, an appropriate value of the energy density of the inflaton/dark-matter field at the transition between the first and second stages of evolution is achieved by ensuring that, immediately after the end of the inflationary period, the dissipation coefficients, responsible for the energy transfer from the scalar fields  $\xi$  and  $\phi$  to the radiation bath, are rapidly suppressed, becoming negligible soon afterwards. When this happens, the scalar field  $\xi$  decouples from radiation and, because its effective mass is larger than the Hubble parameter, begins to oscillate rapidly around its minimum, behaving like dark matter.

In our cosmological model, the behavior of the dissipation coefficients is controlled by two parameters,  $p$  and  $q$ . The first sets the temperature dependence of the coefficients and the second determines how fast these coefficients are suppressed after the end of the inflationary period [see Eq. (16)]. As we have shown, for  $p > 2$ , the dissipation coefficients are suppressed naturally after inflation, with no need for an explicit suppression term, allowing us to set  $q = 0$ . This does not mean, however, that the models with  $p \leq 2$ , requiring  $q \neq 0$ , are less admissible. Actually, such models, have been proposed in recent years [31–34] and are quite sound, both theoretically and observationally.

As already mentioned above, shortly after the end of the inflationary period, the scalar field  $\xi$  begins to oscillate rapidly around the minimum of its potential. We have derived an expression for its energy density during this oscillating phase [see Eq. (29)], showing that it is proportional to  $\exp[(\alpha - \beta)\kappa\phi/2]$ . In the case  $\alpha = \beta$ , this exponential equals unity and dark matter behaves exactly as ordinary baryonic matter, i.e., evolves as  $a^{-3}$ , where  $a$  is the scale factor. But for  $\alpha \neq \beta$  the situation is quite different; the energy density of dark matter depends directly on the dark-energy field  $\phi$ , leading to a non-simultaneous peaking of the energy densities of dark matter and ordinary baryonic matter (see Figs. 6 and 7). Furthermore, in the case  $\alpha \neq \beta$ , the energy density of the dark energy is a non-negligible fraction of the critical energy density throughout the matter-dominated era.

Finally, we have shown that the effective equation-of-state parameter  $w_{\text{eff}}$  tends, asymptotically, to  $-1 + \beta^2/3$ , implying that, for  $|\beta| < \sqrt{2}$ , the universe enters a period of everlasting accelerated expansion (for values of  $|\beta|$  slightly above  $\sqrt{2}$ , this accelerated expansion still takes place, but does not last forever).

In this article, we have proposed a triple unification of inflation, dark energy, and dark matter in a two-scalar-field cosmological model. This is a first approach, intended to show that such a unification is, in principle, possible and that it reproduces, at least qualitatively, the main features of the observed universe. We expect to explore and deepen this model in future publications.



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