

UNIVERSITY OF THE ALGARVE
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**THREE-WAY MULTIVARIATE METHODS
IN THE EVALUATION OF PATTERNS OF
STABILITY AND CHANGE IN THE
EUROZONE COUNTRIES**

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Statement of authorship

I hereby declare to be the author of this work, which is original and unpublished. Authors and studies reviewed are properly cited in the text and listed in the included bibliography listing.

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ABSTRACT

In Multivariate Data Analysis it is usual to have several quantitative data tables, which are obtained in different occasions or conditions and standard two-way methods often fail to find the underlying structures in multiway arrays. With increasing number of application areas, multiway data analysis has become popular as an exploratory analysis tool, particularly in using methods such as STATIS (*Structuration de Tableaux A Trois Indices de la Statistique*) and MFA (Multiple Factor Analysis). The goal of these asymmetric methods is to compare multiple data tables, as well as investigate if there is a common structure between them. On the other hand, within the symmetrical methods, the TUCKER3 model is a three-mode method and permits a visualization of the structural relationships within three-way data tables, allowing a confirmatory analysis of the changes observed. They are similar according to the representation of individuals, variables, conditions and trajectories of individuals throughout conditions. The advantage of MFA compared to STATIS is that no group influences preponderantly the first axis of the compromise euclidean image allowing further interpretation of the axes. In turn, TUCKER3 has advantages over the MFA as the generation of additional trends and the measure of not only the relationships between components but also the interrelations among individuals, variables and conditions, which favor more interpretations. Nonetheless the STATIS and MFA vocation to study common data structures with the use of TUCKER3, by contrast, we intend to expand this information. However, the major aim of applying three-way data analysis is to reveal complex patterns of dependencies between the observations. In fact, the identification of benefits by the three methods is the feature which suggests consider them as complementary between themselves. This investigation intends to clarify the process that articulates growth with economic development in a context of comparable economies, and on a timeline that would not only allow evaluating periods of stability but also of change. In this sense, a set of disaggregated data on economic growth and development in twelve countries of the Eurozone, during the period 2005-2010 – the re-launch of the Lisbon Strategy – is analyzed. The aim is to describe and to compare the countries in their economic dynamics in periods of stability and change, during the three years prior to the 2008 economic and financial crisis and two years later. This theme outline led to the construction of three-dimensional data structures that require multivariate three-way analysis. This is done either in an exploratory perspective, where patterns among grouped variables to be measured from the observed variables are analyzed, or in a confirmatory aspect, starting from theoretical premises that confirm the degree of fit of observed data through constructs. The main findings of the research were that in the three methods there are countries that are similarly characterized and R&D is associated with economic growth. The STATIS method, only analyzed economic growth. Nonetheless it was still possible to analyze the main influences on economic growth as well the similarities in the type of growth in the twelve Eurozone countries. The MFA method distinguishes economic growth of economic development and includes the possibility of evaluating the twelve Eurozone countries, particularizing the role of development in the similarities and in the main influences of the economic identities. The TUCKER3 method examines with more detail the changes and influences of the economic identities, which become more realistic for the twelve Eurozone countries. TUCKER3 was able to identify the 2008 financial crisis, and the main interactions in the Eurozone, where the Benelux countries with Austria, Finland and Ireland revealed positive synergistic effects mostly on economic growth which also produced influence on economic development through Human Development Index (HDI) and, were negatively affected on Number of Patents (NP). The southern countries benefited from a positive synergistic effect from Germany and France on NP. On the other hand, they were negatively affected mostly on economic growth by the same two countries. These levels of dependency also influenced negatively the economic development through HDI. The three-way methods allowed not only to point out useful details for designing a more realistic economic and social diagnosis of the Eurozone for the period 2005-2010, but also to demonstrate the advantage of using information derived from the three-way methods, as a complement to the traditional methods of analysis.

Keywords: Eurozone, economic growth, economic development, STATIS, MFA, TUCKER3

RESUMO

Embora na análise multivariada seja comum existirem várias tabelas de dados quantitativos, obtidas em diferentes ocasiões ou condições, os tradicionais métodos de duas vias não se revelam adequados para encontrar as estruturas comuns subjacentes. Com o crescente número de áreas de aplicação, a análise de dados de vias múltiplas tem-se tornado atrativa como ferramenta de análise exploratória, particularmente pela utilização de métodos como o STATIS (*Structuration de Tableaux A Trois Indices de la Statistique*) e a AFM (Análise Fatorial Múltipla). O objetivo destes métodos assimétricos é comparar várias tabelas de dados e investigar se existe uma estrutura comum entre elas. Embora semelhantes em termos da representação dos indivíduos, das variáveis, das condições e trajetórias de indivíduos através das várias condições, a AFM comparativamente ao STATIS tem a vantagem de nenhum grupo influenciar de forma preponderante o primeiro eixo da imagem euclidiana comprometendo permitindo uma maior interpretabilidade dos eixos. Dentro dos métodos simétricos, o modelo TUCKER3 é um método de três modos que possibilita a visualização das relações estruturais dentro de quadros de dados de três vias, permitindo uma análise confirmatória das mudanças e das interações observadas. Por outro lado o método TUCKER3 apresenta algumas vantagens sobre a AFM como seja a geração de tendências e a medição não só das relações entre os eixos fatoriais, mas também das inter-relações entre indivíduos, variáveis e condições, o que possibilita mais interpretações. Não obstante a vocação dos métodos STATIS e AFM para o estudo de estruturas comuns, pretende-se aumentar essa informação por contraste com os resultados obtidos via TUCKER3. O principal objetivo de aplicar métodos para analisar dados de três vias é o de revelar padrões complexos de dependências entre as observações. Na verdade, a identificação dos benefícios de cada um dos três métodos sugerem considerá-los como complementares entre si. Efetivamente, um dos objetivos desta investigação é mostrar como os dados económicos podem ser interpretados a partir de uma matriz de dados multivariada complexa que, por sua vez, pode ser significativamente melhorada através da aplicação dos métodos de três vias. Deste modo, pretende-se clarificar o processo que articula o crescimento com desenvolvimento económico, num contexto de economias comparáveis, onde as dinâmicas económicas envolvem períodos de estabilidade e de mudança. Tradicionalmente, a análise económica faz uso de um conjunto de ferramentas estatísticas para reproduzir e simular os principais mecanismos de sistemas económicos regionais, nacionais ou internacionais. Assim, a fim de compreender a relação entre as variáveis económicas, aplicam-se modelos matemáticos para ajudar, em consonância com a teoria económica, o processo de tomada de decisão. A maioria destes modelos, que geralmente contêm variáveis macroeconómicas ou agregados, são designados por "económicos" e, geralmente são definidos por estimações mediante uma variedade de métodos de cálculo. No entanto, a teoria macroeconómica não é um campo particularmente consensual de investigação, contendo muitas teorias diversas e conflitantes. Neste contexto, temas relacionados com o crescimento económico ou como o desenvolvimento económico podem, nalguns aspetos, ilustrar esta realidade. Questões quantitativas relativas ao Produto Interno Bruto, à Inovação e as questões sociais, tais como o Bem-Estar, o Índice de Desenvolvimento Humano e a Qualidade de Vida, entre outros, podem produzir interações que não são facilmente capturadas através dos modelos econométricos. De facto, o processo de desenvolvimento económico é geralmente associado à ideia de que uma variação positiva nas variáveis associadas ao crescimento é geralmente acompanhada de mudanças positivas em indicadores sociais como o Índice de Desenvolvimento Humano, resultando em melhoria dos padrões de vida. Porém, isto nem sempre se materializa. A ocorrência destes factos representa, em nossa opinião, uma oportunidade para a utilização de ferramentas de estatística multivariada, tais como os modelos de três-vias, para promover outras abordagens. Na verdade, tem havido pouca convergência científica para ultrapassar as barreiras operacionais e estas abordagens podem contribuir significativamente para melhorar a compreensão das ligações entre várias metodologias de análise. Esta falta de complementaridade deve-se principalmente às diferentes origens disciplinares dos investigadores, aos diferentes métodos utilizados e, no seio da análise multivariada de três-vias, às dificuldades de compatibilidade nalguns dos conceitos utilizados pelas escolas francesa e anglo-saxónica. Este trabalho também representa uma tentativa de reduzir esta lacuna na investigação através da combinação dos três métodos de três-vias, dois da escola

francesa e um da escola anglo-saxónica. O objetivo não é apresentar uma equivalência matemática ou uma notação similar entre as técnicas, mas sim evidenciar a complementaridade da interpretação dos três métodos a qual pode revelar-se extremamente útil para apoiar outras técnicas de análise. Assim, o recurso a um conjunto de dados desagregados sobre os países da Zona Euro, durante o período 2005-2010, coincidente com o relançamento da estratégia de Lisboa, possibilitou a construção de estruturas tridimensionais de dados propícios a uma análise multivariada de três-vias. Seja numa perspetiva exploratória, onde padrões entre condições económicas são analisados ou, num contexto confirmatório mediante premissas teóricas que confirmam a confiança dos dados observados, os métodos propostos possibilitam, em nosso entender, não apenas detalhar a realidade económica e social da Zona Euro, como também destacar informação útil ao processo de decisão em matéria de política económica europeia.

As principais conclusões são de que nos três métodos existem países caracterizados de forma similar e que o investimento em Investigação e Desenvolvimento está associado ao crescimento económico. (i) O método STATIS analisa somente o crescimento económico. No entanto foi possível analisar as principais influências no crescimento económico bem como as similaridades no tipo de crescimento para os doze países da Zona Euro. (ii) A AFM distinguiu crescimento económico de desenvolvimento económico e incluiu a possibilidade de avaliá-los para os doze países da Zona Euro, particularizando o papel do desenvolvimento nas similaridades e principais influências das entidades económicas. (iii) O método TUCKER3 analisou com maior detalhe as mudanças e as influências das entidades económicas, que se tornam mais realistas para os doze países da Zona Euro. Com este método foi possível identificar a crise financeira de 2008, e as principais interações da Zona Euro, onde os países Benelux associados com a Áustria, Finlândia e Irlanda revelaram efeitos sinérgicos positivos, predominantemente no crescimento económico, que produziram influências no desenvolvimento económico através do Índice de Desenvolvimento Humano (HDI) e, por outro lado, foram afetados negativamente no Número de Patentes (NP). Os países do sul sofreram um efeito sinérgico positivo da Alemanha e França no NP. No entanto foram afetados negativamente, essencialmente no crescimento económico, pelos mesmos dois países. Estes níveis de dependência influenciaram também negativamente o desenvolvimento económico através do HDI.

Pensamos que um diagnóstico socioeconómico mais pormenorizado da Zona Euro para o período em causa, resultante da informação dos métodos de três-vias, será vantajoso, sobretudo como complemento aos métodos tradicionais de análise económica. Esta abordagem metodológica descreve, compara e analisa 12 países no que concerne às suas dinâmicas de crescimento e de desenvolvimento durante os 3 anos anteriores à crise económico-financeira de 2008 e nos 2 anos seguintes, recriando assim cenários de estabilidade e de mudança na Zona Euro.

Palavras-chave: zona euro, crescimento económico, desenvolvimento económico, STATIS, AFM, TUCKER3

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LIST OF ABBREVIATIONS

AT	Austria
BE	Belgium
BERD	Business Expenditure in R&D
CDOs	Collateralized Debt Obligations
CRS	Constant Returns to Scale
DE	Germany
DEA	Data Envelopment Analysis
DPCA	Double Principal Component Analysis
EC	European Commission
ECB	European Central Bank
ECSC	European Coal and Steel Community
EEC	European Economic Community
EFSF	European Financial Stability Facility
EL	Greece
EMU	Economic and Monetary Union
EPC	European Patent Convention
EPO	European Patent Office
ES	Spain
ESFS	European System of Financial Supervisors
ESM	European Stability Mechanism
EU	European Union
EU-28	The 28 member states of European Union
EUROATOM	European Atomic Energy Community
EUROSTAT	Statistical Office of the European Union
FI	Finland
FR	France
FRG	Federal Republic of Germany
GCF	Gross Capital Formation

GDP	Gross Domestic Product
HDI	Human Development Index
HDR	Human Development Report
ICI	Interstructure-Compromise-Intrastructure
IE	Ireland
IMF	International Monetary Fund
IT	Italy
LU	Luxembourg
MFA	Multiple Factorial Analysis
NL	The Netherlands
NPISH	Non-Profit Institutions Serving Households
OECD	Organization for Economic Co-operation and Development
PCA	Principal Component Analysis
PPP	Purchasing Power Parity
PT	Portugal
R&D	Research and Development
STATIS	Structuration des Tableaux à Trois Indices de la Statistique
TFP	Total Factor Productivity
TiVA	Trade in Value Added
UNDP	United Nations Development Program
UNPD	United Nations Procurement Division
US	United States of America
USD	United States dollar
VRS	Variable Returns to Scale
WTO	World Trade Organization

CHAPTER 1

INTRODUCTION

“The mental features discoursed of as the analytical, are, in themselves, but little susceptible of analysis. We appreciate them only in their effects.”

Edgar Allan Poe (1809 -1849)

1.1 Overview

The evolution of mathematics and consequently of quantitative methods, associated with computational progress, has enabled the development of statistical language that allows the measurement of social phenomena through scientific multivariate statistical techniques and methods. The integration of information where the interrelationships between variables are explored to their maximum depth, and where the solutions to the real problems become more consistent and useful, produces increased opportunities in the description, representation, knowledge extraction and overall better interpretation of the phenomena.

Traditionally, economic research makes use of a set of statistical tools to reproduce and simulate the main mechanisms of regional, national or international economic systems. This is done in order to understand the relationship between economic variables, applying mathematical models to assist, in consonance with economic theory, the decision-making process. Most of these models, which usually contain macroeconomic variables or aggregates, are designated "econometric" and are generally defined by the data used in their estimation through a variety of possible methods of calculation. Nevertheless, macroeconomic theory is not a particularly consensual field of investigation, containing many diverse and conflicting theories. In that sense, different econometric models can reflect not only different procedures, but also the researcher's point of view. In this context, themes relating economic growth to economic development may, in some aspects, illustrate this reality. Quantitative issues related to gross domestic product, innovation, and social issues such as welfare, the Human Development Index (HDI), and the quality of life, among others, may produce interactions not easily captured in econometric models. The economic development process is usually associated with the idea that a positive variation of variables associated with growth (for example, the Gross Domestic Product (GDP)), or with the nature of innovation or technological capacitation, and is usually accompanied by positive changes in social indicators (such as the HDI) resulting in improvements of life standards. However this does not always materialize.

The occurrence of these facts, in our opinion, poses an opportunity for attaining an advantage by the conciliation of multivariate statistical tools to promote better approaches to overcome methodological difficulties.

In fact, a wide variety of approaches may contribute significantly to improve our understanding of the linkages between growth and economic development. However, possible complementary strands of three-way have rarely been combined. There has been little cross-fertilization, for major operational and methodological barriers have kept any potential convergence to a bare minimum. The main reasons for this lack of complementarity are due to the disciplinary backgrounds of the researchers on multiway analysis, to different methods used in the approaches, and to the compatibility difficulties in some of the concepts employed by the French and Anglo-Saxon schools.

This research represents an attempt to reduce this gap in the research by combining three methods, two of the French school and one of the Anglo-Saxon school in one methodological approach. The aim is to show not a mathematical or notation equivalence between all three methods but a complementarity interpretation that can be useful for these methods and also to other nonmultiway techniques.

The purpose of this investigation is to show how the economic data can be interpreted from a complex multivariate data array which can significantly be improved by the application of three-way methods. This approach not only pointed out useful details for designing a more realistic economic and social diagnosis of the Eurozone for the period 2005-2010, but also demonstrated the advantage of the information derived from the three way methods, as a complement to traditional methods of economic analysis.

In the framework of applied research, this investigation intends to clarify the process that articulates growth with economic development in a context of comparable economies, and on a timeline that would not only allow evaluating periods of stability but also of change. In this sense, a set of disaggregated data on economic growth and development in 12 countries of the Eurozone, during the period 2005-2010 – the re-launch of the Lisbon Strategy – is analyzed. The aim is to describe and to compare the countries in their economic dynamics in periods of stability and change, during the 3 years prior to the economic and financial crisis (2008) and 2 years later. This theme outline led to the construction of three-dimensional data structures that require multivariate three-way

analysis. This is done either in an exploratory perspective, where patterns among grouped variables to be measured from the observed variables are analyzed, or in a confirmatory aspect, starting from theoretical premises that confirm the degree of fit of observed data through constructs. The three-way methods allow us, in our opinion, not only to point out useful details for designing a more realistic economic and social diagnosis of the Eurozone for the period 2005-2010, but also to demonstrate the advantage of using information derived from the three-way methods, as a complement to the traditional methods of analysis.

1.2 The Eurozone

The European Union (EU) is a political and economic union of 28 countries (European Union, 2014a) with a single market through a standardized system of laws (European Union, 2014b) that apply in all member states. The EU has its genesis in the European Coal and Steel Community (ECSC) and the European Economic Community (EEC), created by the Inner Six countries in 1951 and 1958, respectively. The community and its evolutions have grown in size by the admission of new member states and in power by the addition of policy areas to its scope. The Treaty on European Union (1992), also known as Maastricht Treaty established the EU in 1993 and introduced European Citizenship. The most recent major amendment to the constitutional basis of the EU, the Treaty of Lisbon (2007), came into force in 2009.

The Eurozone is an economic and monetary union that was established in 1999 and effectively materialized in 2002. Since December 2014, the Eurozone is composed of 18 member states that use the euro as their legal tender: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain. The Eurozone is the second largest economy in the world and, in an aggregated view, has a population of 330 million inhabitants. France, Germany, and Italy are the three most important economies accounting for the majority of the Union's GDP.

One of the main objectives encompassed in the management of this Economic and Monetary Union and the euro was the achievement of economic stability, because it encourages economic growth, which brings prosperity and employment.

According to the established Economic and Monetary Union (EMU), EU member states coordinate their economic policies with the objective of maintaining economic stability. The European Central Bank (ECB) conducts the EU monetary policy with the aim of promoting low inflation in the Eurozone (European Commission, 2013a). Economic stability and low inflation create the environment prone to sustainable long-term growth, which benefits the Eurozone Member States.

1.3 Technology and innovation as drivers for economic growth

The more developed countries compete, on an international level, between each other and also with less developed countries of low-wage economies that can produce products at a much lower cost and put them on the market. On the current market outline this can pose a competitive disadvantage for the former. Therefore international competition between economies, amongst many other aspects, has been fought between paradigms of advantage by innovation and technology and advantage by the form of low cost at the expense of low wages and other social indicators. In contrast, when the more developed countries were starting their early growth process, they were technologically much more advanced than the rest of the world. They could therefore maintain their advantage by designing and developing new technology at a rate defined by their own long-term economic growth requirements.

In this context, a race for innovation advantage has emerged, where countries compete intensely on to achieve the highest levels of innovation-based economic growth (Wein *et al.*, 2014). Many countries, to increase their competitiveness in this race, have implemented national innovation policies aimed at promoting the ability of companies and organizations in their economy to achieve increased innovation and productivity gains. Alternatively, other countries are trying to attain an advantage by engaging on innovation mercantilist practices that distort global trade by attempting to redirect the location of innovation activity to their territories at the cost of other countries (Ezell, 2011).

East Asian and Northern European countries, along with the United States of America (US), are leading the world in this race for global innovation advantage.

In a comparison of the US versus the EU, the US continues to outperform the EU in innovation capacity. This is illustrated in the EU's own Innovation Union Scoreboard released by the European Commission (EC) (2014), which concluded that the US innovation performance in 2014 is still superior to that of the EU countries.

The focus on innovation by the EU was particularly noticeable with the definition and implementation of initiatives such as the Lisbon strategy (Lisbon European Council, 2000) and the new Europe 2020 Strategy (European Commission, 2010a). The Lisbon

strategy, also referred to as the Lisbon Agenda, is a strategic development plan of the EU approved by the European Council, which proposed the conversion of the economy of the EU into a more competitive and dynamic knowledge-based economy before 2010. This strategic objective should be able to promote lasting economic growth together with a quantitative and qualitative improvement in employment and greater social cohesion.

The EU has made some progress towards the objective of investing 3 % of GDP in Research and Development (R&D). Nonetheless it contrasts with the remarkable growth of R&D intensity in the major Asian research-intensive countries. Despite this progress, most Member States remained far from the national 2010 targets they set for themselves as a result of the Lisbon Agenda in 2005 (Commission of the European Communities, 2005).

From both theoretical and empirical perspectives, it is widely accepted that technology and technological advances are an essential component of innovation and economic growth. Currently there is a convergence of views between various authors with regard to the existence of a positive relationship between the level of technological development and economic growth of countries. Schumpeter (1943), with his classic 1943 book "Capitalism, Socialism and Democracy", started a different take on economics and on the economy. For Schumpeter, technological change was fundamental for economic growth, and creative destruction was central in capitalism. In 1957, with the publication of the article "Technical Change and Aggregate Production Function" by Solow (1957), productivity growth of economies was mainly explained by the importance given to the introduction of technological innovations. In fact, innovation and knowledge began to be regarded as important sources of economic growth in countries, especially in developed countries, which have excelled in terms of growth in production, productivity, and international trade. Technology has been assumed as key for raising standards of living. And, in this matter, economic policies began to promote a process of development associated with technological capacitation and with innovation gains, as well as increased participation in international markets and, above all, the expansion and strengthening of the internal market. In reality the investment in R&D was considered strategic to ensure technological potential and as a result, innovation and economic growth. In addition to extending the possibility of achieving a higher technological level in companies and regions, which enables the introduction of new products or processes, it allows greater

growth and profitability. Nonetheless, the economic environment of the innovation process is fraught with uncertainties and risks and, in the course of investment decision-making in technology, economic operators take on even higher risks than the existing ones.

It is commonly assumed that R&D activities generate knowledge which, if successful, can lead to the introduction of new technologies, processes or products with added value, and suitable for economic exploration. Countries, with their specificities, produce different outcomes from inputs related to innovation knowledge and technology and, as final output, economic growth increases the market value of goods and services produced, leading to the better economic performance of a country. Higher levels of standards of living, education, and health are usually associated with the better-performing countries.

In the EU, the Lisbon Agenda (Commission of the European Communities, 2005) purposed a change of paradigm for the member states, focusing on the creation of knowledge as a catalyzer for economic growth. This initiative was planned under the assumption that the participating countries would dedicate capital to implement it. It wasn't fully fulfilled at the end of the program, but public and private investment in R&D was growing up to the time of the economic crisis. Following the outbreak of the crisis, a majority of Member States maintained or increased their R&D investments, despite fiscal constraints, and overall R&D spending over GDP increased until 2010 (European Commission, 2013b). However, there were gaps between Member State performances. Furthermore, in Member States where the business sector was knowledge-intensive and internationally competitive, the government's strategy to protect R&D spending helped maintain the level of private investment. However, this proved more challenging for countries suffering from sovereign debt crisis. In these countries liquidity constraints combined with an insufficiently innovation-friendly environment and a lower level of business demand for knowledge, prejudiced the effectiveness of the counter-cyclic measures to stimulate business investment (European Commission, 2013b).

Nonetheless, the start of the recovery in 2010 was substantially stronger in countries which had previously invested the most in R&D and innovation, such as Germany, Finland and Sweden (European Commission, 2013b).

1.4 The 2008 global crisis as a turning point of stability and change

The occurrence of crisis can disturb the economic stability of countries, changing their economic output and forcing the reallocation of resources and priorities, since former assumptions associated with the previous scenario, which led to the current planning and strategic objectives, no longer exist. This can have a negative effect on many economic aspects of a country and is necessarily an undesirable situation.

From an economic perspective, stability is concerned, above all, with the control of the economic and financial situation of a country by 1) preventing economic and financial crises, and large swings, in economic activity, 2) promoting price stability, the reduction of social inequality, and 3) improving the purchasing power of the low-income population.

Economic stability benefits society, in particular social cohesion and the underprivileged (European Commission, 2013c). Volatile changes in inflation and interest rates increase the gap between the richer and poorer groups and regions, as those with more economically privileged have better conditions to protect themselves. With stable inflation and interest rates, the underprivileged are better protected against a downfall in their wealth, their savings and their purchasing power.

In this context, economic stability refers to the absence of large fluctuations in the macro-economy, therefore an economy with reasonably constant output growth and a low and stable inflation rate is considered economically stable. On the other hand, an economy characterized by large recessions, a severe business cycle, a very high or volatile inflation, or with financial crises is considered economically unstable.

Naturally a dynamic economy necessarily involves instability to some extent, as well as some structural change. Nonetheless, the challenge for policymakers is to minimize instability, without reducing the economy's ability to improve living standards through rising productivity and employment.

From a stability perspective, as with consumers and companies, governments and their electorates, Eurozone citizens benefit from economic stability. Low inflation is a strong well-managed Eurozone achievement that makes government borrowing less expensive.

This translates into a reduction in interest repayments on national debt, releasing large amounts of taxpayers' taxes, previously allocated to repay the interest, for other national priorities. Economic stability also makes the Eurozone more resilient to external economic shocks, such as unexpected economic changes that may emerge and unsettle national economies, and also turbulence on global currency markets or worldwide oil price rises. The Eurozone, with its size and structure, would be better suited to absorb such external shocks avoiding or minimizing job losses and lower growth (European Commission, 2013c).

The 2008 financial crisis, considered by many economists to have been the worst financial crisis since the Great Depression of the 1930s, erupted in August 2007, first in the US, when products not recorded on the balance sheet that had been set up by banks to manage portfolios of complex structured credit securities ran into liquidity problems. This situation escalated as uncertainties increased about the nature and degree of exposure of banks to these so-called toxic assets. In this context the culmination was the bankruptcy of Lehman Brothers in 2008, an event which triggered an unparalleled surge of volatility across financial markets and a generalized decline of asset prices. This event exposed fundamental weaknesses in financial systems and had repercussions worldwide. The European banking system was also heavily impacted by the US sub-prime crisis which threatened the total collapse of financial institutions. This was prevented by the bailout of banks by national governments. Nonetheless stock markets dropped worldwide.

The condition of the financial system was reflected in real economic performance, contributing to a sharp economic slowdown across the developed economies, as credit tightened and international trade declined. This crisis played a significant role in business failures and reduced consumer wealth, and a caused a weakening in economic activity contributing to the 2008–2012 global recession. Governments and central banks responded with unprecedented fiscal stimulus packages, expansionary monetary policies and institutional bailouts.

This scenario had negative consequences for economic growth but also for economic development as it increased unemployment and negatively affected living standards.

1.5 Challenges for economic growth and development of the EU

Economic growth is a common goal for any policy maker, as a positive change in the output, or production, of a country or of an economy corresponds to a general increase in the output of a country's products and services contributing to its prosperity. This growth is usually evaluated and quantified by the rate of variation of the GDP. Traditionally economic growth depends on factors such as market dimension, investment and technical progress.

Economic development is also an essential goal for any country's policy maker, as it corresponds to society's ability to satisfy its needs and to attain a determined level of well-being. Economic development usually increases economic health and social welfare, and ensures a stable political environment. There will be economic development when growth is translated into a qualitative spectrum. For example, when it is extended to the various regions of the country, it contributes to reduce asymmetries of income distribution, respects citizens and promotes social well-being. The most widely-used method of measuring economic development is the HDI which takes into account three goals of development: longevity, knowledge and standard of living.

In this context, economic growth is a necessary condition but is not sufficient to attain economic development and, this is likewise necessary for economic growth, while it can also limit it. In reality, the allocation of resources to maintain or improve quality of life can reduce the produced quantities, while on the other hand, a reduction of inequality, the improvement of life quality in general terms, and education for the population are favorable factors for economic growth. In sum, the final objective of a country is, in reality, economic development, and economic growth is a means to attain it.

In terms of economic growth, the EU competes directly with the US and, on another level, with other high-growth-rate parts of the world, such as the developed Asian economies (Japan, South Korea, Singapore and Taiwan) (Lin and Rosenblatt, 2012; Knox *et al.*, 2014).

Regarding economic development, the EU countries have attained considerable performances and very high values in the HDI, according to the United Nations Development Program (UNDP). Nonetheless, there are major differences in social

indicators between EU countries and other countries outside its shores, and there are still intra-EU discrepancies (Bubbico and Dijkstra, 2011).

The fundamentals and main goals of promoting lasting economic growth together with a quantitative and qualitative improvement in employment and greater social cohesion, as defined by the Lisbon Strategy were aligned in a context of awareness of constraints and the means of implementation of the European Strategy, for the decade 2000-2010 (European Commission, 2010a). It is important to refer that during this period it became perceptible that the goals were not being integrally reached, to a large extent due to the evolution of the economic and financial crisis that occurred, and to the subsequent change of economic paradigm.

1.6 Problematic of mechanisms and methods for economic development and growth analysis

Economists have done extensive research on the determinants of economic growth. Scientific research on this subject has been developed, focusing on the differences between countries regarding their technological capabilities, political and institutional frameworks, and ultimately, on their results in terms of economic growth.

While various models have been developed, there are few widely agreed-upon results (Klenow and Rodriguez-Clare, 1997). This lack of consensus is natural to a degree because of the different visions and assumptions involved, but it is also unfortunate because the increase in the growth of countries economies is a main global policy objective.

When economists use statistical methods of analysis, other differences between countries that are difficult to measure can be pertinent, and the results are not as direct. We can show that factors such as R&D investment and level of education are strongly correlated with technological capacitation, but it is not so easy to demonstrate that they are correlated with current growth.

In a superficial approach, finding the determinants of growth may seem simple. From this perspective, it would be necessary to do no more than to identify what the prosperous

countries did that other countries failed to do. Yet this undertaking has not been easy because not all fast-growing countries have identical policies, whereas some high growth and low growth countries may have similar policies.

This kind of analysis is frequently done using temporal data, which can give rise to potentially inappropriate time averaging issues, when studying countries. Also, if using only one observation per country in a statistical analysis, this will require the inclusion of a large number of countries which can be problematic since the parameters of empirical growth models may vary widely across groups of countries. Another problem can arise when studying a heterogeneous group of countries and the richer countries are removed from the sample and its effect on conditional convergence (Bilbao-Osorio and Rodriguez-Pose, 2004). Other methodological difficulties arising from conditions of structural change were detected in the econometric work of Foster (1991:1999) about the dynamics of evolutionary systems.

Another statistical issue that makes empirical growth research difficult is the problem of reverse causality. We are rarely sure whether the variables expected to cause growth do so, or whether they are themselves caused by growth. In this sense, we can assume that economic development helps growth, nonetheless economic growth itself may causes economic development. Another difficult issue is to determine, with reasonable confidence, which particular variable can be excluded.

1.7 A three-way multivariate approach

The study of evolution and change are key aspects in the analysis to characterize the addressed theme. However, empirical research works on this context are not common, perhaps due to analytical and methodological difficulties already detected in previous econometric works about the stability and change associated with these dynamic evolving systems.

The application of three-way methods can respond to the requirements of this research in the sense that the addressed theme bypasses multiple data structures, involving a set of matrices that result from a set of individuals (countries), a set of variables (macroeconomic technology, R&D, and those of a social nature) on different experimental occasions or conditions (studies or perspectives).

Non-three-way methods operate by flattening the data cube, or stringing out its contents. The idea is to transform three-way data into two-way data by eliminating one of the ways. Instead of studying and representing the interactions between three types of ways, only two are analyzed (Kroonenberg, 2008).

In the three-way methods there are two main families, the French school (Escofier and Pagès, 1984; Escofier and Pagès, 1998; Pagès, 2002), and the Anglo-Saxon school (Tucker, 1963 and 1966; Kroonenberg and Leeuw, 1980).

The French school methods are focused on comparing configurations of the same individuals or the same variables at different temporal or spatial horizons and to find a common structure which is stable and representative of all data tables and of asymmetric nature because the 3rd way (usually occasions or conditions) has a different treatment from the other ways (individuals and variables). These methods lie within a perspective called Interstructure-Compromise-Intrastructure (ICI), a designation which corresponds to the three steps that are undertaken in *Struturation des Tableaux à Trois Indices de la Statistique* (STATIS) (L'Hermier des Plantes, 1976; Lavit, 1988 and Lavit *et al.*, 1994) and in Multiple Factorial Analysis (MFA) (Escofier and Pagès, 1985).

The Anglo-Saxon school methods are focused on the differences, are symmetric in nature and were based on the seminal works of Tucker (1966) who created a method of treating

the three modes of a data structure equally by introducing a new perspective based on the idea that each way is associated with an underlying factor structure, and that there is not only one group of factors, but three. In other words, the three data inputs are associated, each one with another via (not observable) that can be regarded as an optimum set of factors or categories. The objects, variables and occasions are considered as linear combinations of the "idealized" objects, "latent" variables and "prototype" occasions respectively. Relations between the three types of factors are considered in a table or matrix estimated by the model and called the core matrix. However, the original TUCKER model had the disadvantage of not being able to give approximations of the data in the sense of ordinary least squares, which is why the TUCKER3 (Tucker 1963, 1964, 1966) model was developed.

1.8 The aims of the research

The methodological proposal and its refinements will enable a richer, three-dimensional approach to studies of data arrays over time periods in relation to the extraction of relevant information contained in multiple data sets through simultaneous factorial spatial representations. In this way, intending a substantial improvement in the interpretation of relevant information, we propose, organized into four sections, the following twelve objectives:

I. In the context of economic growth and development in the Eurozone

1. Identify the similarities and differences of evolution patterns over time in the countries of the Eurozone related economic growth and economic development;
2. Identify the roles of the different socio-economic aggregates on the construction of economic growth and economic development dimensions;
3. Clarify the relationship between R&D investment, Patent production and Human Development Index and the Gross Domestic Product components and their association with economic growth and economic development;
4. Identify the relationships of Eurozone countries with economic growth, particularizing the role of economic development in the main influences of the socio-economic entities;
5. Highlight an overall structure of Eurozone countries and to particularize the interactions for all countries on analysis;

II. From the perspective of stability and change effects over 2005-2010

6. To detect periods of stability and change in the period on analysis and to relate them with Eurozone countries performance in order to identify specificities on the processes of economic growth and economic development;
7. Discern trends and identify different types of Eurozone countries, within the period on analysis, with respect to economic growth and economic development in different macroeconomic contexts;
8. Highlight an overall structure of Eurozone and detail the similarities and the influences of the economic entities, emphasizing the interactions of the Eurozone;

III. 2005-2010 interactions in Eurozone countries

9. To detect the interactions between different countries and to analyze the level of positive and negative synergistic effects;
10. Identify economic dependence between different growth and development structures of the euro countries;

IV. Relevance in the use of the three-way methods

11. Propose a methodological approach to solve the problem of describing not only the stable part of the dynamics of economic structures between growth and development in different countries in the Eurozone over 2005-2010, but also to distinguish the interactions and changes in the economic dynamics of the evolutionary systems;

12. Come up with a three-way approach not only to point out useful details for designing a more realistic economic and social diagnosis of the Eurozone for the period 2005-2010, but also to demonstrate the advantage of the information derived from these methods, as a complement to traditional methods of economic analysis.

1.9 The relevance of the research

The problematic of economic growth and its relationship with economic development and innovation has been a topic of political, economic, and methodological interest. Various countries of the world have sought, where possible and with different priorities and approaches, to increase their advantage over other competing countries through allocation of investments for innovation, therefore promoting economic growth and social development. In this context, the relationship between R&D expenditure, innovation and economic growth, and the level of development of countries is of key interest to policy makers when defining and evaluating their initiatives to assure economic growth.

The approaches to three-way methods can go beyond the typical inter-country comparison, common in the literature and existing research, also taking into account the technological and economic changes that have occurred over time.

Three-way methods have a strongly exploratory character, which means that it is possible to find patterns among the elements of the three ways, without postulating specific configurations a priori and without applying tests to these patterns.

This is particularly useful because, in the context of dynamic economic turmoil, it is difficult to specify such patterns beforehand, partly because hypothesis testing supposes that something is known about the distributions of the scores which, for multiway data, is very often not the case.

1.10 The research structure

This thesis is structured as follows.

Chapter 1. The introduction contextualizes the research object, the motivations and argumentation that justified it, and also presents the objectives to be achieved and the organization of the document.

Chapter 2. The literature review, where, by means of a literature survey, the topics regarding the theoretical framework, events, concepts and reviews used in the investigation are laid out.

Chapter 3. The methodology dedicated entirely to the description of the three-way multivariate analysis methods STATIS, MFA and TUCKER3, and of the adopted methodological procedures where the empirical contributions of this research are highlighted.

Chapter 4. The results, where the empirical application is presented and the main results of the three methods are discussed and compared.

Chapter 5. The conclusions, limitations and suggestions for further investigations, where conclusions are stated, the main limitations are identified and future research suggestions are outlined.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Understanding the nature of economic growth always proved essential to elaborate more effective development strategies. The relationship between economic growth and economic development, with special attention to the period 2005-2010 concerning Eurozone economies, led to the need for a framework that would highlight economic vulnerability and structural failures underlying the effects of the 2008 crisis.

The qualitative shift in the economic situation in the Eurozone, marked by the progressive interpenetration of the global crisis, broke the continuity of an economic model of stability. Space was thus generated for debate on the logic of economic accumulation, on the rules of economic systems and on the role of the state and public policies. And, inherently, new challenges have been set, marked by a strong context of uncertainty in relation to growth / development.

The failure of European institutions to address and respond to global systemic problems, accentuated by the different rhythms of the economies of the Eurozone, changed the European agenda for economic development, which should be based on sustainability.

From stability emerged change phenomena, further aggravating existing asymmetric vulnerabilities of research and development levels, patents, technology transfer and, of course, human development.

Currently, to address issues that call into question the "certainties" for economic growth, which are no longer effective with the new trends in sustainability of development, there is a need for new interpretations of growth. And in this context, the traditional quantitative methods of economic analysis can benefit from new methodological approaches that can assist to build models that not only include the relevant variables, but also their new dynamics.

Understanding the economic growth and development processes in their current dimensions, both at the macro and local level, requires a more complete diagnosis to know in which direction it will proceed.

And, we believe that three-way multivariate methods can contribute to the realization of this diagnosis.

2.2 The European Union context with focus on the Eurozone

The EU has its roots in the European Coal and Steel Community (ECSC) (Treaty establishing European Coal and Steel Community, 1951), and the European Economic Community (EEC) (EC, 1957) Belgium, France, Italy, Luxembourg, the Netherlands and West-Germany) (Treaty establishing the European Economic Community, 1957), formed by the Inner Six countries, Belgium, France, Italy, Luxembourg, the Netherlands, and West-Germany and the European Atomic Energy Community (EUROATOM) (Treaty establishing European Atomic Energy Community, 1957) formed in Rome by Belgium, FRG, France, Italy, Luxembourg and the Netherlands) in 1951 and 1957, respectively. In 1965, the Merger Treaty was signed in Brussels creating the set of institutions for the three communities, usually referred to as the European Communities (Treaty establishing a Single Council and a Single Commission of the European Communities, 1965), although commonly known as the European Community. The Communities enlarged with the entry of Denmark, Ireland, and the United Kingdom in 1973¹, Greece in 1981², Spain and Portugal in 1986³.

In 1985, the Schengen Agreement launched the foundations for the creation of Europe's borderless Schengen Area in 1995 (EC, 1985). In 1986, the Single European Act was signed (EC, 1986).

In 1990, East Germany became part of the Community as part of the reunited Germany.

In 1992, the Treaty of Maastricht marked a new stage in European integration, as it raised concrete and definitive lines of action in terms of not only economic integration but for policy of the Member States. With the Treaty of Maastricht, the Economic and Monetary Union (EMU) was created, and prequels an advanced stage of economic integration supported on a single market that involved close coordination of economic and fiscal policies. At this stage, the adoption of the euro as the common currency of the economic system of the EU is raised. This single currency could be adopted by Member States which comply with the terms of the convergence criteria that comprised low and stable

¹ Act concerning the conditions of accession and the adjustments to the Treaties –Accession of the Kingdom of Denmark, Ireland and the United Kingdom (1972).

² Act concerning the conditions of accession and the adjustments to the Treaties –Accession of the Hellenic Republic (1979).

³ Act concerning the conditions of accession and the adjustments to the Treaties – Accession of the Kingdom of Spain and the Portuguese Republic (1985).

inflation, stable exchange rate, and healthy public finances (Treaty on European Union, 1992).

The Copenhagen criteria for candidate members to join the EU was agreed upon in June 1993 (Treaty on European Union, 1992) by the European Council in Copenhagen. To comply with the criteria a country must attain a stable democracy that respects human rights and the rule of law, and a market economy, to fulfill the obligations of membership and to recognize EU law (European Council in Copenhagen, 1993).

The EU was established when the Maastricht Treaty, which had been signed in 1992, came into force on November 1st 1993 (Treaty on European Union, 1992) and in 1995 Austria, Finland, and Sweden were integrated in the EU⁴.

The Treaty of Amsterdam amended the Treaty of the EU, which established the European Communities and related acts, was signed on October 2nd 1997, and entered into force on May 1st 1999. It made significant changes to the Treaty of Maastricht. The Treaty of Amsterdam laid emphasis on citizenship and the rights of individuals, and democracy with increased powers for the European Parliament, created a community employment policy, a community area of freedom, security, and justice, the basis for a common foreign and security policy and the reform of the institutions of the enlargement process (Treaty of Amsterdam⁵, 1997).

The Treaty of Nice⁶ was signed by the European leaders on February 26th 2001 and came into force on February 1st 2003. It amended the Treaties of Maastricht and Rome. It strengthened the institutions of the EU in their operation and effectiveness, it also gave greater emphasis to economic regulation. It reformed the institutional structure of the EU preparing it for the enlargement, a goal which was originally intended to have been achieved by the Amsterdam Treaty, but which was not addressed at the time.

⁴ Act concerning the conditions of accession and the adjustments to the Treaties – Accession of the Republic of Austria, the Republic of Finland and the Kingdom of Sweden (1994).

⁵ Treaty of Amsterdam amending the Treaty on European Union the Treaties establishing the European Communities and related acts (1997).

⁶ Treaty of Nice amending the Treaty on European Union, the Treaties establishing the European Communities and certain related acts (2001).

In 2004, the EU underwent a considerable enlargement when Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia, and Slovenia became member states⁷. On January 1st 2007⁸, Romania and Bulgaria joined the EU.

The Lisbon Treaty (2007) ends the European Community, changes the former structure of the EU and establishes a new balance of powers between the EU and Member States. Modifications are made to the functioning of European institutions and the decision-making processes. The main objective was to improve decision-making in an enlarged Union comprising 27 Member States. It also proposes the reforming various internal and external policies of the EU. Endows the institutions of the capacity to legislate and gives them competencies in new policy areas.

On December 1st 2009, the Lisbon Treaty (Lisbon Treaty, 2007) entered into force and alongside other aspects, it changed the legal structure of the EU. The main EU institutions are the European Commission that concentrates executive EU tasks even so the European Council having some restricted capabilities in that area, the European Parliament that divides the analyses and amendment of EU legislation with the Council of the European Union, the Court of Justice of the European Union that interprets and applies the EU law and treaties, the European Central Bank that leads the EU monetary policy, and the Court of Auditors that scrutinizes the EU budget.

The Lisbon Treaty, provides a new institutional framework to the EU promoting a more democratic, transparent and effective policy. It also contributes to greater impact, coherence and visibility of EU action on the world stage.

Two of the fundamental goals of the European Economic Community were the development of a common market, the single market, and to implement a customs union between its member states. The economic process of the EU rests on the progressive removal of barriers to trade and the free movement of factors that involves a process of extensive regional integration. Theoretically this process can have a substantial impact on the structure production of these countries, by facilitating the reallocation of production

⁷ Act concerning the conditions of accession and the adjustments to the Treaties - Accession of the Czech Republic, the Republic of Estonia, the Republic of Cyprus, the Republic of Latvia, the Republic of Lithuania, the Republic of Hungary, the Republic of Malta, the Republic of Poland, the Republic of Slovenia and the Slovak Republic to the European Union (2003).

⁸ Act concerning the conditions of accession and the adjustments to the Treaties - Accession of the Republic of Bulgaria and Romania to the European Union (2005).

factors to the most efficient activities. In this context, the single market, on the Schengen space, comprises the free circulation of goods services, capital, and people within the EU, (EC, 2014a). The customs union, which started with the Community Customs Code, and on its context, all goods upon entry on the market are subjected to a common external tariff. Once goods have been admitted into the market there are no additional import quotas, customs duties, or to any discriminatory taxes (EC, 2014b).

Free movement of capital allows movement of investments such as immobiliary purchases and buying of shares between countries (EC, 2014c).

The free circulation of persons implies that EU citizens can move freely to live, work, study, or retire in any member states. This required the simplification of administrative formalities and the recognition of professional and educational/academic qualifications between states.

The free movement of services and of establishment allows business persons and professionals or legal persons, operating legally in a Member State, to exert a stable and continuous economic activity in another Member State or offer and provide their services in other Member States, on a temporary basis, while remaining in their country of origin (EC, 2014a).

The EU policy pursues undistorted competition within the single market. The EU Commission as the competition regulator for the single market, addressing antitrust issues of anticompetitive agreements and abuse of dominant market position, approving mergers, dismantling cartels, promoting economic liberalization, and preventing state aid (EC, 2014d).

The European single currency became an objective of the EEC in 1969 but it was only in 1993, with the Maastricht Treaty that member states were legally obliged to start the monetary union (until January 1st 1999). The single European currency, converted from accounting currency to actual currency on January 1st 2002, when euro notes and coins were distributed and national currencies began to be retired of those EU Member States which have adopted it. At this time Greece had also joined the Eurozone and, together with the 11 founding member states, Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain adopted the Euro as currency. In 2007, Slovenia also adopted the euro, followed in 2008 by Cyprus and Malta,

Slovakia in 2009, Estonia in 2011, and Latvia in 2014. Table 2.1 presents the Eurozone member states by date of admission.

Table 2.1 – Eurozone member states by date of admission until December 31st 2014

Date of admission on the Eurozone	Country
January 1st 1999	Austria
January 1st 1999	Belgium
January 1st 1999	Finland
January 1st 1999	France
January 1st 1999	Germany
January 1st 1999	Ireland
January 1st 1999	Italy
January 1st 1999	Luxembourg
January 1st 1999	The Netherlands
January 1st 1999	Portugal
January 1st 1999	Spain
January 1st 2001	Greece
January 1st 2007	Slovenia
January 1st 2008	Cyprus
January 1st 2008	Malta
January 1st 2009	Slovakia
January 1st 2011	Estonia
January 1st 2014	Latvia

Source: (European Commission, 2014)

http://ec.europa.eu/economy_finance/euro/adoption/euro_area/index_en.htm

At the core of the Eurozone is the creation of the euro which, despite its symbolic value, was created to facilitate a single market to help the mobility of citizens and goods, providing price transparency, reducing exchange rate problems, creating a single financial market, low interest rates, price stability, and to protect against shocks by the large amount of intra-trade of the Eurozone. The euro, and the monetary policies of Eurozone members, are under the control of the European Central Bank (ECB) (Treaty on European Union, 1992).

The ECB controls the monetary policy, assuring 1) stability of prices and inflation, 2) the balance of exchange rates, 3) that the national authorities adequately supervise financial markets and institutions, and 4) that the payment systems work well. It comprehends all EU national central banks and is controlled by its General Council, which includes the President and Vice-President of the ECB, and the governors of the national central banks of all 28 EU member states (Treaty of Lisbon, 2007).

In the EU, economic decisions are a matter of common interest, there is no single institution responsible for overall economic policy, but the powers are divided between

the Member States and EU institutions. Since the financial crisis emerged in 2007, the Eurozone has established and used provisions to assign emergency loans to member states depending on the implementation of economic reforms. The Eurozone has also implemented some limited fiscal integration, for example with review of national budgets of member states. The issue is highly political and includes, as from 2011, which future provisions are to be agreed for Eurozone reform.

In 2009, following of the global financial crisis, the European Commission proposed the European System of Financial Supervisors (ESFS) which is composed by three authorities: the European Banking Authority, the European Insurance and Occupational Pensions Authority, and the European Securities and Markets Authority. In addition to this framework, there is a European Systemic Risk Board, also under the responsibility of the ECB. The objective of this financial control framework is to ensure the economic stability of the EU (EC, 2014e).

2.3 Discrepancies of economic growth and development on the EU

After a strong stimulus in the decades following World War II, the EU ceased its path of catching up with the US in the 80s. Since then, the gap in living standards between the economic areas on either side of the Atlantic has widened. Since the onset of the crisis in 2008, the US, despite marginal growth, has pulled further away from Europe in economic terms (Balcerowicz *et al.*, 2013). EU economic growth since the 2008 global financial crisis has been not only worse than in the US, but also than in Japan during the initial four years. Nonetheless, EU exports have increased more than in the US relative to the 2008 GDP. This increase was the sole basis of improvement in the European trade balance during that period. In this matter the EU is not less competitive than the US. On the other hand, Europe is outperformed on investment by the US, Fixed investment fell 14.9% in the EU between 2008 and 2012. However there are significant discrepancies of GDP in Purchasing Power Parity (PPP) per capita within individual EU states. For example EU Member States Austria, Sweden, Denmark, and Germany have a GDP per capita of more than 20% above the EU average and at the other extreme Bulgaria has a level 55% below the EU average (EUROSTAT, 2014a). Structural Funds and Cohesion Funds exist to support the development of underdeveloped regions of the EU. Several funds provide

emergency support for candidate members to reform their countries to comply to the EU's standard (Phare, ISPA, SAPARD and TACIS) (EC, 2008; EC, 2010b).

In terms of unemployment, the current economic crisis has, however, damaged much of the progress achieved in Europe since 2000. At the beginning of 2000, 9.1% of the total labor force was unemployed in the EU-28. In the first months of 2005 a period of steadily declining unemployment started, lasting until the first quarter of 2008 (EUROSTAT, 2014b). At that time, EU-28 unemployment hit a low of 6.8%, before rising severely in the genesis of the economic crisis. Between the second quarter of 2008 and mid-2010 the unemployment level raised, taking the rate up to 9.6%, at that time the highest value recorded since the start of the series in 2000 (EUROSTAT, 2014b).

From the second quarter of 2011 until the first quarter of 2013 unemployment had a steady increase taking it to a record rate of 10.9%. From this period onward the rate started to decrease, falling to 10.7% at the end of 2013 (EUROSTAT, 2014b). The Eurozone had a similar performance as in the EU-28. However, between 2000 and the beginning of 2004 the unemployment rate in the Eurozone was below the EU-28 level and, between 2005 and the beginning of 2008, unemployment declined faster in the Member States with own currency. In the Eurozone, during the economic crisis, unemployment increased substantially, with the exception of the period between mid-2010 and mid-2011, when it declined. In 2013 the unemployment level reached its maximum with 19.2 million in the second quarter, before decreasing in the second part of the year (EUROSTAT, 2014b).

From a human development perspective (HDI), which emphasizes aspects other than economic activities and their growth, and which are necessary for development, including life expectancy, literacy and enrolment rates, freedoms and capabilities (Anand and Sen, 1994), Europe globally reveals little variation. The HDI for all EU Member States ranks high or very high on human development. However, between Eurozone members, there are still differences in the values of the index, although there is no classification below 0.8 (very high human development) (United Nations Development Program (UNPD), 2013).

With insubstantial growth across much of the globe, stimulating new sources of growth has become a global policy priority. Science, technology and innovation are vital mechanisms for encouraging sustainable growth. Investment in innovation remains a

priority, largely by means of R&D support measures. In 2012, OECD governments had an average investment of 0.8% of their GDP in direct funding of R&D at home or abroad with Korea and Finland investing surpassing 1% (OECD, 2013). In 2011, France, Korea, the Russian Federation and Slovenia provided the most combined support for business R&D as a percentage of GDP. In Australia and Canada indirect financing of business R&D exceeded direct financing by a factor of five (OECD, 2013).

The World Trade Organization (WTO) Trade in Value Added (TiVA) indicators demonstrate that countries have become more dependent on imports from a various economies in order to sustain or improve their export performance. In this context, in China, from 1999 up to 2009, gross exports increased 12-fold at current prices to USD 1300 billion, and the foreign value-added content of exports tripled to more than 30% to 20% of the value-added of exports from OECD countries, half of it from Japan and Korea (OECD, 2013).

The interdependency of countries grows, where consumers in one country impact on the jobs in other countries. In the year of the crisis 2008, 20% to 45% of business sector jobs in the majority of European economies and 20% of jobs in China respond to foreign demand (OECD, 2013). For the US the OECD estimates suggest that in 2008, over 10 million business sector jobs were sustained by foreign consumers, 2 million of which to respond to East and Southeast Asian consumers.

Emerging economies increasingly play a role in science and innovation. In the global landscape of scientific research, the emergence of new players has changed the structure of world collaboration networks. In 2011 China was the second-largest R&D performer after the United States, ahead of Japan, Germany, and Korea (OECD, 2013:14).

In the European context, science, technology and innovation are considered critical factors for growth. In this sense, EU policies aim to increase the innovation capacity and decrease the gaps in economic and living standards within the EU, as well as making Europe more competitive in relation to other countries outside the Union.

In the EU, the Lisbon Agenda purposed a change of paradigm for the state members, focusing on knowledge creation as a catalyzer for economic growth. This initiative was planned and projected such that the participating countries would dedicate capital to

implement it. It was not fully achieved at the end of the program, but public and private investment in R&D grew up to the economic crisis. Following the outbreak of the crisis, a majority of Member States maintained or even increased their R&D investments, in spite fiscal constraints, and overall R&D spending over GDP even increased until 2010 (EC, 2013a). The EU and Eurozone global Research and Development expenditure as % of GDP is presented in Table 2.2 and Figure 2.1.

Table 2.2 – Research and development expenditure (% of GDP)

Countries	Years					
	2005	2006	2007	2008	2009	2010
European Union	1,82	1,84	1,84	1,94	2,04	2,03
Eurozone	1,84	1,87	1,87	2,00	2,10	2,10

Source: Adapted from World Bank, 2014

Period 2005-2010 <http://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS/countries?page=1&display=map>

EU research and technological framework programs sponsor research conducted by consortia from all EU members to work within a single European Research Area (EC, 2002b). This enables researchers, research institutions, and businesses to better move and compete strengthening EU Member States' research systems, increasing their competitiveness and allowing them to cooperate more effectively.

Figure 2.1 – R&D expenditure % of GDP per country 2005-2009



Source: Adapted from World Bank, 2014

Period 2005-2009 <http://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS/countries?page=1&display=map>

However, there are gaps between Member States' performances. Furthermore, in Member States with knowledge-intensive and internationally competitive business sector, the government's strategy to protect R&D expenditure helped maintain the levels of private investment. However, this proved more difficult for member states suffering from sovereign debt crisis. In those countries, liquidity limitations combined with a less innovation-friendly environment, and a low level of business demand for knowledge debilitated the effectiveness of government counter-cyclic efforts to stimulate business investment (EC, 2013c).

In recent decades investment in research and development in the pursuit of increased innovation and technological potential has been a path championed by policy makers to ensure European economic growth. The EU Member States have established policies to promote research activities mainly through public investment. This is particularly

noticeable with the definition and implementation of initiatives such as the Lisbon strategy and Europe 2020 Strategy. The Lisbon strategy is a strategic development plan of the EU (European Parliament, 2010) approved by the European Council, which proposed the conversion of the economy of the EU into a knowledge-based economy to be more competitive and dynamic before 2010. This is a strategic objective which should be able to promote lasting economic growth together with improvement in employment levels and achieving social cohesion. However, according to the Presidency conclusions of the Lisbon European Council, the achievement of this goal requires a strategy to prepare the transition to a society and an economy founded on knowledge through policies that cover the needs of the information society and research and development, as well as accelerating structural reforms to increase competitiveness and innovation.

On the other hand, the growth strategy for the EU for 2010 to 2020 was set by the European Commission in March 3, 2010 (EC, 2010a) denominated the "EUROPE 2020- a strategy for smart, sustainable and inclusive growth" is primarily aimed at revitalizing the economy of the EU. The Europe 2020 strategy has established three mutually reinforcing priorities and provides a vision of Europe's social market economy for the 21st century, namely:

- i. Smart growth: developing an economy based on knowledge and innovation;
- ii. Sustainable Growth: promoting a more efficient economy in terms of use of resources, greener and more competitive and,
- iii. Inclusive growth: fostering an economy with high levels of employment that ensures social and territorial cohesion.

Also here, in the European context, science, technology, and innovation are considered as key factors for growth. Effectively, European policies aim to increase the innovation capacity of the Member States and consequently decrease economic gaps within the EU, as well as making Europe more competitive in relation to other countries outside the Union.

2.4 The influence of the 2008 global crisis on stability and change in Eurozone countries

Many have been the achievements of economic integration in the EU despite the adverse impact of the international financial crisis which started in the United States in 2007. In this context, the international financial crisis represented a negative external shock being that the system was affected by the growing speculation bubble in the US financial system that quickly contagious major European banks (Carmassi *et al.*, 2009).

In this sense, the structural weakness of the economies of some Member States exposed the euro to speculative attacks. To counter this risk, the EU institutions and the twenty-seven Member States decided on May 9th, 2010, to create a financial stabilization mechanism of 750,000.00 million. The central question for the future was how to achieve closer coordination and greater economic solidarity between Member States, which should ensure good governance of their public finances and reduce its budget deficit (Arghyrou and Kontonikas, 2012).

The European debt crisis erupted in the wake of the subprime derived financial crisis which originated on the US, propagated through the banking system and became global. The EU economy is susceptible to economic actions worldwide, from speculators and other external occurrences. The most obvious sign of a financial crisis in developed countries is a downturn in equity markets. After the US equities markets had collapsed, we saw the fall of several economies worldwide within a short time.

The European debt crisis is an enduring prolonged debt crisis taking place in a group of Eurozone member states. These member states were unable to refinance or repay their government debt or to bail-out banks under their national supervision, and required the assistance of other institutions like the European Central Bank (ECB), the European Financial Stability Facility (EFSF), or the International Monetary Fund (IMF).

Crises, in general, emerge from the convergence of external and internal factors, which occur in the economy. Some economic conditions are more prone to promote crisis than others, but most crises have common market failure causes (Allen and Gale, 1999). An important cause of many crises for developed countries, in recent decades, is

related to regulation issues or financial liberalization originated by permissive government regulations of banking and financial institutions (Lane, 2012).

Reinhart and Rogoff (2009) made a wide study of indicators on context of crisis, and found that multiple indicators are involved. It has been shown, in recent years, that frequently banking crisis prequel the financial crisis. Reinhart and Rogoff (2009) described the start of a banking crisis by bank runs, or on the other hand if there are no bank runs, closures, mergers, takeovers, or large-scale government assistance of major institutions. In this context, Goodhart (2007) explains that financial institutions have become less prudent and have undertaken excessive risk in the belief that the Central Banks would secure them against financial collapse. The coexistence of low-interest rates and cheap money can result in asset bubbles, which when burst, have severe consequences to the investors, for which the primary driver was the prevalence of non-performing loans. There are several causes for this, such as commodity or real estate booms and collapses, currency crisis, unfounded sense of value derived from erroneous ratings, and sophisticated financial products.

In the 1990s politicians encouraged subprime households to strive for loans after setting targets for lending increases for mortgage institutions (Oberg, 2009).

Subprime loans were placed on Collateralized Debt Obligations (CDOs), among other products, which with the downturn of real estate market prices, contracted balance sheet margins. To clarify the magnitude of the problem, the IMF made an estimate of potential losses for the period 2007-2010 from US-originated derivatives held by banks of \$2.7 trillion (Crotty, 2009).

The significant real estate decline, that began in 2006, was decisive to the US financial crisis, contributing to the number of non-performing loans. Reinhart and Rogoff (2008) show that real estate prices went down during all banking crises studied. The context of low-interest rates, adjustable rate mortgages, lack of consumer protection or financial education, and the lack of due diligence by institutions, intensified the problem. Overvaluations in real estate were unsustainable and in 2008, trillions of dollars vanished from the economy.

Commodity prices impacted on financial crises where, for example, food prices rose due to the production of ethanol from corn and other grain crops (World Bank,

2012). However, the financial collapse of September 2008 that led to a worldwide credit restriction resulted in a price collapse for most commodities (Griffith-Jones and Ocampo, 2009).

Another factor that contributes to the crisis is the lack of reasonable assessments of loans from the rating agencies. Goodhart (2007) points out the question of lack of actual competition in the rating business with only the two big American rating agencies, Moody's and Standard and Poor's, and of Fitch the European agency. In this context, the fact that the rating reports from the rating agencies are paid for by the investment banks, creates a possible conflict of interest. With a lack of supervision from the government, rating agencies may have insufficient stimulus to protect the consumer. The World Bank (2009) reported that in 2007, there were 12 triple-A rated companies in the world, nonetheless 64,000 structured finance instruments had a triple-A rate. This top rating on a broad range can be a signal that ratings are rendered without a straightforward and robust criterion, especially when so many originated from the derivative market.

Another problem associated with crisis is the decline in trade that can occur during the crisis, due to lack of credit and business. Looking at the trade between nations experiencing a financial crisis, both may be affected, so the balance of trade may be uncertain. A country with isolated crisis will face a larger balance of trade problem and it was demonstrated that exports decreased considerably during balance-of-payment crises (Reinhart and Rogoff, 2009).

In several countries, private debts arising from the property bubble were included in sovereign debt as a result of bank bailouts and government responses to the weakened economies. The fact that the Eurozone has a currency union without a real fiscal union contributed to the crisis and undermined the ability of European leaders to respond. European banks' own relevant quantities of sovereign debt which brings concerns regarding the solvency of banking systems or sovereigns are negatively strengthening. Concerns intensified in the beginning of 2010, leading European nations to implement a series of financial support measures such as the EFSF and ESM (Baber and Zahoor, 2013).

Various states in Europe had to bailout their more fragile banks with recapitalization loans, to assure the financial stability of the economy. From January 2009 onwards 10

Central and Eastern European banks asked for public-funded bailout and the European Commission forecast of a 1.8% decline in EU economic output for that year, made the outlook for the banks even worse (Wagstyl, 2009; Shambaugh *et al.*, 2012). These bank recapitalizations contributed to the deterioration of debt-to-GDP ratios of several EU member states during the financial crisis, but cannot be the sole cause for the emergence of the subsequent sovereign debt-crisis in European states.

The major contributing factors for the four sovereign debt crisis erupting in Europe were low economic growth, competitive weakness, liquidation of banks and sovereigns, large debt-to-GDP ratios, and considerable liability stocks of government, private, and non-private sectors (Petraakis *et al.*, 2013).

In response to the global financial crisis, fiscal policy was loosened in most EU countries during 2008-2009, through the operation of automatic stabilizers as well as through fiscal stimulus packages, even though the majority of EU countries had no margin for such a loosening (Balcerowicz *et al.*, 2013). This was a policy that translated into complications to the economic situation from 2010.

From 2010 up to 2012, the fiscal balance improved in the EU, but the cyclically adjusted deficit underperformed the relative to the before crisis values (Balcerowicz *et al.*, 2013). It is important to emphasize that two-thirds of the reduction in the cyclically adjusted deficit to GDP ratio in the EU in 2012, relative to 2009, was achieved through taxes and almost all of the expenditure cuts were in the government side.

The Eurozone sovereign debt crisis was characterized by a context of high government structural deficits and increasing debt levels. Those states adversely besieged by the crisis faced a substantial interest rate rise for government bonds, as a result of investor lack of confidence about their future debt sustainability, culminating in four Eurozone states requiring sovereign bailout programs (Greece, Ireland, Portugal and Cyprus), delivered by the IMF and European Commission, with technical support from the European Central Bank (IMF, 2014).

Nonetheless, the Maastricht Treaty of 1992, obliged the EU members states to limit their deficit spending and debt levels, and in the early 2000s, some members were failing to stay under the boundaries of the Maastricht criteria and adopted the securitizing of future government revenues to reduce their debts and deficits, sidestepping good practice and

disregarding international standards that allowed the sovereigns to under-evaluate their deficit and debt. From late 2009 forward, after Greece made their debt and budget deficit explicit, uncertainties about sovereign defaults in certain European states were increased. This led to the downgrading of the government debts for these states by rating agencies (EUROSTAT, 2014a).

2.5 Economic Growth, Patents, Research and Development and Human Development

Since the seminal contributions of Adam Smith and Marx, all current economic thought, in the innovation-economic growth relation, produced unanswered questions. The Keynesian paradigm of macroeconomic adjustments dominated the academic and economic world in the postwar decades. It considered technological change just as technical progress within production functions, ie as a trend over time without clarifying, in its true dimension, the relationship between productivity and technological change. Following the seminal contributions of Solow (1956, 1957) and Swan (1956), the neoclassical model became the mainstream approach to the analysis of growth, at least within academia. The neoclassical growth theory introduced technical progress in its analysis (Abramovitz, 1956; Solow, 1957; Solow, 1956), and implicitly incorporated the assumption that technological progress can be expressed as an exogenous factor, reflected in residual terms. For a long time, this mainstream economic theory which considered technological progress as exogenous. It would be necessary to wait for the new growth theory (Romer, 1986; Lucas, 1988), which clarified the need to introduce technology as endogenous factor affecting growth, so the old theoretical approaches of authors such as Schumpeter, of long-term economic cycles, dynamic analysis, endogenous technical progress, enterprise and innovation, could be revisited.

Endogenous growth theorists assume a macroeconomic growth as investments conducent to technological improvements. Thus begin the new models of economic growth, where growth is driven by technological change, arising from a decision of investment made by agents to maximize their utility (Romer, 1990). Variables related to technological

progress (Romer, 1990) are regarded in economic analysis as key explanatory of economic growth. Variables like Education, Research and Development, and Innovation, will then be the central element in explaining this growth, relegating the role of capital investments into the background (Romer, 1986).

Economics growth in innovation economics results from 1) knowledge, 2) regimes and policies promoting entrepreneurship and innovation, 3) technological spillovers, 4) externalities between collaborative firms, 4) and systems of innovation that create innovative prone environments (Antonelli, 2003; Johnson, 2008).

Innovation economics contrasts with neoclassical economics on perspectives regarding the determinants for economic growth, and the assumptions of context between economic actors. Despite the differences in economic thought, both perspectives (Neoclassical and Innovation) recognize relations between economic growth, innovation, technological progress and the optimization of the utilization of factors.

A wide range of studies investigating the innovation-performance link lead to mixed results and indicate this relation may be more refined and complex than commonly supposed (Salge and Vera, 2009).

In the current economic dynamics, it is widely recognized and accepted the leading role of technological innovation, development and economic growth. Both on theoretical and empirical perspectives it is widely accepted that technology and technological advances are a key component of innovation and economic growth. There is a convergence of views between various authors regarding a positive relationship between the level of technological development and economic growth of the countries. Since 1957, with the publication of the article "Technical Change and Aggregate Production Function" by Robert Solow, the productivity growth of economies is mainly explained by the importance given to the introduction of technological innovations.

Using differing conceptual and methodological approaches, various studies have emphasized different growth determinants and offered various insights to its relations. In fact, innovation and knowledge began to be regarded as important sources of economic growth in countries (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Audretsch and Feldman, 1996; Cantwell and Iammarino, 1998; Furman, Porter and Stern, 2002; Bilbao-Osorio and Rodríguez-Pose, 2004), especially in developed

countries, which have excelled in terms of growth in production, productivity, and international trade. Grossman and Helpman (1994), also observed that technology has been the key to raising the standards of living. And, in this matter, economic policies began to promote a process of development associated with technological capacitation and with innovation gains (Kim, 1980; Wade, 1990; Nelson, 1993; Fagerberg *et al.*, 2007; Fagerberg and Srholec, 2008), as well as increased participation in international markets and, above all, the expansion and strengthening of the internal market. In reality the investment in research and development was considered strategic to ensure technological potential and as a result, innovation and economic growth (Trajtenberg, 1990). In addition to extending the possibility of achieving a higher technological level in companies and regions, which enables the introduction of new products or processes, it allows greater growth and profitability (Romer, 1990; Lichtenberg, 1992).

Nevertheless, the capacity for innovation and knowledge is not totally immune to the development and economic growth of a country (Rodríguez-Pose, 1999). The economic environment of the innovation process is fraught with uncertainties and risks and, in the course of investment decision-making in technology, economic operators take on even higher risks than existing ones. And, in this context, it is the private sector of the economy that best reflects this reality. The association to innovation policies of public sector human resources and infrastructure of research proves to be instrumental in the creation of an environment conducive to the Research and Development process itself.

2.5.1 Gross Domestic Product as a measure of Economic Growth

Gross domestic product is the market value of goods and services produced within a selected geographic area in selected intervals of time. The concept of GDP gained a dissemination of its usage in the 90s. The international standard for measuring GDP was defined in the book System of National Accounts of 1993, created by the International Monetary Fund, European Union, Organization for Economic Co-operation and Development, United Nations, and World Bank (Commission of the European Communities – Eurostat *et al.*, 1994).

GDP is widely used to measure the economic performance of a country. The most frequent use of GDP estimates is to calculate the growth of the economy between periods. The evolution of GDP growth supports the evaluation of economic policies and determines if an economy is in recession. Economic growth is the increase in the market value of the goods and services produced by an economy, conventionally per year. It is usually measured as the percent rate of increase in real GDP (Worldbank, 2013). An increase in growth can be caused by more efficient use of inputs or by increases in inputs such as capital, population, and even territory.

Regarding the GDP components and related indicators of economic output, imports and exports, domestic private and public consumption and investments, *“as well as data on the distribution of income and savings can give valuable insights into the driving forces in an economy and thus be the basis for the design, monitoring and evaluation of specific EU policies”* (EUROSTAT, 2014a).

From an expenditure-based perspective, GDP is total final expenditures at purchasers' prices (including the free on-board price value of exports of goods and services), the free on-board price value of imports of goods and services (Commission of the European Communities – Eurostat *et al.*, 1994). On the expenditure approach of GDP, final expenditures are expenditures on goods and services purchased for final use, that is, for final consumption or for gross capital formation. Final consumption consists of the goods and services used by individual households or the community to satisfy their individual collective households and general government. Gross Capital Formation (GCF) comprises gross fixed capital formation, change in inventories and acquisitions less disposals of valuables. Gross capital formation expenditures are originated by resident producers of goods and services, incorporated and unincorporated enterprises, general government and non-profit institutions. Final expenditures exclude expenditures on intermediate consumption, which comprises the goods and services, other than fixed assets, that are used or consumed as inputs by a production process.

Therefore GDP is: total final consumption expenditure, (which comprises final consumption expenditure of households, final consumption expenditure of final consumption expenditure of non-profit institutions serving households), plus gross capital formation (which comprises gross fixed capital formation, change in inventories, acquisitions less disposals of valuables) plus balance of exports and imports.

In the system of national accounts, only households, Non-Profit Institutions Serving Households (NPISH) and governments have final consumption. The corporations have intermediate consumption. Private final consumption expenditure is the expenditure on goods and services for the direct satisfaction of individual needs, where government consumption expenditure includes goods and services produced by government, as well as purchases of goods and services by government supplied to households as social transfers in kind. Gross capital formation is defined as the sum of gross fixed capital formation and the change in inventories. The external balance is the difference between exports and imports of goods and services. The balance can be called a surplus (positive) or a deficit (negative) (EUROSTAT, 2014a). The ratio of GDP *per capita* is better suited to compare countries performances. Moreover, Furman *et al.* (2002) consider that GDP *per capita* can be an estimate of the size of the technological development of a country.

2.5.2 R&D expenditure as a measure of countries' research and development intensity

Research and development (R&D) refers to creative work performed systematically to increase the stock of knowledge (including knowledge of man, culture and society), and using this knowledge to conceive new applications (OECD, 2002).

R&D expenditure is used as an indicator of an economy's degree of investment in generating new knowledge. In this perspective several countries have defined targets for this indicator to support focus policy decisions and public funding. In this context one of the key objectives of the EU during recent decades has been to stimulate increasing levels of investment, in order to boost the EU's competitiveness. The Lisbon strategy set an objective of allocating 3% of the gross domestic product (GDP) to R&D activities by 2010 (EC, 2011). The target was not reached and the 3% target remained as one of five key targets within the Europe 2020 strategy formalized in 2010 (European Parliament, 2010).

EUROSTAT's statistics on R&D expenditure are compiled using guidelines in the Frascati manual, published in 2002 by the OECD. R&D expenditure is a basic measure

that covers intramural expenditure (OECD, 2002), that is, all expenditures for R&D performed within a statistical unit or sector of the EU economy.

Expenditure on research and development (R&D) is one of the most commonly used measures of innovation inputs. This broad definition of an innovation comprises a wide range of possible innovations. An innovation can be categorized as the implementation of one or more types of innovations, of product or process nature. This includes new or improved goods, services, or processes, new marketing methods, or a new organizational method (OECD, 2005).

The literature usually considers R&D expenditures primarily as an input indicator of the efforts made on establishing R&D that might theoretically lead to output. Investment in R&D can be regarded as a country's effort to create new knowledge and to capitalize on that existing.

Correlation of R&D input, for example R&D expenditure, with R&D output, through patents has been identified (Griliches, 1990, 1998; Hausman *et al.*, 1984; Hitt *et al.*, 1997).

R&D investment has been assumed as a means of achieving a higher standard of technology in countries, which would allow them to introduce new and superior products and processes, resulting in higher levels of income and growth. R&D efforts can also indicate the innovative competences of companies that are found to affect their performance, particularly in high-tech industries (Duysters and Hagedoorn, 2001; Henderson and Cockburn, 1994).

It is therefore worth analyzing how different levels of investment in R&D have affected innovation activity.

2.5.3 Patents as a measure for knowledge transfer to technology

Patents reflect a country's inventive activity and denote the country's capacity to exploit knowledge and translate it into potential positive economic returns. In this context, patent systems have been developed to protect organizations against the prospect that the knowledge they develop will be imitated by others, and thus decide not to invest in

research. The patent system assures a temporary control of a part of the knowledge that has been conceived by the patent applicant, leaving other aspects of this knowledge for public use and to be used to create or upgrade technologies (David, 1993).

The European Patent Organization (EPO) is an intergovernmental organization created on 7 October 1977 supported on the European Patent Convention signed in Munich in 1973. It comprises the European Patent Office and the Administrative Council. Inventions can be protected in Europe by national patents granted by the competent national authorities or by European patents granted by the EPO. EU Member States have their own patent authorities which process applications for national patents. The protection granted by a national patent is limited to the territory of the state (EUROSTAT, 2014c).

In 2012 Member States and the European Parliament decided on a legislative initiative with two Regulations and an international Agreement, which created the foundations for the unitary patent protection in the EU (EUROSTAT, 2014c). The patent package implemented enhanced cooperation between most EU Member States. With all Agreements and the Regulations in force, it is possible to obtain a European unitary patent, a legal title ensuring standardized protection for an invention across the EU Member States on a centralized and expedite process, providing cost advantages and reducing administrative weight.

Patent counts are generally accepted as one of the most appropriate indicators that allow researchers to compare the inventive or innovative performance in terms of new technologies, processes or products (Acs and Audretsch, 1989; Aspden, 1983; Bresman *et al.*, 1999; Cantwell and Hodson, 1991; Freeman and Soete, 1997; Griliches, 1998; Napolitano and Sirilli, 1990; Patel and Pavitt, 1995; Pavitt, 1988). As with most other indicators the use of raw patent counts is subject to discussion regarding its bias and limitations (Archibugi, 1992; Cohen and Levin, 1989; Dosi, 1988; Griliches, 1998). Even some authors who have been critical of patents as a performance indicator, such as Arundel and Kabla (1998) and Mansfield (1986), consider that patents can be an appropriate indicator for that purpose.

2.5.4 HDI as a measure of human development achievements

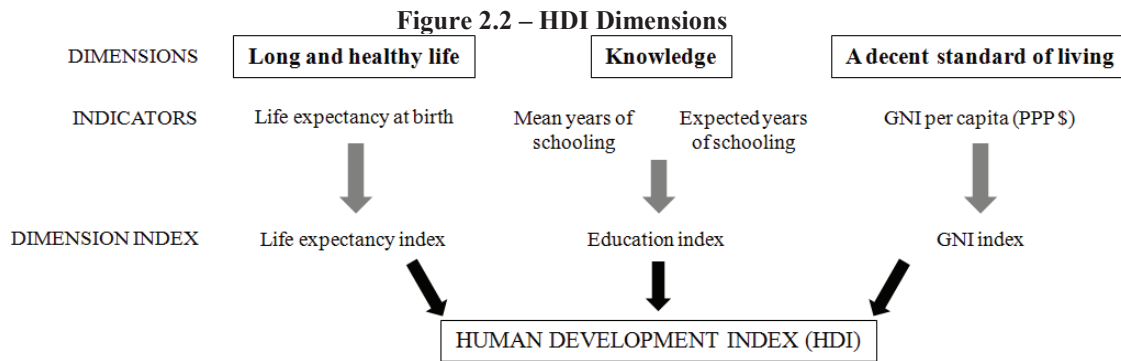
The human development index (HDI) was created by Mahbub ul Haq and Amartya Sen in 1990 (UNDP, 2010) and was published by the United Nations Development Program (UNDP, 2013). Since the United Nations Development Program's (UNDP) first global Human Development Report (HDR) in 1990, most countries have registered significant human development (UNDP, 2014).

The main idea behind HDI is that a simple composite measure of human development can promote the change of evaluation of development from a exclusively economic advances measurement but also on improvements in human well-being. This index was purposed to change the focus of policy makers from economic growth alone to human and development (Fukuda-Parr, 2003; UNDP, 2010).

HDI measures a country's average achievements in three dimensions of human development: the first, a long and healthy life; the second, access to knowledge; and third a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions. The insight for the HDI was the creation of a index to serve as a reference for social and economic development (UNDP, 2010). The HDI is a composite statistic of life expectancy, education, and income indices used to rank countries into four tiers of human development (very high, high, medium and low).

Since 1990, the United Nations Development Program (UNDP) has calculated the HDI for the countries according to availability and quality of data allowed. The HDI aims to operationalize the “capabilities” of a country’s population (ul Haq 2003, Sen 1999, UNDP 2010).

A schematic describing the HDI dimensions is presented on Figure 2.2.



Adapted from HUMAN DEVELOPMENT REPORT 2013 – (UNDP, 2013)

Over time, countries may experience HDI changes, as a result of any of the elements of the index. Though the index does not incorporate the entire construct of human development (Ranis *et al.*, 2005) the demographic and development literature has long reflected about the relation between mortality, income, and education (Bédia and Dumont, 2008; Cutler *et al.*, 2006; Deaton, 2003; Elo and Preston, 1996; Kitagawa, 1977; Lucas, 1988; Morand, 2004; Preston, 1980; Preston, 1975).

2.6 Traditional methods for economic development and growth analysis

In most of the papers reviewed the causal relation between R&D investment and the economic performance of countries is practically abandoned. The importance of measuring the R&D expenditure and relating it to technological change and to productivity growth, came to the attention of the economists from Solow's seminal theory that economic growth could be driven by technological improvements.

Various research works were developed, during the 70s, because of the lowering of productivity across several countries. This concern for R&D measurement was especially acute as the slowdown in R&D expenditure was assumed as the main explanatory factor for such a slowdown (Griliches, 1980).

Griliches (1980), using the manufacturing sector for analysis, attempted to examine whether the slowing of US productivity in the 70s could be attributed, to a decline in the

growth of R&D expenditures, considered that the 70s were not a favorable period to evaluate R&D investment impact on growth, since the oil price shocks affected most of OECD countries (Hall and Mairesse, 1995), and concluded that the instrument of measurement could not isolate that relationship from other influential factors.

Griliches (1988) choose an econometric analysis to support the hypothesis that no clear empirical relation could be found between US R&D expenditures and productivity slowdowns and confirmed that R&D coefficients declined in consonance with its statistical significance, reaching values close to zero in the period of 1969-1977.

This decline of the R&D coefficients in that period could have been, according to Griliches (1980), a reflection of the large energy price shocks, the resulting fluctuations in capacity usage, the substantial increase in uncertainty about the future, and the relative prices may have turned away many firms from their long-run production frontiers. Thus, data did not show movements along the technological frontier, and hence they cannot be attributed to a variable that shifts this frontier outward.

So relating this decrease to productivity slowdown in the 1970s is not reliable because national accounts had overlooked both R&D spillovers and R&D social returns. That distorted the impact of R&D capital on productivity. The result of this error was the over-evaluation of slowdown in R&D.

Coe and Helpman (1995) investigated the extent to which a country's total factor productivity depended upon its domestic R&D capital and foreign R&D capital. Using data from the period of 1970-1990, an econometric analysis was performed, in which cumulative R&D expenditure was a proxy for the stock of knowledge. For each country on analysis, two types of stock of knowledge were constructed: one from domestic R&D expenditure nature and another, which the exclusive source was R&D expenditure of the country's trade partners. The results demonstrated that a country's total factor productivity depended on domestic R&D capital stock and of the R&D capital stock of its trade partners.

Four years later, Coe, Helpman, and Bayoumi (1999) conducted a research, analyzing the influence of R&D, international R&D spillovers and trade on a country's Total Factor Productivity (TFP) and output growth. The resulting macroeconomic model confirmed

the existence of a positive relation between R&D and growth in industrialized and developing countries.

Bilbao-Osorio and Rodríguez-Pose (2004) argue that there are several factors that must be taken into account when assessing the effectiveness of R&D investment. This author gave great focus on social filters, that is, the social components of a region that can be explanatory factors of why not all regions are capable of converting their R&D into innovation and innovation efforts in economic growth. These social filters, peculiar to each region, translate to regional disparities, originated by different capacities to convert technology and investment in R&D to profit. Rodríguez-Pose and Bilbao-Osorio evolved his perspective on the role played by “social filters”, by studying the differences in the innovation and economic growth performance between peripheral and non-peripheral regions in Europe. The methodological model used to assess how R&D could be linked to the genesis of innovation was highly explanatory, however, more suitable for innovation in peripheral regions. The main conclusion about relationship between innovation and economic growth was that R&D investment impact on innovation and economic growth depends on the sectors and the regions where it applied. In the case of public and private sectors considered in the study the research activities of the private sector higher rates of return than the research of the public sector. Specificities of the private sector make it more prone to engage on research that can be straightforwardly commercialized in the market (Bilbao-Osorio and Rodríguez-Pose, 2004). In the non-peripheral regions, private funded research seems be an innovation driver, while in peripheral regions it is the research from higher education institutions which achieves better results.

The research on government-funded R&D carried out by Walwyn (2007) provided an interesting understanding of the relationship between R&D and economic growth, taking the Finnish mobile phone industry, as the empirical context. The Finnish case allowed to isolate that relationship from other variables that could that also affect economic growth. Using economic data from 1990 to 2001, the author analyzed the relationship between government-funded R&D and Business Expenditure in R&D (BERD), between BERD and GDP and, between electrical BERD and electrical value added. The Patterson-Hartmann model used in the research relates R&D investments with product revenue and allows decision makers to use investment budgets and product investment waves to simulate scenarios of probable outcomes from R&D investment and to control the time-

lag between the translation of the investment into revenue (Hartmann, 2003). The results of Walwyn's research, demonstrated the extremely high return on R&D investment achieved by the Finnish government and identified the government R&D funding as very attractive when directed to a promising sector. Its policy option was a main driver to industry research investment in Finland and laid the ground for future private sector investment in R&D.

Sharma and Thomas, (2008) examined the efficiency of the R&D process on a group of 22 developed and developing countries using Data Envelopment Analysis (DEA). The R&D technical efficiency was examined using a model with patents granted to residents, gross domestic expenditure on R&D, and the number of researchers. Under CRS (Constant Returns to Scale), Japan, the Republic of Korea and China were considered to be efficient. From the VRS (Variable Returns to Scale) perspective, Japan, the Republic of Korea, China, India, Slovenia and Hungary were found efficient. The emergence of some developing nations on the efficiency frontier indicated that these nations can be used as benchmarks for their efficient use of R&D resources. It is highlighted in this study that inefficiency in the R&D resource usage indicates the underlying potential that can be used for the development and growth of nations.

Petrariu *et al.*, (2013) tried to show the link between innovation and economic growth in the Central and Eastern European countries and considered, in its analysis, R&D spending, patenting, the number of researchers, firms characteristics, mergers, and acquisitions as proxies to measure innovation. Their findings were consistent with the existing literature in which innovation makes significant contribution to national competitiveness and economic growth and that the gap between the Western and Eastern economies can be reduced with investment in innovation. They concluded that development level of a country is the key for innovation, and the allocation of funds to R&D is the main source to support it. Welfare is central to enhance the innovation in a country. Nonetheless the innovation factors that influence growth fail to make a significant contribution for the Central and Eastern European countries and the rapid economic growth of these countries was not based on the innovation process.

In Table 2.3 a review of innovation and economic growth related studies is presented.

Table 2.3 Literature review of innovation and economic growth related studies

Authors	Methodology	Specification	Focus
Griliches (1980)	Econometric Analysis: with estimation of the elasticity of aggregate or sectorial output regarding changes in the R&D capital.	- R&D capital; - TFP; - LPROD; (log output per man-hour);	To relate the decline in R&D expenditure to decline in productivity growth.
Coe and Helpman (1995)	Econometric analysis: Cointegration equations; Regression model; Inclusion of dummy variables.	- R&D capital stock (domestic/foreign) - TFP	To analyze the extent to which a country's TFP is dependent on domestic and foreign R&D capital.
Rousseau and Rousseau (1997; 1998)	DEA approach with CRS	Inputs : GDP, active population and R&D expenditure Outputs: publications and patents	To use DEA as a tool to construct performance indicators for governments.
Coe, Helpman and Bayoumi (1999)	Multicountry macroeconomic simulation model. MULTIMOD version comprising 12 linked econometric models: TFP is endogenously determined by R&D spending, R&D spillovers and trade. Cobb-Douglas production function.	- TFP; - R&D capital (domestic/foreign); - R&D expenditure; - Output growth.	To assess the contribution of R&D spending, international R&D spillovers and trade.
Bilbao-Osorio and Rodríguez-Pose (2004)	Two statistical models: R&D→Innovation; linear regression model; modified Cobb-Douglas function; Cross-section OLS regression. Innovation→economic growth: linear regression model; Cross-section OLS regression.	R&D → Innovation: - GDP; - R&Dpriv./pub./higher ed.; - Economic structure; - Employment rate; - Patents; Innovation → Economic Growth - Innovation and innovation growth (patents); - Skills; - Economic structure; - Employment rate; - Economic growth;	To relate R&D investment with innovation and innovation with economic growth.
Walwyn (2007)	Patterson-Hartmann Model Simulating methodology	- GOVERD; - BERD; - Electrical BERD; - Electrical value added; - GDP;	To examine the return on investment from government funded R&D
Sharma and Thomas (2008)	DEA approach with CRS as well as VRS.	Inputs: R&D expenditures, researchers, gross domestic product, population output: patents granted, publications counts	To examine efficiency with VRS: Japan, Republic of Korea, China lie on the efficiency frontier with CRS, Japan, Republic of Korea, China, India, Slovenia and Hungary
Petrariu <i>et al.</i> (2013)	Panel data regression models with fixed effects variable both within among cross section and time series: Innovation → economic growth State of the economy/resources → Innovation	GDP growth, Education, GDF for Research, FDI on GDP, Unemployment, HDI, GDP hab, Researchers Number to 1,000,000 people, Patents Number.	Innovation has a direct and positive impact on economic growth The condition of the economy and resources allocation affect Innovation Estonia, Latvia, Lithuania, Poland, Germany, Czech Republic, Slovakia, Hungary, Romania, Moldova, Bulgaria, Croatia, Macedonia and Serbia

Source: adapted from Cardoso and Teixeira, (2009)

2.7 The three-way tables and techniques

Generally, traditional statistical methods, when studying actual phenomena of multivariate and dynamic nature, often fail to capture simultaneous changes that occur in the behavior of the variables analyzed. And in this context, multivariate statistics has been introducing developments in the evolutionary study of the phenomena, enabling the study of multidimensional structures present in multiple data sets.

Effectively the measurement of a variable on a set of individuals, which is in the statistical standpoint of view the simplest situation, allows the building of just one vector of observations with a single input or via, the individuals. If, in addition, a set of variables is considered, the information can be organized as a data matrix and two ways are obtained, one for the individuals and the other for the variables. But if for each previous matrix a repeat measurement at different times, experimental situation, or subject perspective is performed, we are facing a three-dimensional array, or in other words, of three ways: individuals, variables, and times or situations or perspectives.

Typically, in the pursuit of data explanation and for the single-way situation, the distribution of the data is modeled. For two-way data, explicative/explanatory models or covariation models may be constructed, and for three-way data models there are three-way models. The main purpose of these three-way models is to analyze multiple data tables where the observations are, for example, objects or individuals on which several measurements were made on several occasions. The term occasion can also be used to refer to different moments in time or at different measurement conditions, i.e. the different experimental conditions. In these models it is usual to associate the data set to a graphical representation that allows us to distinguish and display the essential trends of the studied phenomena.

As previously mentioned, the three-way data analysis is undertaken when information is organized into structures in which each data is represented by three-ways or indices:

1. An index to identify individuals that are the object of study;
2. Another index for the variables that were measured on these individuals, and
3. A third index, to identify the various occasions where these measurements were

performed.

According to Kiers (1991) these structures usually correspond to two types of data organization:

I. Cubic or triadic data for observations on the same set of individuals, for which the same set of variables was measured on different occasions;

II. Data of multiple groups, when one of the ways is composed of different sets, and in this case, there may occur:

a. Observations of the same individuals in different sets of variables at different times or experimental situations;

b. Observations of different sets of individuals, on the same variables at different times or experimental situations.

Though a structure of three-way data can be considered as a succession of two-way tables, the statistical methods traditionally that analyze the latter are not suited for three-way data treatment because they do not allow simultaneous analysis of the various data frames, measured by occasions. The purpose is to detect the similarities and differences in the configurations of individuals' and variables' trajectories. In other words, the methods of multivariate two-way data do not identify the patterns of the internal structure present between the elements of the three-way configuration data.

According to Kroonenberg (2008):

“...the non-three-way methods operate by flattening the data cube, or stringing out its contents. In both cases the purpose is to turn three-way data into two-way data by eliminating one of the ways. Instead of looking at the interactions between three types of ways, only two need to be analysed. The described flattening consists of producing a two-way matrix from a three-way array by removing one of the ways. It is typically done by taking the averages over all “slices” in the data cube, so that one is left with one “slice” containing means. These may be averages over all years, so that one loses sight of trends over time, but one may also take averages over all observations, so that individual differences disappear below the horizon.”

This flattening can be done by stringing out – creating a two-way matrix out of a three-way array by either laying out all the data “slices” in one long row, so that the relation between similar variables at different time points is neglected, or laying out all data “slices” in one tall column, so that the connections between subjects’ scores at different moments in time are lost. The technical term for this procedure is matricization. In all these cases, the data and their analysis are shrunk from three-way to two-way. Sometimes this may do no harm, because it is possible that a three-way analysis leads to the conclusion that no three-way analysis is necessary: for instance, if nothing changes over time, or if all subjects may be viewed as having been randomly drawn from one single population. However, if this is not the case, the flattening or stringing out of three-way data leads to an unnecessary and sometimes unacceptable simplification...” (Kroonenberg, 2008: 7)

In recent decades Methods of Multivariate Data Analysis have shown their effectiveness in the study of large amounts of data. The evolution of information technology has accelerated this process, allowing the treatment of voluminous amounts of information which, without the aid of computers, would become very time consuming or even impossible.

Nonetheless, Multivariate Data Analysis started with Pearson (1901) with the study of vectors and plans (spaces) that would allow the adjustment of a set of points in a Euclidean space. Subsequently, and in this follow-up, Spearman (1904) laid the foundations of factor analysis, through psychology. However, only later did Principal Component Analysis (PCA) emerge with Hotelling (1933), in the wake of developments of the previous works of Pearson and Spearman.

Multivariate Data Analysis then began to be used in various areas, including psychology, economics and biology, with the contribution of Benzécri (1976). Harman (1967) and Morrison (1967) contributed to developments related to the PCA the factor analysis and discriminant analysis.

In the 70s and 80s, Escoufier (1973), Bouroche (1975), L'Hermier des Plantes (1976), Robert and Escoufier (1976), Jaffrenou (1978), Foucart (1981) and Escoufier and Pagès

(1985) among others, began to develop their studies in methods of analyzing multidimensional tables, thus giving rise to the fundamental framework of the French school. Of these methods those that stand out are Double Principal Component Analysis (DPCA) of Bouroche (1975), the STATIS method and the Dual STATIS method (Escoufier, 1973; L'Hermier des Plantes, 1976; Lavit, 1988; Lavit *et al.*, 1994) and the Multiple Factor Analysis (MFA) of Escoufier and Pagès (1985).

On another branch of Three-Way Methods different approaches arose that led, in turn, to the Anglo-Saxon school from which we can highlight the family of TUCKER models Tucker (1966), the PARAFAC model (Parallel Factor Analysis) of Harshman (1970), the CANDECOMP model (Canonical Decomposition) Carrol and Chang (1970) and TUCKALS2 and TUCKALS3 models and Kroonenberg and Leeuw (1980).

However, Kiers (1991) divided the three-way methods for data analysis into asymmetric and symmetric. Symmetric, of which the TUCKER and TUCKALS models are examples, treat the three ways of a data structure equally, while the asymmetric such as MFA and STATIS, are developed according to the ICI approach (Interstructure-Compromise- Intrastructure) and treat the third way differently.

2.7.1 Asymmetric methods

Asymmetric methods are developed along three basic stages: the Interstructure, the Compromise, and the Intrastructure.

The goal of Interstructure analysis, also known as global analysis, is to compare the data tables to each other and to recognize homogeneous groups. When performing these comparisons it is important not only to have in consideration the concept of proximity between tables, but also to look for a graphic representation on which it is possible to interpret the proximity between two points as corresponding to two similar tables, seen in the sense of distance (usually the Euclidean distance). In summary, the analysis by Interstructure reveals proximities between tables without, however, it being possible to interpret in detail the elements that generate the similarities or differences between these tables.

The analysis of Compromise is to summarize tables in one of the same nature as the original tables. It is the space generated by the Compromise that allows an overall summary of all the tables.

Since Interstructure analysis is insufficient to compare the tables in detail, the obtained Compromise allows representation of the Compromise positions of each of the elements (individuals and / or variables) that constitute the different data tables. The Compromise elements' positions correspond to their average positions. This procedure is called Analysis of the Intrastructure.

It is common in Intrastructure analysis to include the concept of trajectory. This concept has its origin in studies where the tables are generated temporally and what is sought is the description of the evolution of the phenomenon under analysis. However, the concept is also applicable to interpretable tables generated from different experimental situations. The goal is, in general, to know in detail the evolution of each of the elements that constitutes the tables.

The most representative techniques of asymmetric methods are STATIS and the MFA method.

2.7.1.1 STATIS

STATIS (*Structuration des Tableaux à Trois Indices de la Statistique*), proposed by L'Hermier des Plantes (1976) and developed by Lavit (1988) and Lavit *et al.* (1994), has its theoretical basis in the work of Escoufier (1973, 1976).

This is an exploratory method for analyzing multivariate data, which is based on linear algebra and in particular on Euclidean vector spaces, and which aims to compare configurations of the same individuals, or of the same variables, quantitative or qualitative, in different temporal or spatial horizons in order to find a common structure, stable and representative of all tables. Thus, quantitative data can be presented in T data tables collected at different temporal or spatial circumstances, called "occasions" or "studies" about the same individuals, but in which the variables may differ over the frames.

Analyses from tables of this nature seek essentially to study the structure of individuals, in order to check if it is common and stable to different data tables. It comprises the following four fundamental steps:

1. Interstructure analysis, which accounts for a comprehensive comparison of data tables;
2. Compromise analysis, which describes the common structure of several data tables;
3. Analysis of the Intrastructure, which allows highlighting of the individuals responsible for similarities (or differences) between tables;
4. Finally, the trajectories that describe the evolutionary behavior of each individual or variable are traced from the Compromise of the image.

In short, the STATIS method allows us not only to capture the trajectories of individuals and variables over time or conditions, but also to identify the variables and individuals that contribute to stability.

2.7.1.2 Multiple factor analysis

Multiple factor analysis (MFA) was developed by Escofier and Pagès (1985) and generally applies to the simultaneous treatment of a series of tables with the same individuals characterized by the same or different sets of variables, quantitative or qualitative.

It is also an exploratory method for analyzing multivariate data, which is based on linear algebra and vector Euclidean spaces, as in the STATIS method, also looking for a common structure, stable and representative of all the tables and its implementation is based on two essential steps:

1. The preliminary step is to perform a Principal Component Analysis (PCA) on each of the groups of variables. The purpose of this step focuses on the acquisition of the first eigenvalue of each analysis, in order to be used as a weighting factor in the next step, and on the evaluation of possible effects of the dimensionality of each group in the subsequent analysis;

2. In the second step on a PCA normative is carried out in the global table which considers each table by the inverse of the first eigenvalue obtained in the previous step, in order to balance the influence of groups of variables. Then concatenated so that all sub-tables are weighted and finally a new PCA is carried out on the corresponding global matrix. This approach makes it possible to obtain a common framework called Consensus space, which allows the study and analysis of the evolution and relations of the different individuals and variables, allowing one to obtain graphical representations and quality indices that facilitate the interpretation.

2.7.2 Symmetric methods

Principal Component Analysis (PCA) is the basic foundation of multivariate three-way techniques. Its purpose is to reduce the number of variables to analyze, replacing them with a smaller number of linear combinations, obtained in a way so that the least possible reduction in the total variability occurs. And for the least possible loss of information to happen, it resorts to the least squares method to find the best fit for the initial set of data, trying to minimize the sum of squares of the differences between the estimated and the observed data. However, the estimation procedure of ordinary least squares to assign equal weights to the observations, which is equivalent to the constancy of variance (homocedasticity), causes a higher concentration of the regressed data, which cannot succeed in the presence of multiple data tables. In other words, the frequency distributions of data from multiple tables are present, sometimes with quite different standard deviations, generating unstable factor structures from each other.

The presence of a high dispersion of data (heterocedasticity) led Tucker (1966) similarly to the case of a three-way data structure. It introduced a new perspective, based on the idea that each way is associated with an underlying factor structure, and that there is not only one set of factors, but three. In other words, the three data inputs are connected, each one with another way (not observable) that can be regarded as an optimum set of factors or categories. The objects, variables and occasions are considered as linear combinations of the "idealized" objects, "latent" variables and "prototype" occasions, respectively. Relations between the three types of factors are considered in a table or matrix estimated by the model and called core matrixes.

However, the original TUCKER model has the disadvantage of not providing approximations of the data in the sense of ordinary least squares, which is why other methods were developed. It was in an effort to overcome this limitation that Kroonenberg and Leeuw (1980) developed two methods based on the original model of Tucker (1966), called TUCKALS3 and TUCKALS2.

However, the difficulties in the interpretation of the results of these two methods favored methods based on easier assumptions. Carroll and Chang (1970), on the one hand and Harshman (1970), on the other, independently developed a model that decomposes the three-way tables with entries fully folded in a simpler form. The first authors named it the CANDECOMP (Canonical Decomposition) and Harshman designated it PARAFAC (Parallel Factor Analysis).

2.7.2.1 TUCKER3

Although for Kroonenberg and Leeuw (1980), the goal of the two methods is to decompose or factorize the matrix formed by the original data, the TUCKER3 (1966) model is more general, since it makes the adjustment to the original Tucker model through the method of ordinary least squares. It also reduces the three ways considered, which differentiates it from the TUCKER2 model, which only caters for individuals and for the variables.

Thus, in the TUCKER3 model, the core matrix represents the weight of a specific combination of components of the three ways. In the TUCKER2 model, the core matrix is reduced to a combination of only two ways.

In both methods, the next step is to maximize the inertia explained by an adjustment method of least squares, similar to the PCA.

2.7.2.2 PARAFAC and CANDECOMP

In these methods, according to Carroll and Chang (1970), on the one hand and also to Harshman (1970), the difference in the conception from Tucker (1966) is that it only shows the determination of a unique group of factors from observed data, and that can be interpreted by the three-way, simultaneously. The drawback lies in the fact that the axes can be interpreted in one way, since the method does not allow rotation of the axes.

This method can be considered as a special case of the TUCKER2 model in the sense that now the principal components for the variables and individuals are defined, whereas in PARAFAC and CANDECOMP, models are defined for both simultaneously. The resolution of this, as in previous cases, is to maximize the explained inertia, i.e., to minimize the loss of information.

2.7.3 Comparison of Methods

The STATIS is a method that allows the finding of a common structure that summarizes the data tables. It is thus an average-Compromise structure of individuals, with individual-Compromise being characterized according to its position in the planes formed by the factors that take meaning by relating to the variables of all the tables.

The method that considers more aspects of the information given is the MFA, that allows an analysis of the positions of individuals-Compromise by factors that take meaning in accordance with their correlation with the variables of each table (variable-partial). In addition, MFA allows a characterization of each individual in each table because of the simultaneous representation of all individuals (individuals, partial).

Previous methods such as PCA-based methods allow an easily relation of the individuals and variable planes and, therefore, the coordinates (correlations) of significance of variables with the factors that determine the positions of the individuals. The same occurs in the TUCKER3, which identifies independent form factors for tables, variables, and individuals, which allows the description for each channel separately, and the connecting element designated core matrix. This matrix contains the weights of the three-way combinations, that is, the amount by which each of the combinations contributes to the

data, but the interpretation of results is difficult when trying to establishing relationships between individuals, variables and times. That is not so for the TUCKER2 model, which with the associated inconvenience only identifies the underlying two-way (variables and individuals) model, without occasions and therefore does not generally compare the starting tables.

In turn, the PARAFAC CANDECOMP models and identifies one system factors for all routes, that is, three matrices allow us to obtain components that summarize the linear relationship between individuals, variables and times (no central core matrix or compromise matrix). However, they have the drawback that the number of factors identified in all paths has to be the same, which considerably limits the method.

In summary, within the asymmetric methods which seek solutions to common factor and stable structures, the MFA model is the method that enables a closer look at a three-way table, followed by the STATIS method.

Regarding the symmetrical methods which analyze factorial solutions of unstable structures, TUCKER3 model followed by PARAFAC and CANDECOMP models are those which result in smaller loss of results, respectively.

In all cases, these numerical results are materialized and are summarized in representations of factor structures of tables of individuals, variables and occasions that give a broader but also relation perceptive view of the studied phenomenon.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter is structured into five sections. The first section describes the PCA. The second section describes the STATIS method. The third section presents the MFA methodology. The fourth section describes the TUCKER3 model, and finally, the fifth section establishes the comparison between these three-way methods of analysis.

3.2 Principal Component Analysis

The Principal Component Analysis (PCA) was initially developed by Pearson (1901) with his study of vectors and spaces and later evolved with Spearman (1904) and was formalized by Hotelling (1933) as a dimension-reduction technique to graphically summarize and describe a data matrix. This method comprises transforming a set of correlated variables, in a new set of uncorrelated variables, which are linear combinations of the original variables, that is, the principal components. The first principal component retains the largest possible variance and, therefore, explains most of the inertia of the data table. The remaining component are constructed under the constraint of orthogonality to the previous components and to retain the largest possible inertia. The factor scores created for the new variables are interpreted geometrically as projections of the observations onto the principal components. This technique captures the information in the first components, with the least possible loss of information, and expresses the information of the original data as a set of new orthogonal variables called principal components.

3.2.1 Main features

Consider the data matrix $X_{(n \times p)}$, defined by n individual p variables:

$$X = \begin{bmatrix} x_1^1 & \cdots & x_1^p \\ \vdots & \ddots & \vdots \\ x_n^1 & \cdots & x_n^p \end{bmatrix} \quad (3.1)$$

Where

- x_i^j is the value that the i -th individual takes the j -th variable, with $i = 1, \dots, n$ and $j = 1, \dots, p$
- the j -th variable is represented by the vector of \mathbb{R}^n

$$x^j = \begin{pmatrix} x_1^j \\ x_2^j \\ \vdots \\ x_n^j \end{pmatrix} \quad (3.2)$$

- the i -th individual is described by the vector \mathbb{R}^n

$$x_i' = (x_i^1 \quad x_i^2 \quad \dots \quad x_i^p) \quad (3.3)$$

Consider the matrix D , a positive diagonal matrix, where significant values represent the weightings of the n individuals:

$$D = \begin{bmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & p_n \end{bmatrix} \quad (3.4)$$

with $0 < p_i < 1$ and $\sum_{i=1}^n p_i = 1$.

The weightings are assigned according to the relative importance of each individual in the data set. Usually, all individuals have the same importance and the matrix D can be represented by $D = \frac{1}{n} I_n$ where I_n is the identity matrix of order n .

The vector composed of arithmetic means for each variable, denominated by centroid or center of gravity is defined by:

$$g' = (\bar{x}^1 \quad \bar{x}^2 \quad \dots \quad \bar{x}^p) \quad (3.5)$$

with $\bar{x}^j = \sum_{i=1}^n p_i x_i^j$. In matrix form we have that:

$$g = X' D 1_n \quad (3.6)$$

Let Y be the matrix of centered data associated to X with $y_i^j = x_i^j - x^j$

Then:

$$Y = X - 1_n g' = X - 1_n (X' D 1_n)' = X - 1_n 1_n' D' X$$

where by:

$$Y = (I_n - 1_n 1_n' D') X \quad (3.7)$$

In the particular case where $D = \frac{1}{n} I_n$, we have that:

$$Y = \left(I_n - \frac{1_n 1_n'}{n} I_n \right) X \quad (3.8)$$

The covariance between the variables x^j and x^k is expressed by:

$$S_{jk} = S(x^j, x^k) = \sum_{i=1}^n p_i (x_i^j - \bar{x}^j) (x_i^k - \bar{x}^k) \quad (3.9)$$

where $S_j^2 = S^2(x^j)$, $j = 1, \dots, p$

Thus the variance and covariance matrix is defined by:

$$V = \begin{bmatrix} S_1^2 & S_{12} & \dots & S_{1p} \\ S_{21} & S_2^2 & \dots & S_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ S_{p1} & S_{p2} & \dots & S_p^2 \end{bmatrix} = Y' D Y = X' D X - g g' \quad (3.10)$$

being V a positive defined symmetric matrix.

Be Q_{1/S^2} , the diagonal matrix of the inverse of the variances of the variables:

$$Q_{1/S^2} = \begin{bmatrix} 1/S_1^2 & 0 & \dots & 0 \\ 0 & 1/S_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/S_p^2 \end{bmatrix} \quad (3.11)$$

The Z matrix, with centered and reduced data is defined by:

$$Z = Y Q_{1/S} \quad (3.12)$$

with $z_i^j = \frac{x_i^j - \bar{x}^j}{S_j}$

The matrix of correlations between p variables, designated by R is represented by:

$$R = \begin{bmatrix} 1 & r_{12} & \dots & r_{1p} \\ r_{21} & 1 & \dots & r_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ r_{p1} & r_{p2} & \dots & 1 \end{bmatrix} \quad (3.13)$$

with

$$r_{jk} = r(x^j, x^k) = \frac{S_{jk}}{S_j \cdot S_k} \quad (3.14)$$

Thus:

$$R = Q_{1/s} V Q_{1/s} = Q_{1/s} Y' D Y Q_{1/s} = Z' D Z \quad (3.15)$$

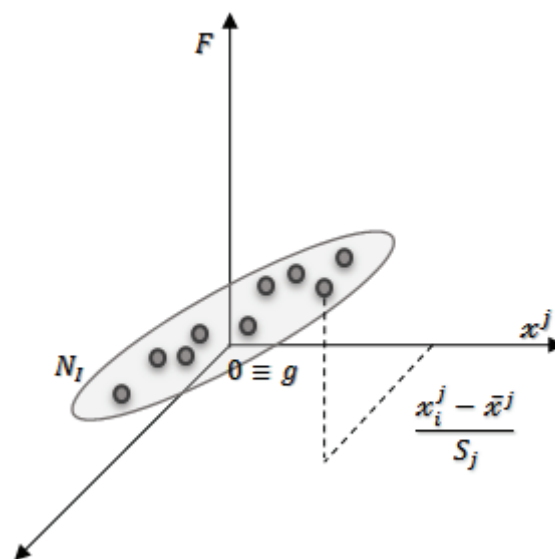
When the initial data are centered and reduced R coincides with V because:

$S_j^2 = 1, j = 1, \dots, p$ that causes the $Q_{1/s} = I_p$.

3.2.2 Individuals

Let F be the space of individuals, the dimension p , which contains each of the n $x'_i = (x_i^1, x_i^2, \dots, x_i^p), i = 1, \dots, n$. This set forms a cloud of points in F , designated N_I , in which, if the data is centered, the center of gravity g coincides with the origin of the axes (Figure 3.1).

Figure 3.1 – Cloud of individuals in F space



Source: adapted from Carvalho (2005)

The analysis of the data frame, from the perspective of individuals, is equivalent to visualization of the same, along the rows.

The space F shall be fitted with a metric for determining the distances between any individuals.

Given Q , a $p \times p$ matrix symmetric, defined positive, we specify the scalar product in F as:

$$(x_i, x_j) = x_i' Q x_j \tag{3.16}$$

Thus, the distance between the x_i and x_j individuals and is given by:

$$d(x_i, x_j) = \|x_i - x_j\|_Q = \sqrt{(x_i - x_j)' Q (x_i - x_j)} \tag{3.17}$$

The matrix Q is the metric associated with the scalar product between individuals.

The matrix of scalar products between individuals is designated as $W = X Q X'$ whose elements are defined by:

$$w_{ij} = \langle x_i, x_j \rangle_Q = x_i' Q x_j \tag{3.18}$$

In PCA the most commonly used metrics are:

- $Q = I_p$, which is to use the Euclidean scalar product:

$$\langle x_i, x_j \rangle_{I_p} = x_i' I_p x_j = x_i' x_j = \sum_{k=1}^p x_i^k x_j^k \tag{3.19}$$

- $Q = Q_{1/S^2}$ defined in (3.11). With this metric, the distance between two individuals does not depend on the units of measurement.

Property 3.1: Let Q be a symmetric positive defined matrix ($p \times p$). Since Q can be decomposed as $T'T$ where T is a triangular matrix (Cholesky decomposition), than:

$$\langle x_i, x_j \rangle_Q = x_i' I_p x_j = x_i' x_j = \sum_{k=1}^p x_i^k x_j^k \tag{3.20}$$

That is, everything processed as if it had used the metric $Q = I_p$ on the transformed data, that is, about $X T'$.

In the case $Q = Q_{1/S^2}$ it is verified that $T = Q_{1/S}$ and therefore:

$$\langle x_i, x_j \rangle_Q = x_i' I_p x_j = x_i' x_j = \sum_{k=1}^p x_i^k x_j^k \quad (3.21)$$

Thus, we use the I_p metric on $X Q_{1/S}' = X Q_{1/S}$. If the matrix X is already centered, then by (3.12), $X Q_{1/S}$ corresponds to the centered and reduced data matrix.

If at the beginning the data matrix X is centered and reduced (Y calculated from (3.8) and Z (3.12)), we use the I_p metric.

If only center, we use the metric Q_{1/S^2} , as defined in (3.11).

The distance between two individuals, either by a procedure or the other, is equal and is no more than the Euclidean distance. Thus, the closer two individuals are of each other, the greater similarity is.

The centering of the data does not change the inherent problematic of this type of analysis, it only makes the origin of the axes to coincide with the center of gravity g .

If the units of measure in which the variables are expressed are very different, it is appropriate to reduce the data, that is, divide each of the observations of the column j by the standard deviation S_j . Thus, all variables will have the same variability and hence the same influence on the calculation of distances between individuals.

Total inertia is defined, for the cloud of individuals, as the weighted average of the squares of the distances of individuals to the cloud's center of gravity:

$$I_g = \sum_{i=1}^n p_i \|(x_i - g)\|_Q^2 = \sum_{i=1}^n p_i (x_i - g)' Q (x_i - g) \quad (3.22)$$

Naturally, if the data is centered, then $g = 0$.

The inertia of the cloud of individuals relative to the point a is expressed by:

$$l_a = \sum_{i=1}^n p_i \|x_i - a\|_Q^2 \quad (3.23)$$

Under the Huygens' theorem:

Theorem 3.1:

$$l_a = l_g + \|g - a\|_Q^2 \quad (3.24)$$

Note that:

- $l_g = tr(QV) = tr(VQ)$ (where $tr(A)$ denotes the trace of the matrix A);

$$\begin{aligned} tr(QV) &= tr(QY'DY) \\ &= tr\left(Q \sum_{i=1}^n p_i y_i y_i'\right) \\ &= tr\left(\sum_{i=1}^n Q p_i y_i y_i'\right) \\ &= tr\left(\sum_{i=1}^n p_i \underbrace{Q y_i}_{(p \times 1)} \underbrace{y_i'}_{(1 \times p)}\right) \\ &= tr\left(\sum_{i=1}^n p_i y_i' Q y_i\right) \\ &= \sum_{i=1}^n p_i y_i' Q y_i \stackrel{(3.22)}{\cong} l_g \end{aligned} \quad (3.25)$$

- $l_g = tr(WD) = tr(DW)$ se $g = 0$;

$$\begin{aligned} &\stackrel{(3.9)}{=} tr(QV) \\ &\stackrel{(3.10)}{\cong} tr(Q X' D X) \\ &= tr(D X Q X') \\ &\stackrel{(3.18)}{\cong} tr(D W) \\ &= tr(W D) \end{aligned} \quad (3.26)$$

- If $Q = I$, then $l_g = tr(QV) = TrV$ (ie is the sum of the variances of the variables p);
- If $Q = Q_{1/S^2}$ then $l_g = tr(R) = p$.

$$\begin{aligned}
 l_g &= tr(Q_{1/S^2} V) \\
 &= tr(Q_{1/S^2} V Q_{1/S}) \tag{3.27}
 \end{aligned}$$

$$\begin{aligned}
 &\stackrel{(3.15)}{=} tr(R) = p
 \end{aligned}$$

3.2.3 Variables

Let E be the space variables of dimension n , which contains each one of the p variables $x^j = (x_1^j, x_2^j, \dots, x_n^j)$, for $j = 1, \dots, p$. This set forms a cloud of points in E , designated N_j .

The analysis of the data frame, from the standpoint of variables, is equivalent to the visualization under columns.

The metric used in this space is the matrix of weights D defined in (3.4) $n \times n$ symmetric and positive defined. This option is straightforward because:

- The scalar product of two centered variables is equal to its covariance:

$$\langle x^j, x^k \rangle_D = (x^j)' D x^k = \sum_{i=1}^n p_i x_i^j x_i^k = S(x^j, x^k) \tag{3.28}$$

- The length of a centered variable is equal to its standard deviation:

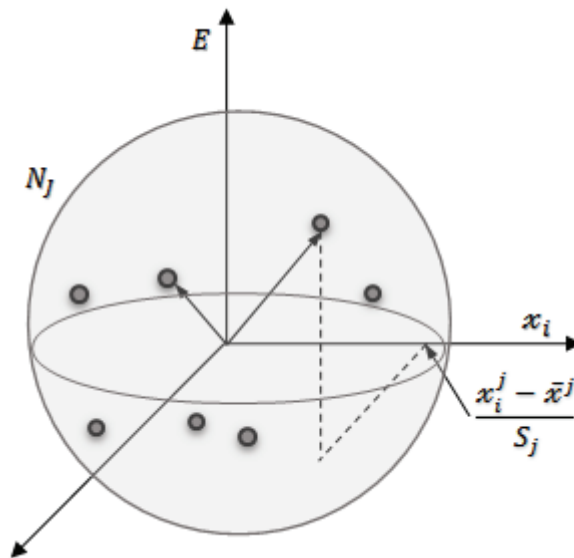
$$\|x^j\|_D = \sqrt{\langle x^j, x^j \rangle_D} = \sqrt{S(x^j, x^j)} = \sqrt{S^2(x^j)} = S(x^j) \tag{3.29}$$

- The cosine of the angle between two centered variables is equal to the linear correlation coefficient between them:

$$\cos(x^j, x^k) = \frac{\langle x^j, x^k \rangle_D}{\|x^j\|_D \|x^k\|_D} = \frac{S(x^j, x^k)}{S(x^j) S(x^k)} = r(x^j, x^k) \quad (3.30)$$

According to the metric used in E space, each variable can be represented as a vector of dimension n , where the group of extremities of each vector forms the N_j cloud. If the data is centered and reduced (normed PCA) each variable has norm equal to 1, so the N_j cloud lies within a hypersphere of radius 1 (Figure 3.2) limited to \mathbb{R}^3 . The j -th variable is a vector of \mathbb{R}^n whose components are $\frac{x_i^j - \bar{x}^j}{S_j}$ with $i = 1, \dots, n$.

Figure 3.2 – Cloud of variables in space E



Source: adapted from Carvalhido (2005)

If the data is only centered, the length of the variables is equal to its standard deviation, and this is a non normed PCA.

Thus, a trio formed by (X, Q, D) can be characterized by two different objects:

- $X Q D' = W$, as defined in (3.18);
- $X' D X = V$, as defined in (3.10), if the data is centered.

3.2.4 Projection of individuals in a subspace

If $p > 3$, the direct study of N_I cloud becomes impossible because of visual limitations to higher dimensional spaces than 3. Hence the interest of PCA to provide the approximate planar imaging optima of cloud N_I situated in a higher dimension space. To the reduction in dimension of the space of individuals, of p for a number q , is naturally associated a loss of information, which much be minimized.

It is then necessary to find a basis $\{u_k; k = 1, \dots, q\}$ of vector \mathbb{R}^p , called principal axes, Q -ortonormated, which define the principal planes on which the N_I cloud is represented.

Admitting the existence of this base, let $F_{u_k}(i)$ be the projection of the individual x_i over the u_k axis (Figure 3.3):

$$F_{u_k}(i) = \langle x_i, u_k \rangle_Q \quad (3.31)$$

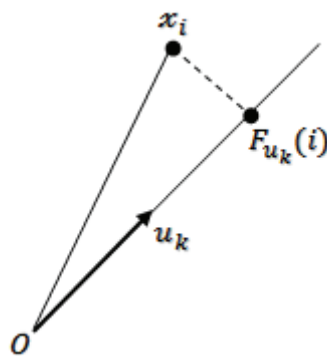
and $F_u(i)$ the vector of projections of the individual x_i over axes u_1, \dots, u_q :

$$F_u(i) = \sum_{k=1}^q F_{u_k}(i) \cdot u_k = \sum_{k=1}^q \langle x_i, u_k \rangle_Q \cdot u_k \quad (3.32)$$

and h cloud center of gravity projected on these same axes that is.,

$$F_u(g) = h$$

Figure 3.3 –Projection of the individual x_i on the u_k axis



Source: adapted from Carvalhido (2005)

The projection deformation is defined as:

$$\mathcal{D} = \sum_{i=1}^n p_i \|x_i - F_u(i)\|_Q^2 \quad (3.33)$$

We have

$$\mathcal{D} = \iota_g + \|g - h\|_Q^2 - \sum_{i=1}^n p_i \|F_u(i) - h\|_Q^2 \quad (3.34)$$

As can be easily shown:

$$\begin{aligned} & \stackrel{(3.33)}{\cong} \sum_{i=1}^n p_i \|x_i - F_u(i)\|_Q^2 \\ &= \sum_{i=1}^n p_i \|x_i - h\|_Q^2 \\ & \quad - \sum_{i=1}^n p_i \|F_u(i) - h\|_Q^2 \quad (\text{Pythagoras Theorem}) \end{aligned} \quad (3.35)$$

$$\stackrel{(3.23)}{\cong} \iota_h - \sum_{i=1}^n p_i \|F_u(i) - h\|_Q^2$$

$$\stackrel{(3.24)}{\cong} \iota_g + \|g - h\|_Q^2 - \sum_{i=1}^n p_i \|F_u(i) - h\|_Q^2$$

The projection deformation is minimal $g = h$ when the inertia of the projected cloud, $\sum_{i=1}^n p_i \|F_u(i) - h\|_Q^2$ is maximum.

Assuming that the data is centered, the cloud of inertia on the axes designed u_1, \dots, u_q is:

$$\begin{aligned} \sum_{i=1}^n p_i \|F_u(i)\|_Q^2 &= \sum_{i=1}^n p_i \left\| \sum_{k=1}^q \langle x_i, u_k \rangle_Q \cdot u_k \right\|_Q^2 \\ &= \sum_{i=1}^n p_i \left\langle \sum_{k=1}^q \langle x_i, u_k \rangle_Q \cdot u_k, \sum_{k=1}^q \langle x_i, u_k \rangle_Q \cdot u_k \right\rangle_Q \\ &= \sum_{i=1}^n p_i \sum_{k=1}^q \langle x_i, u_k \rangle_Q^2 \end{aligned} \quad (3.36)$$

$$\begin{aligned}
 &= \sum_{k=1}^q \sum_{i=1}^n p_i \langle u_k, x_i \rangle_Q \cdot \langle x_i, u_k \rangle_Q \\
 &= \sum_{k=1}^q \sum_{i=1}^n p_i u'_k Q x_i \cdot x'_i Q u_k \\
 &= \sum_{k=1}^q u'_k Q \left(\sum_{i=1}^n p_i x_i x'_i \right) Q u_k \\
 &\stackrel{(3.10)}{\cong} \sum_{k=1}^q u'_k Q V Q u_k
 \end{aligned}$$

The VQ matrix is Q-symmetric, ie for any pair of vectors u_k and u_l of \mathbb{R}^p , we have that:

$$\langle u_k, VQ u_l \rangle_Q = u'_k Q V Q u_l = \langle VQ u_k, u_l \rangle_Q \quad (3.37)$$

thus, by a result of Linear Algebra, VQ is diagonalizable and admits a Q-orthonormal base of eigenvectors associated with the eigenvalues $\lambda_1, \dots, \lambda_q, \dots, \lambda_p$. And because of the principal axes being Q-ortonormated, the inertia of the cloud projected on the axis $u_k, u'_k Q V Q u_k$, is maximum for the highest eigenvalue of VQ.

Let then $\{u_k; k = 1, \dots, q\}$, the eigenvectors VQ.

The principal axes are also V^{-1} - orthogonal. And, since they are Q-orthogonal, we have that:

$$\langle u_k, u_l \rangle_Q = 0 = u'_k Q u_l = 0 \quad (k \neq l) \quad (3.38)$$

But since these axes are eigenvectors of VQ,

$$VQ u_l = \lambda_l u_l \quad (3.39)$$

multiplying by V^{-1} in both members,

$$Q u_l = \lambda_l V^{-1} u_l \quad (3.40)$$

then, replacing in (3.38), we have that:

$$u'_k \lambda_l V^{-1} u_l = 0 \Leftrightarrow u'_k V^{-1} u_l = 0 \Leftrightarrow \langle u_k, u_l \rangle_{V^{-1}} = 0 \quad (3.41)$$

By the definition of total inertia of the point cloud given in (3.22) and the fact that $\lambda_1, \dots, \lambda_p$ are eigenvalues of VQ we have that:

$$I_g = tr (VQ) = \sum_{k=1}^p \lambda_k \quad (3.42)$$

and, (3.36), the inertia of the projected cloud is:

$$\sum_{i=1}^n p_i \|F_u(i)\|_Q^2 = \sum_{k=1}^q u'_k Q V Q u_k \stackrel{(3.39)}{\cong} \sum_{k=1}^q \lambda_k \quad (3.43)$$

Therefore the projections of the individual x_i dimensional subspace q form a cloud in which each point has coordinates q . The coordinates of these projections in u_k axis form the vector F_{u_k} , which belongs to the space E (that is, an element \mathbb{R}^n) which is designated as main component.

This creates the new data matrix $n \times q$:

$$\begin{bmatrix} \langle x_1, u_1 \rangle_Q & \langle x_1, u_2 \rangle_Q & \dots & \langle x_1, u_q \rangle_Q \\ \langle x_2, u_1 \rangle_Q & \langle x_2, u_2 \rangle_Q & \dots & \langle x_2, u_q \rangle_Q \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_n, u_1 \rangle_Q & \langle x_n, u_2 \rangle_Q & \dots & \langle x_n, u_q \rangle_Q \end{bmatrix} \quad (3.44)$$

wherein.

$$F_{u_k} = (\langle x_1, u_k \rangle_Q \ \langle x_2, u_k \rangle_Q \ \dots \ \langle x_n, u_k \rangle_Q)' = X Q u_k \quad (3.45)$$

with $k = 1, \dots, q$.

The principal components have the following properties:

- Each principal component can be expressed as a linear combination of variables x^1, x^2, \dots, x^p (centered);

$$F_{u_k} \stackrel{(3.44)}{\cong} \sum_{j=1}^p q_j \cdot u_k^j \cdot x^j \quad (3.46)$$

where q_j is the j -th element of the diagonal matrix (metric) Q defined previously.

The main components are zero mean variables;

$$\begin{aligned}
 F'_{u_k} D \mathbf{1}_n &\stackrel{(3.44)}{\cong} (X Q u_k)' D \mathbf{1}_n \\
 &= u'_k Q' X' D \mathbf{1}_n \\
 &= u'_k Q' (X' D \mathbf{1}_n) \quad (Q \text{ is symmetric})
 \end{aligned} \tag{3.47}$$

And as the data is centered $g = X' D \mathbf{1}_n = 0$, so $F'_{u_k} D \mathbf{1} = 0$.

The principal components have λ_k variance and are uncorrelated with each other:

$$\begin{aligned}
 S(F_{u_k}, F_{u_l}) &\stackrel{(3.28)}{\cong} F'_{u_k} D F_{u_l} \\
 &\stackrel{(3.44)}{\cong} (X Q u_k)' D (X Q u_l) \\
 &= u'_k Q X' D X Q u_l \\
 &\stackrel{(3.10)}{\cong} u'_k Q V Q u_l \\
 &\stackrel{(3.39)}{\cong} u'_k Q \lambda_l u_l \\
 &= \lambda_l (u'_k Q u_l) \\
 &= \lambda_l \cdot \begin{cases} 0 & \text{se } k \neq l \\ 1 & \text{se } k = l \end{cases}
 \end{aligned} \tag{3.48}$$

- The main components can be obtained by diagonalization of the matrix WD.

$$\begin{aligned}
 V Q u_k &= \lambda_k u_k \\
 (XQ)(VQ u_k) &= \lambda_k (XQ) u_k \\
 X Q X' D F_{u_k} &\stackrel{(3.44)}{\cong} \lambda_k F_{u_k}
 \end{aligned} \tag{3.49}$$

$$W D F_{u_k} = \lambda_k F_{u_k}$$

To the principal axis u_k is associated the linear form z_k , such that:

$$z_k = Q u_k \tag{3.50}$$

called principal factor.

The principal factors also have some properties that are noted.

The principal factors are Q^{-1} – *ortonormated*::

$$\begin{aligned}
 \langle z_k, z_l \rangle_{Q^{-1}} &= z'_k Q^{-1} z_l \\
 &\stackrel{(3.50)}{\cong} (u'_k Q') Q^{-1} (Q u_l) \\
 &= (u'_k Q) Q^{-1} (Q u_l) \quad (Q \text{ is symmetric}) \\
 &= u'_k Q u_l = \langle u_k, u_l \rangle_Q \\
 &= \begin{cases} 0 & \text{se } k \neq l \\ 1 & \text{se } k = l \end{cases} \quad (\text{the principal axes are } Q \text{ – ortonormated})
 \end{aligned}
 \tag{3.51}$$

The principal factors are V -*orthogonal*:

$$\begin{aligned}
 \langle z_k, z_l \rangle_V &= z'_k V z_l \\
 &\stackrel{(3.50)}{\cong} (u'_k Q') V (Q u_l) \\
 &= \\
 (u'_k Q) V (Q u_l) &\quad (Q \text{ is symmetric}) \\
 &= u'_k Q (V u_l) \\
 &= u'_k Q (\lambda_l u_l) \quad (\text{the principal axes are eigenvectors of } VQ) \\
 &= \lambda_l \langle u_k, u_l \rangle_Q
 \end{aligned}
 \tag{3.52}$$

$$\lambda_l = \begin{cases} 0 & \text{se } k \neq l \\ 1 & \text{se } k = l \end{cases} \quad (\text{the principal axes are } Q \text{ – ortonormated})$$

The principal factors are eigenvectors of matrix QV associated with eigenvalues $\lambda_1, \dots, \lambda_p$:

$$\begin{aligned}
 QV z_k &\stackrel{(3.50)}{\cong} Q V Q u_k \\
 &= Q \lambda_k u_k \quad (\text{the principal axes are eigenvectors of } VQ) \\
 &\stackrel{(3.50)}{\cong} \lambda_k z_k
 \end{aligned}
 \tag{3.53}$$

It is important however to distinguish between two situations:

The diagonalization of the V matrix of variance and covariance (assuming $Q = I_p$) amounts to a non normed PCA, which should be applied when the variables have the same units of measure;

The diagonalization of the correlation matrix R or of matrix $V \cdot Q_{1/s^2}$ (if the data is centered only) corresponds to a normed PCA, which is applied with variables that are not of the same nature, i.e. do not have the same units of measure.

3.2.5 Projection of the variables in a subspace

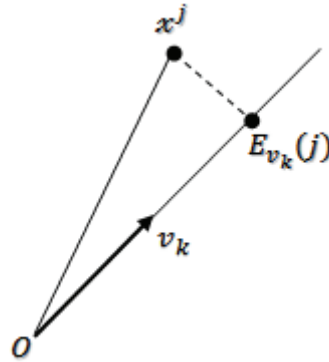
The projection of the variables in a subspace of \mathbb{R}^n is analogous to the projection of individuals in a subspace of \mathbb{R}^p . Let $\{v_k; k = 1, \dots, r\}$ be a vector base of \mathbb{R}^n , D-ortonormated defining the subspace (dimension r) on which the projection of the variables will be made. The criterion applied in 3.2.4 to minimize projection deformation (or maximize the inertia of the projected cloud) also applies, but with a different meaning, for N_j cloud is not centered and all its points are located in the hypersphere of radius 1. It is of interest to consider the angles formed by the vectors representing variables and not the distance between the cloud point, as defined in (3.10).

Be $E_{v_k}(j)$ the projection of x^j on the axis v_k such that:

$$E_{v_k}(j) = \langle x^j, v_k \rangle_D \quad (3.54)$$

e $E_v(j)$ the vector projections of variable x^j on the axes over v_1, \dots, v_r (Figure 3.4), such that:

$$E_v(j) = \sum_{k=1}^r E_{v_k}(j) \cdot v_k = \sum_{k=1}^r \langle x^j, v_k \rangle_D \cdot v_k \quad (3.55)$$

Figure 3.4 –Projection of x^k in v_k axis

Source: adapted from Carvalhido (2005)

The inertia of the cloud projected the axes v_1, \dots, v_r is deduced similarly will expression (3.36):

$$\sum_{j=1}^p \frac{1}{S_j^2} \|E_{v_k}(j)\|_D^2 = \sum_{k=1}^r v_k' D W D v_k \quad (3.56)$$

The WD matrix is D -symmetric, therefore it admits a D -orthonormal base of eigenvectors associated with the eigenvalues $\delta_1, \dots, \delta_r, \dots, \delta_n$. It is concluded then that the cloud of inertia projected for the axis v_k , $u_k' D W D v_k$ is maximum for the highest eigenvalue of WD , since $v_k' D v_k = 1$ and WD eigenvectors are v_k .

Finally, the coordinates of the projections of the variables x^1, \dots, x^p in v_k axis form the vector E_{v_k} of F space:

$$E_{v_k} = X' D v_k \quad (3.57)$$

3.2.6 Duality and transition relations

Determining v_k axes of the cloud of variables is identical to the determination of u_k axis of the cloud. For this is only necessary, in the results of projection of individuals in a subspace, to replace the X matrix by the matrix transposed X' and the Q matrix by the matrix D .

It was seen, upon the projection of individuals in a subspace, that $W D F_{u_k} = \lambda_k F_{u_k}$ and upon the projection of the variables in a subspace that $W D u_k = \delta_k$. Thus:

$$E_{v_k} = X' D v_k \quad (3.58)$$

$\lambda_k = \delta_k$ and the inertias of the projected clouds N_I and N_J on the respective principal axes are equal;

The VQ and WD arrays do not have the same dimension, therefore the eigenvalues not common to the two arrays are zero (Escofier and Pagès, 1998);

F_{u_k} and v_k are collinear vectors of \mathbb{R}^n associated with the same eigenvalue and as $\|v_k\|_D = 1$ and $\|F_{u_k}\|_D = \sqrt{\lambda_k}$, we have that:

$$F_{u_k} = \sqrt{\lambda_k} u_k \quad (3.59)$$

and the dual relationship of this is:

$$E_{v_k} = \sqrt{\lambda_k} u_k \quad (3.60)$$

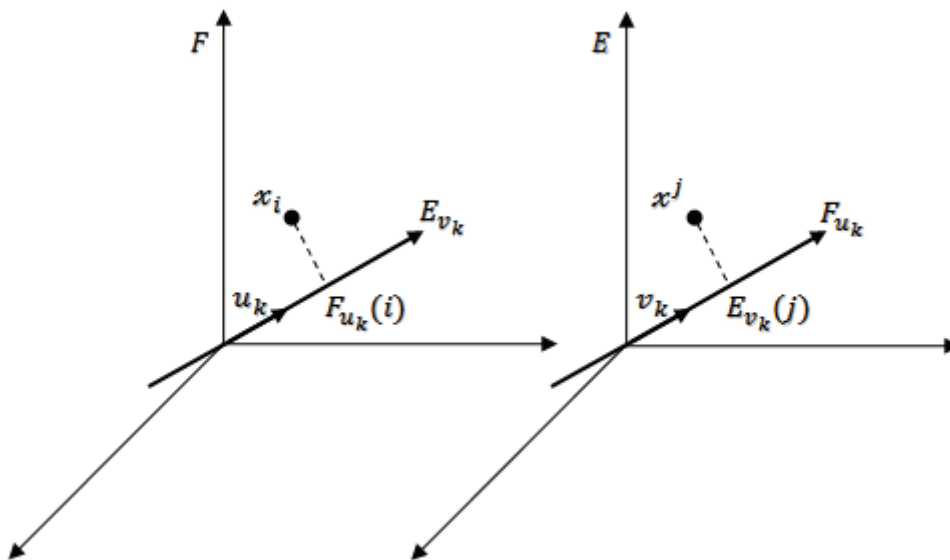
The projections of N_I on u_k are the coordinates of F_{u_k} , collinear with v_k and projections of N_J on v_k are the coordinates of E_{v_k} , collinear with u_k . These dual relationships are outlined in Table 3.1 and Figure 3.5.

Table 3.1: Duality relations between the spaces of individuals and variables

Clouds	N_I	N_J
Space	F	E
Metric	Q	D
Principal Axes	u_k	v_k
Equation	$X'D X Q u_k = \lambda_k u_k$	$X D X' Q v_k = \lambda_k v_k$
Norm	$\ u_k\ _Q = 1$	$\ v_k\ _D = 1$
Orthogonality	$\langle u_i, u_j \rangle_Q = 0 \ (i \neq j)$	$\langle v_i, v_j \rangle_D = 0 \ (i \neq j)$
Principal Components	$F_{u_k} = X Q u_k$	$E_{v_k} = X' D v_k$
Equation	$X Q X' D F_{u_k} = \lambda_k F_{u_k}$	$X D X' Q E_{v_k} = \lambda_k E_{v_k}$
Norm	$\ F_{u_k}\ _D = \sqrt{\lambda_k}$	$\ E_{v_k}\ _Q = \sqrt{\lambda_k}$
Orthogonality	$\langle F_{u_k}, F_{u_l} \rangle_D = 0 \ (k \neq l)$	$\langle E_{v_k}, E_{v_l} \rangle_Q = 0 \ (k \neq l)$

Source: adapted from Carvalhido (2005)

Figure 3.5 – Duality relations between principal axes and principal components



Source: adapted from Carvalhido (2005)

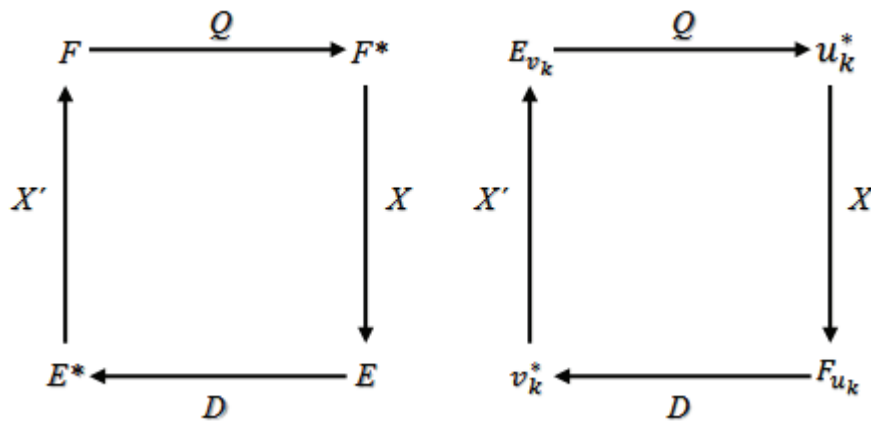
The XQ matrix defines an application of F to E and $X'D$ matrix defines an application of E on F . These matrices interconnect the axes and the principal components of the clouds. The Q matrix defines an isomorphism of F in F^* , dual space of the space of individuals.

If F is provided with Q^{-1} , Q is an isomorphism application Euclidean spaces, because the distances and the shapes are kept. Similarly, the matrix D defines an isomorphism of E on E^* , dual space of the space of variables.

The X matrix defines an application of F^* in E and the matrix X' defines an application of E^* in F .

The duality scheme introduced by Cailliez and Pagès (1976) shown in Figure 3.6, is a mathematical framework characterized by a triplet of three matrices, X , D , and Q . It establishes relationships between the matrices allowing to pass from the axes of a cloud to the axes of the other.

Figure 3.6 – Duality scheme of PCA



Source: adapted from Carvalhido (2005)

Note that the principal axes of the u_k^* cloud, dual of N_J^* , verify the relation:

$$Q X' D X u_k^* = \lambda_k u_k^* \tag{3.61}$$

and the principal axes of v_k^* of N_J^* , dual and N_J , verify the relation:

$$D X Q X' v_k^* = \lambda_k v_k^* \tag{3.62}$$

It is important now establish the transition relations between the main components F_{u_k} and E_{v_k} . The first relationship (3.63) expresses the projection of the individual x_i on the axis u_k as a linear combination of the projections of all the variables. The second relationship (3.64), dual of the first, expresses the projection of x^j on the axis v_k as a linear combination of the projections of all individuals.

$$\begin{aligned}
 (3.31) \\
 F_{u_k} &\stackrel{\hat{=}}{=} \langle x_i, u_k \rangle_Q \\
 &= x'_i Q u_k \\
 &= \frac{1}{\sqrt{\lambda_k}} \sum_{j=1}^p \frac{x'_i - \bar{x}^j}{s_j} \cdot E_{v_k} (j)
 \end{aligned} \tag{3.63}$$

$$\begin{aligned}
 (3.54) \\
 E_{v_k} &\stackrel{\hat{=}}{=} \langle x^j, v_k \rangle_D \\
 &= (x^j)' D v_k \\
 &= \frac{1}{\sqrt{\lambda_k}} \sum_{i=1}^n \frac{1}{p_i} \cdot \frac{x^j_i - \bar{x}^j}{s_j} \cdot F_{u_k} (i)
 \end{aligned} \tag{3.64}$$

3.2.7 Quality and interpretation of results

3.2.7.1 Reconstitution formulas

We have seen in (3.44) that $F_{u_k} = X Q u_k$. Now, multiplying both sides by u'_k and adding from $k = 1, \dots, p$, follows:

$$\sum_{k=1}^p F_{u_k} u'_k = X Q \sum_{k=1}^p u_k u'_k \tag{3.65}$$

By a result of Linear Algebra (if $\{u_1, u_2, \dots, u_n\}$ is a Q-orthonormal basis, then $\sum_{k=1}^n u_k u'_k = Q^{-1}$ and it is verified that the expression (3.65) is equivalent to:

$$X Q Q^{-1} = X \tag{3.66}$$

Thus it can be obtained an approximate reconstruction of the centered data frame, using only the q first principal components:

$$X \cong \sum_{k=1}^q F_{u_k} u'_k \quad (3.67)$$

For a reconstitution of the variance and covariance matrix, consider:

$$\begin{aligned} V &\stackrel{(3.22)}{\cong} X' D X \\ &\stackrel{(3.65)}{\cong} \left(\sum_{k=1}^p F_{u_k} u'_k \right)' D \left(\sum_{k=1}^p F_{u_k} u'_k \right) \\ &= \left(u_1 F'_{u_1} + \dots + u_p F'_{u_p} \right) D \left(F_{u_1} u'_1 + \dots + F_{u_p} u'_p \right) \\ &= u_1 [F'_{u_1} D F_{u_1}] u'_1 + \dots + u_p [F'_{u_p} D F_{u_p}] u'_p \end{aligned} \quad (3.68)$$

given that:

$$\begin{aligned} u_k [F'_{u_1} D F_{u_1}] u'_1 &= u_k \cdot \langle F'_{u_p}, F_{u_l} \rangle_D \cdot u'_l \\ &= u_k \cdot S \left(F'_{u_p}, F_{u_l} \right) \cdot u'_l \stackrel{(3.48)}{\cong} 0 \end{aligned} \quad (3.69)$$

for $k \neq l$ as the principal components are uncorrelated.

Returning to expression (3.68), it follows that:

$$V = \lambda_1 u_1 u'_1 + \dots + \lambda_p u_p u'_p \quad (3.70)$$

given:

$$F'_{u_k} D F_{u_k} = S \left(F_{u_k}, F_{u_k} \right) \stackrel{(3.48)}{\cong} \lambda_k \quad (3.71)$$

Using the q first principal components, one obtains an approximate reconstruction of the matrix V :

$$V \cong \sum_{k=1}^q \lambda_k u_k u'_k \quad (3.72)$$

3.2.7.2 Quality measures

Since, by (3.42), $l_g = tr(VQ) = \sum_{k=1}^p \lambda_k$, it follows that:

$$\frac{\lambda_k}{\sum_{k=1}^p \lambda_k} \quad (3.73)$$

represents the proportion of inertia explained by the principal axis u_k and:

$$\frac{\sum_{k=1}^q \lambda_k}{tr(VQ)} \quad (3.74)$$

It is the proportion of inertia explained by the q first principal axes. These latter quantities are quality indicators of N_I cloud representation in u_k and in projection subspace (of dimension q), respectively.

Previously has already been shown in (3.48) that the variance of the k -th principal component is equal to the eigenvalue λ_k :

$$S^2(F_{u_k}) \stackrel{(3.9)}{=} \sum_{i=1}^n p_i (F_{u_k}(i))^2 = \lambda_k \quad (3.75)$$

By definition, $F_{u_k}(i)$ is the projection of the individual x_i on the principal axis u_k . Therefore, the absolute contribution of the individual x_i to the formation of principal axis u_k is:

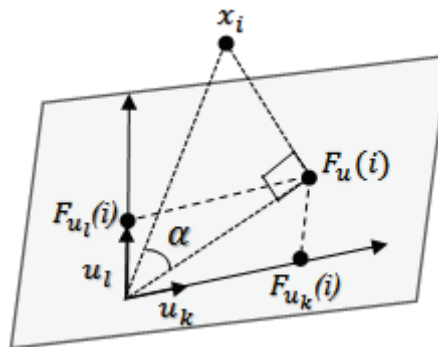
$$CTA_i^k = \frac{p_i (F_{u_k}(i))^2}{\lambda_k} \quad (3.76)$$

Note that:

- $\sum_{i=1}^n CTA_i^k = \sum_{i=1}^n \frac{p_i (F_{u_k}(i))^2}{\lambda_k} = 1$;
- The absolute contribution of an individual to the formation of principal axis u_k , that is, for the variance explained by u_k axis, allows us to highlight the individuals who present characteristics related to the phenomenon, translated by principal component that corresponds to it;

- It is that the contribution of the individual x_i to the formation of principal axis u_k is only relevant if it exceeds its weight (p_i) on the sample (Saporta, 1990);
- If an individual has an excessive contribution to the formation of the first axes, that could be a factor of instability in the PCA. This individual should then be supplementary, that is, it should not intervene in determining the main elements, and only subsequently be included in the graphical representations.
- To measure the quality of representation of an individual x_i in the principal plane (u_k, u_l), formed by the principal axes u_k and u_l , it is necessary to consider the angle α between this individual and this principal plane (Figure 3.7).

Figure 3.7 –Projection of the individual x_i in the (u_k, u_l) plane



Source: adapted from Carvalhido (2005)

We have that:

$$\cos^2 \alpha = \frac{\|F_u(i)\|_Q^2}{\|x_i\|_Q^2} = \frac{(F_{u_k}(i))^2 + (F_{u_l}(i))^2}{\sum_{j=1}^p q_j (x_i^j)^2} \quad (3.77)$$

This ratio is denominated by of the relative contribution of plan (u_k, u_l) to the individual x_i and is denoted by $\rho_i^{k,l}$. Note that the closer x_i is of the plane, also the closer will be to $F_u(i)$, the lower the amplitude of the angle α , that is, the closer $\cos^2 \alpha$ will be to 1 and the better the quality of their representation.

Let us define the relative contribution of major axis u_k to individual x_i, ρ_i^k , as:

$$\rho_i^k = \frac{\|F_{u_k}(i)\|_Q^2}{\|x_i\|_Q^2} = \frac{(F_{u_k}(i))^2}{\sum_{j=1}^p q_j (x_i^j)^2} \quad (3.78)$$

This contribution reflects the proximity between the individual x_i and the principal axis u_k , so the closer to 1 is ρ_i^k , the better the quality of representation of individual x_i on axis u_k . Note that:

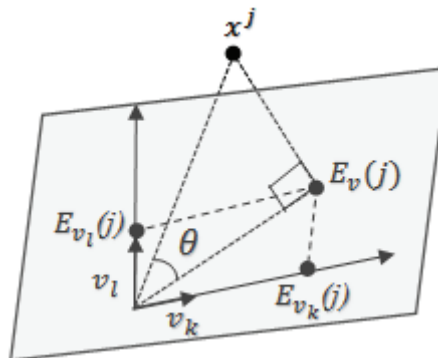
$$\rho_i^{k,l} = \rho_i^k + \rho_i^l \quad (3.79)$$

Similarly, θ is the angle between the variable x^j and its projection in the principal plane (v_k, v_l) (Figure 3.8) and,

$$\cos^2 \theta = \frac{\|E_v(j)\|_D^2}{\|x^j\|_D^2} = \frac{(E_{v_k}(j))^2 + (E_{v_l}(j))^2}{\sum_{i=1}^n p_i (x_i^j)^2} \stackrel{(3.60)}{\cong} \frac{(\sqrt{\lambda_k} u_k^j)^2 + (\sqrt{\lambda_l} u_l^j)^2}{S^2(x^j)} \quad (3.80)$$

being u_k^j the j -th element of the vector u_k .

Figure 3.8 –Projection of the variable x^j in the (v_k, v_l) plane



Source: adapted from Carvalho (2005)

Let us define the relative contribution of the main component F_{u_k} to the variable x^j as:

$$\gamma_k^j = \frac{(\sqrt{\lambda_k} u_k^j)^2}{S^2(x^j)} \quad (3.81)$$

Being the principal components F_{u_k} the new variables, it is important to know how to interpret them. One way of doing this is to determine the degree of linear correlation between each main component and each of the initial variables x^j

$$\begin{aligned} r(x^j, F_{u_k}) &= \frac{S(x^j, F_{u_k})}{S(x^j) S(F_{u_k})} \\ &\stackrel{(3.48)}{=} \frac{(x^j)' D F_{u_k}}{S(x^j) \sqrt{\lambda_k}} \\ &= \frac{\lambda_k u_k^j}{S(x^j) \sqrt{\lambda_k}} \\ &= \frac{\sqrt{\lambda_k} u_k^j}{S(x^j)} \end{aligned} \quad (3.82)$$

Note that:

$$\gamma_k^j = [r(x^j, F_{u_k})]^2 \quad (3.83)$$

The following relationship:

$$\|u_k\|_Q^2 = 1 \Leftrightarrow u_k' Q u_k = 1 \Leftrightarrow \sum_{j=1}^p q_j (u_k^j)^2 = 1 \quad (3.84)$$

Comes the absolute contribution of x^j for forming the main component F_{u_k} :

$$CTA_k^j = q_j (u_k^j)^2 \quad (3.85)$$

So we have that the contributions of the variables are proportional:

$$\gamma_k^j = \lambda_k \cdot CTA_k^j \quad (3.86)$$

with λ_k as proportionality factor.

3.2.7.3 Circle of correlations

If the data is centered and reduced, the graphical representation of the variables can be carried out using the linear correlation coefficient between F_{u_k} and x^j :

$$r(x^j, F_{u_k}) \stackrel{(3.82)}{=} \sqrt{\lambda_k} u_k^j \quad (3.87)$$

Whereas the axes associated with a pair of principal components (F_{u_k}, F_{u_l}) , the points are represented:

$$(r(x^j, F_{u_k}), r(x^j, F_{u_l})) \quad (3.88)$$

Where each variable x^j is represented by the point of abscissa $r(x^j, F_{u_k})$ and ordinate $r(x^j, F_{u_l})$ the so-called circle of correlations contained in the principal plane (v_k, v_l) (Figure 3.9).

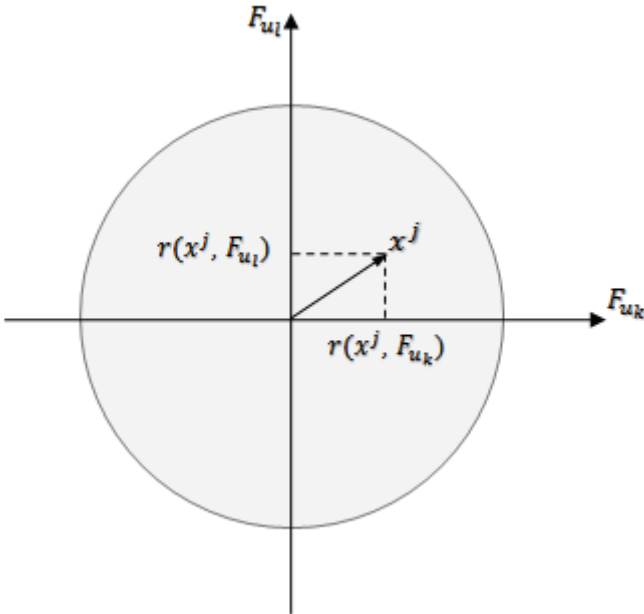
The quality of representation of the variable on the axis represented by F_{u_k} will only be reasonable if their representation in the circle of correlations, approaches the border since $\gamma_k^j = [r(x^j, F_{u_k})]^2$ and the linear correlation coefficient varies between -1 and 1.

Consider the example in Figure 3.10. The principal component F_{u_k} is highly correlated with the variables x^1 (positively) and x^2 (negatively) and not correlated with variables x^3 and x^4 . In opposition, the principal component F_{u_l} is highly correlated with the variables x^3 (positively) and x^4 (negatively) and not correlated with the variables x^1 and x^2 .

Each variable (centered and reduced) is represented in \mathbb{R}^n by a point of the hypersphere of radius 1 and projected in the plane defined by a pair of principal components within a circle of radius 1.

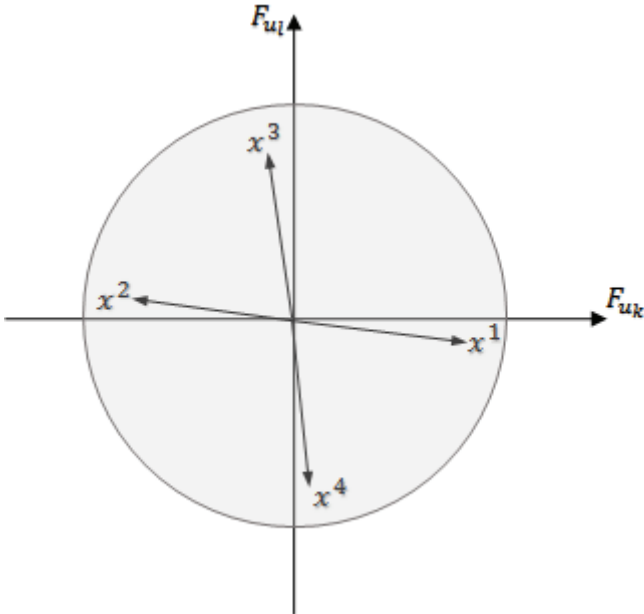
In a straightforward approach, the closer a variable is to the border of the circle of correlations, the better his the reconstruction of the variable from the first two components and the closer to the center of the circle a variable is, the less important it is for the plotted components.

Figure 3.9 –Coordinates of the variable x^j in the correlations circle



Source: adapted from Carvalhido (2005)

Figure 3.10 –Correlations circle interpretation



Source: adapted from Carvalhido (2005)

3.2.7.4 Number of axes to consider

Since we already know how to reduce the size of the space of p individuals to q (with $q \leq p$), the question that arises is how many components to retain, that is, what will be the value of q . It has been shown in (3.74) that:

$$\frac{\sum_{k=1}^q \lambda_k}{\sum_{k=1}^p \lambda_k} \quad (3.89)$$

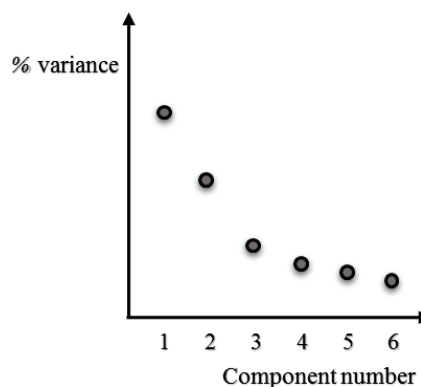
represents the percentage of inertia explained by q first principal axis and acts as a global quality indicator for the PCA.

A commonly used empirical criterion is to determine the lowest value of q that ensures a percentage of explained variance of not less than an amount fixed and usually is around 80%. This criterion can lead to retention components with significant weight only to a variable.

Kaiser (1961) proposes another method for centered and reduced data to exclude the principal components whose values are less than 1, that is, components whose eigenvalue is smaller than the average. However, this procedure can lead to exclude relevant information.

A straightforward approach proposed by Cattell (1966) (Figure 3.11) is a graphical representation of the percentage of variance explained by each component in descending order to see if there is a point such that the graph is almost parallel to the first axis, and to exclude the components which are onward.

Figure 3.11 – Scree plot



Source: Adapted from Carvalhido (2005)

3.3 STATIS

The STATIS method (*Structuration de Tableaux À Trois Indices de la Statistique*) was first introduced by Escoufier (1973) and L'Hermier des Plantes (1976) in the Probability and Statistics Laboratory of the University of Montpellier II, around 1976, and later developed by Lavit (1988b). This method is not restricted to the analysis of one data frame, as was the case of the PCA, but allows simultaneous exploration of multiple data frames (quantitative) collected from one of the following situations:

1. T data frames collected at different times on the same subjects; variables can vary over the frames; In this case the method is characterized by T studies $(X_t, Q_t, D), t = 1, \dots, T$;
2. The same variables are described throughout the T frames of data collected at different times on individuals, which may differ along the frames; In this case the method is characterized by T studies $(X_t, Q, D_t), t = 1, \dots, T$.

A different strategy corresponds to each of these situations; The first shows the proximity between individuals (STATIS) and the second favors the relationships between variables (Dual STATIS). However, it should be noted here in that the Dual STATIS will not be the object of attention in this study, since the produced empirical contributions concern the traditional method, that is, in STATIS.

Nevertheless, these two methods of analysis comprise the same three fundamental steps:

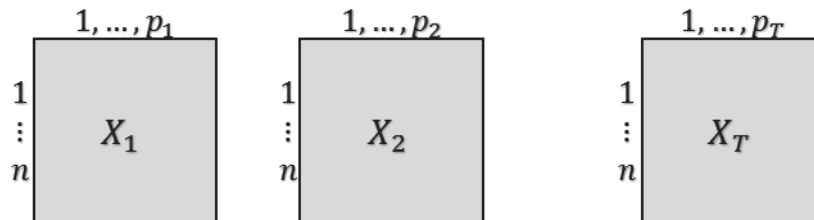
1. The **interstructure**, characterized by the global comparison of data frames (or studies). It is intended, at this stage, to determine the existence of similarities (or dissimilarities) between studies. However one cannot assess which are the individuals (or variables) responsible;
2. The graphical representation of interstructure on a factorial plan allows assessment of the possibility of establishing a **compromise** matrix. As the name implies, this is a representative matrix of the set of initial data frames and features through a Euclidean image, the common structure to the set of initial data;

3. The definition of compromise, in turn, allows the representation of the positions of different individuals described in the data frames, i.e., it allows the analysis of the **intrastructure**. Upon detection of associations between the original variables and the various individuals is possible to interpret the positions of individuals in compromise space, identifying those responsible for the similarities (or differences) between studies. And, from the compromise image, the trajectories are drawn that describe the evolutionary behavior of each individual.

3.3.1 The procedure

The representation in Figure 3.12 considers T data frames consisting of n individuals on p_t quantitative variables, $t = 1, \dots, T$.

Figure 3.12 – T data tables



Source: Adapted from Carvalhido (2005)

The condition t to X_t table is represented by the array of dimension $n \times p_t$

$$X_t = \begin{pmatrix} (x_1^1)^t & (x_1^2)^t & \dots & (x_1^{p_t})^t \\ (x_2^1)^t & (x_2^2)^t & \dots & (x_2^{p_t})^t \\ \vdots & \vdots & \ddots & \vdots \\ (x_n^1)^t & (x_n^2)^t & \dots & (x_n^{p_t})^t \end{pmatrix} \quad (3.90)$$

where the j -th variable is the vector of \mathbb{R}^n :

$$(x^j)^t = \begin{pmatrix} (x_1^j)^t \\ (x_2^j)^t \\ \vdots \\ (x_n^j)^t \end{pmatrix} \quad (3.91)$$

and the i -th individual is the vector \mathbb{R}^{p_t} :

$$((x_i)^t)' = \left((x_i^1)^t, (x_i^2)^t, \dots, (x_i^{p_t})^t \right) \quad (3.92)$$

3.3.2 The interstructure

This first step is the overall comparison of T frames. This requires defining a representative object for each study, a metric on representative objects of each study and, finally, define a Euclidean image of representative objects associated with scalar products already defined. Thus, it is possible to characterize a study (X_t, Q_t, D) , $t = 1, \dots, T$ for an object:

$$W_t = X_t Q_t X_t' \quad (3.93)$$

where Q_t is the metric associated with the space of individuals of table X_t , and D is the metric associated with the variable space defined in (3.4). The W_t object is a dimension $n \times n$ matrix called the matrix of scalar products between individuals of the X_t table.

As already mentioned, to graphically represent the T studies it is required to define a distance (metric) between them, and for this is it necessary to define a scalar product between the objects:

$$\langle W_t, W_{t'} \rangle_{HS} = tr (W_t D W_{t'} D) \quad (3.94)$$

The Hilbert-Schmidt scalar product was introduced initially by Escoufier (1973), to induce a distance (Euclidean) between W_t and $W_{t'}$ objects, given by the following expression:

$$\begin{aligned}
 d_{HS}(W_t, W_{t'}) &= \|W_t - W_{t'}\|_{HS} \\
 &= \sqrt{\langle W_t - W_{t'}, W_t - W_{t'} \rangle_{HS}} \\
 &\stackrel{(3.94)}{\cong} \sqrt{\text{tr}((W_t - W_{t'}) D)^2}
 \end{aligned} \tag{3.95}$$

Note that the distance between these objects can also be established as follows:

$$\begin{aligned}
 d_{HS}(W_t, W_{t'}) &= \|W_t - W_{t'}\|_{HS} \\
 &= \sqrt{\langle W_t - W_{t'}, W_t - W_{t'} \rangle_{HS}} \\
 &= \sqrt{\|W_t\|_{HS}^2 + \|W_{t'}\|_{HS}^2 - 2 \langle W_t, W_{t'} \rangle_{HS}}
 \end{aligned} \tag{3.96}$$

The norm of the W_t object is thus defined as follows:

$$\|W_t\|_{HS} = \sqrt{\langle W_t, W_t \rangle_{HS}} \stackrel{(3.94)}{\cong} \sqrt{\text{tr}(W_t D W_t D)} = \sqrt{\text{tr}(W_t D)^2} = \sqrt{\sum_{i=1}^n (\lambda_i^t)^2} \tag{3.97}$$

with λ_i^t the i -th eigenvalue of $W_t D$.

The S matrix of scalar products between objects W_t and $W_{t'}$ of dimension $T \times T$, has the general term:

$$S_{tt'} = \langle W_t, W_{t'} \rangle_{HS} \tag{3.98}$$

In the same way that happened in the PCA of the W_t objects have very different norms, it is appropriate to normalize them, that is, to consider the $W_t / \|W_t\|_{HS}$ objects; In this case, the scalar product matrix \tilde{S} has a general term:

$$\tilde{S}_{tt'} = \frac{\langle W_t, W_{t'} \rangle_{HS}}{\|W_t\|_{HS} \|W_{t'}\|_{HS}} \tag{3.99}$$

The correlation coefficient vector between W_t and $W_{t'}$ objects, or RV coefficient, proposed by Robert and Escoufier (1976) is denoted by:

$$RV(t, t') = \left\langle \frac{W_t}{\|W_t\|_{HS}}, \frac{W_{t'}}{\|W_{t'}\|_{HS}} \right\rangle_{HS} = \frac{S_{tt'}}{\sqrt{S_{tt}}\sqrt{S_{t't'}}} = \tilde{S}_{tt'} \quad (3.100)$$

The RV coefficients are very useful in the interpretation of the interstructure, since they possess the following properties:

- The RV coefficients allow obtaining the distance between two normed objects:

$$\begin{aligned} d_{HS} \left(\frac{W_t}{\|W_t\|_{HS}}, \frac{W_{t'}}{\|W_{t'}\|_{HS}} \right) &= \left\| \frac{W_t}{\|W_t\|_{HS}} - \frac{W_{t'}}{\|W_{t'}\|_{HS}} \right\|_{HS} \\ &= \sqrt{2 - 2 RV(t, t')} \end{aligned} \quad (3.101)$$

- If $RV(t, t') = 1$ the distance above is null and:

$$\frac{W_t}{\|W_t\|_{HS}} = \frac{W_{t'}}{\|W_{t'}\|_{HS}} \quad (3.102)$$

This means that Euclidean image of t study subjects is deduced from the Euclidean image of the study t' by the homothety of reason $\|W_t\|_{HS} / \|W_{t'}\|_{HS}$.

There are two important reasons for using the scalar product of Hilbert-Schmidt:

1. Decomposing $\|W_t - W_{t'}\|_{HS}^2$ in the space of individuals (ie, with Q_t and $Q_{t'}$ metrics) one obtains:

$$\|W_t - W_{t'}\|_{HS}^2 = \sum_{i=1}^n \sum_{j=1}^n p_i p_j \left(\langle (x_i)^t, (x_j)^t \rangle_{Q_t} - \langle (x_i)^{t'}, (x_j)^{t'} \rangle_{Q_{t'}} \right)^2 \quad (3.103)$$

So $\|W_t - W_{t'}\|_{HS}^2$ is equal to the weighted sum of the squares of the differences between the scalar products between individuals X_t table and the scalar products between individuals $X_{t'}$ table.

2. Expressing $\langle W_t, W_{t'} \rangle_{HS}$ in the space of the variables and using the "metric identity" to calculate scalar products between individuals (i.e., $Q_t = I_{p_t}$ and $Q_{t'} = I_{p_{t'}}$), we have :

$$\langle W_t, W_{t'} \rangle_{HS} = \sum_{k=1}^{p_t} \sum_{l=1}^{p_{t'}} (\langle (x^k)^t, (x^l)^{t'} \rangle_D)^2 \quad (3.104)$$

Then $\langle W_t, W_{t'} \rangle_{HS}$ is equal to the sum of the squares of covariances between X_t table variables and the $X_{t'}$ table variables.

If W_t objects and $W_{t'}$ are orthogonal, we have that $\langle W_t, W_{t'} \rangle_{HS} = 0$, then $RV(t, t') = 0$ and we can say that the covariances between the variables of X_t and the variables $X_{t'}$ are null.

To construct the Euclidean image of T studies it is necessary affect each of these a weight called π_t . Thus, the matrix of weights of the studies is:

$$\Delta = \begin{pmatrix} \pi_1 & 0 & \dots & 0 \\ 0 & \pi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \pi_T \end{pmatrix} \quad (3.105)$$

In the case of assigning the same importance to the studies we have that $\Delta = \frac{1}{T} I_T$. If there is a table that should intervene in the Euclidean image without contributing to the analysis and to the construction of the axes, that table should be considered supplementary, i.e. it should be assigned a zero weight.

From there it is necessary to apply an PCA to the matrix S ; To this end it is required to calculate the eigenvalues and eigenvectors of the $S\Delta$ matrix. Let then:

- $\tau_1, \tau_2, \dots, \tau_T$ the eigenvalues of matrix $S\Delta$ associated with the Δ – *ortonormated* eigenvectors $\gamma_1, \gamma_2, \dots, \gamma_T$, respectively;
- A_1, A_2, \dots, A_T the points associated with W_1, W_2, \dots, W_T , respectively, in the Euclidean image.

The A_t coordinates of the i -th axis are the components of the vector $\sqrt{\tau_i}\gamma_i$ (dimension t) with $t = 1, \dots, T$, by analogy with the expression (3.60).

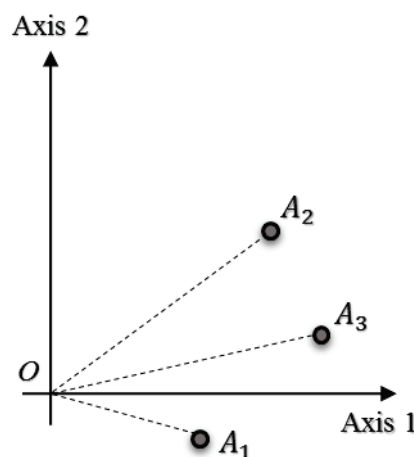
In practice, the Euclidean image is limited to the first two points, resulting in a flat Euclidean image approximated of the T studies, associated with the scalar products between objects: the main plane. Thus, the distance between the points A_t and $A_{t'}$ is the best approximation between W_t and $W_{t'}$ objects in the sense of the scalar product of Hilbert-Schmidt and two points sufficiently close to the main plane show a common structure of individuals to the corresponding tables.

The following theorem, whose proof is in Lavit (1988b), allows an idea of how the A_t points will position themselves in the respective Euclidean image.

Theorem 3.2: *A symmetric matrix with all positive terms admits an eigenvector associated with largest eigenvalue all of whose coordinates have the same sign.*

Now representing the A_t points ($t = 1, \dots, T$) in the main plane formed by the first two axes, these will be located all the first and fourth quadrants, assuming that the coordinates of the first eigenvector are all positive (Figure 3.13)

Figure 3.13 – Representation of objects in the main plane



Source: Adapted from Carvalhido (2005)

Note that the coefficient $RV(t, t')$ represents the cosine of the angle formed by the vectors $\overrightarrow{OA_t}$ e $\overrightarrow{OA_{t'}}$:

$$RV(t, t') \stackrel{(3.100)}{\cong} \frac{\langle W_t, W_{t'} \rangle_{HS}}{\|W_t\|_{HS} \|W_{t'}\|_{HS}} = \cos(\overrightarrow{OA_t}, \overrightarrow{OA_{t'}}) \quad (3.106)$$

3.3.3 The compromise

The study of interstructure evidenced the similarities between the various studies, without explaining them. It is time to look for a new table which summarizes the set of objects and which is of the same nature as them. This table is called compromise and is no more than a weighted average of W_t objects:

$$W = \sum_{t=1}^T \alpha_t W_t \quad (3.107)$$

If the objects are normed the compromise is defined by:

$$\|W\|_{HS} = \sum_{t=1}^T \pi_t \|W_t\|_{HS} \quad (3.108)$$

The determination of the α_t coefficients depends on two criteria:

- The W compromise is the most correlated object with W_t objects (in the sense of the scalar product of Hilbert-Schmidt);
- W must be an object of the same nature as the W_t objects, ie:

$$\|W\|_{HS} = 1 \quad (3.109)$$

Be γ_1 the eigenvector of $S\Delta$ associated with higher value eigenvalue τ_1 referred to as:

$$\gamma_1 = \begin{pmatrix} \gamma_1^1 \\ \gamma_1^2 \\ \vdots \\ \gamma_1^T \end{pmatrix} \quad (3.110)$$

whose coordinates are all of the same sign (let us admit that they are all positive) by virtue of Theorem 3.2.

Now the α_t coefficients are determined by the following formula (Lavit, 1988b):

$$\alpha_t = \frac{1}{\sqrt{\tau_1}} \left(\sum_{l=1}^T \pi_l \|W_l\|_{HS} \right) \pi_t \gamma_1^t \quad (3.111)$$

Therefore the expression that defines the compromise is given in the following formulas:

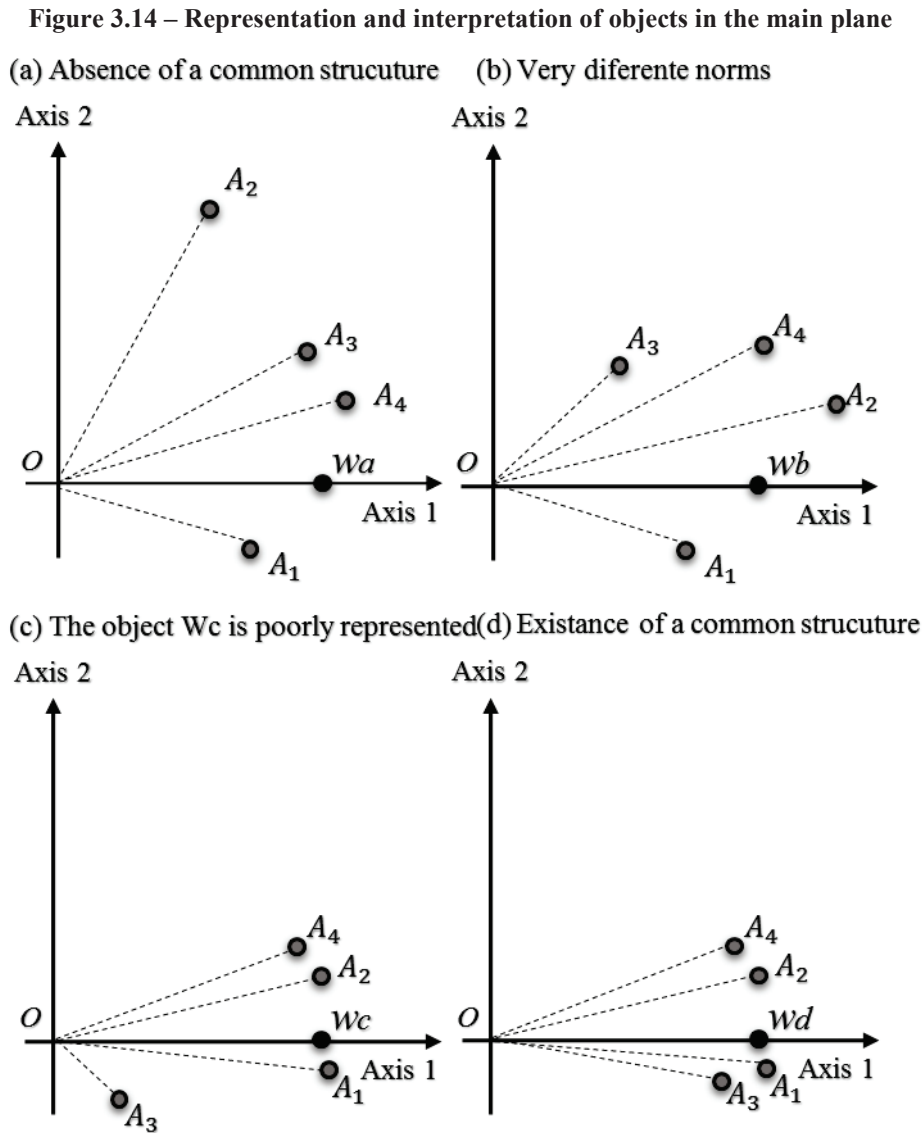
$$W = \sum_{t=1}^T \left[\frac{1}{\sqrt{\tau_1}} \left(\sum_{l=1}^T \pi_l \|W_l\|_{HS} \right) \pi_t \gamma_1^t \cdot W_t \right] \quad (3.112)$$

The coordinate of the W compromise over the i -th axis is obtained by linear combination of the coordinates $\sqrt{\tau_i} \gamma_i^t$ of A_t points over the i -th axis:

$$\sum_{t=1}^T \alpha_t \sqrt{\tau_i} \gamma_i^t \quad (3.113)$$

As the eigenvectors of the $S\Delta$ matrix are Δ -ortonormated (since $S\Delta$ is Δ -symmetric) and by definition of α_t , all compromise coordinates will be zero, except the first one. Thus, it can be concluded that the W compromise object will be located on the first axis of the Euclidian image.

Note that the interpretation of the Euclidean image of objects is only valid if the RV coefficients are high between the respective studies. The figures shown in Figure 3.14, can assist in this interpretation.



Source: Adapted from Carvalhido (2005)

In Figure 3.14 (a), the compromise object is just a weighted average of the objects and does not reflect a structure of individuals common to the objects, since they are very different, and consequently the RV coefficients are weak.

The norms of the objects in Figure 3.14 (b) are very different, as the objects of higher norms contribute more to the construction of the compromise.

The W_c object intervenes little in the construction of compromise on Figure 3.14 (c), i.e. the X_c table has a different structure from the other. A possible alternative is to consider it supplementary.

The objects considered in Figure 3.14 (d) have very similar norms and high RV coefficients. In this case, there is a common structure for individuals to the multiple tables and the compromise correctly translates that structure.

At this moment the conditions are met to make two types of representation:

1. The cloud representation of n individuals characterized by T frames to obtain the compromises Euclidean image;
2. The representation of the correlations of the various tables with the compromise axes, pursuing the interpretation of these axes and of the positions of individuals on the respective compromise plane.

The compromise is designated by W ($n \times n$) matrix, which is centered by the weights of individuals. Indeed, being the W_t tables centered and fixing the column j , we have that:

$$\begin{aligned} \sum_{i=1}^n p_i W_{ij} &= \sum_{i=1}^n p_i \sum_{t=1}^T \alpha_t (W_t)_{i,j} \\ &= \sum_{t=1}^T \alpha_t \sum_{i=1}^n p_i (W_t)_{i,j} = 0 \end{aligned} \tag{3.114}$$

Applying a PCA to the cloud of individuals of the matrix W , we obtain the Euclidean image of compromise. Let then:

- $\mu_1, \mu_2, \dots, \mu_n$ be the eigenvalues of the WD matrix associated with the (D-ortonormated) $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ eigenvectors, respectively;
- B_1, B_2, \dots, B_n the points associated with individuals in Euclidean image of compromise.

The coordinates of these points over k -th axis are the components of the vector $\sqrt{\mu_k} \varepsilon_k$ (of dimension n) with $k = 1, \dots, n$.

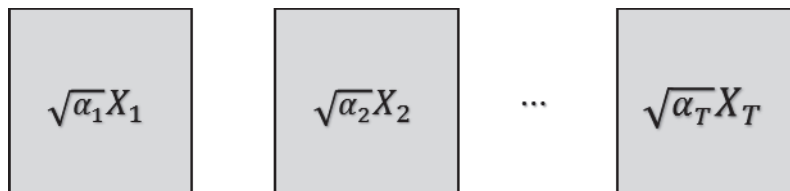
If the interstructure study demonstrated the existence of a common structure to the individuals tables (as in Figure 3-14 (d)), then it is appropriate to make the Euclidean representation of the approximated image of the compromise, by restricting to the first two or three axes, according to the percentage of inertia explained by them.

The d_{B_i, B_j} distance between B_i and B_j points in this image corresponds to the Euclidean distance between i and j individuals in the period $[1, T]$, and it is deduced from the distance between i and j individuals in each study:

$$d_{B_i, B_j}^2 = \sum_{t=1}^T \alpha_t \left\| (x_i)^t - (x_j)^t \right\|_{Q_t}^2 \quad (3.115)$$

Note that the obtained compromise Euclidean image is equivalent to what would have been obtained by implementing an PCA on the data table of $n \times \sum_{t=1}^T p_t$ built from the juxtaposition of tables $\sqrt{\alpha_1}X_1, \dots, \sqrt{\alpha_T}X_T$, or arranging these same as columns (Figure 3.15).

Figure 3.15 – Juxtaposition of the STATIS tables



Source: Adapted from Carvalhido (2005)

As the compromise is centered by the weights of the individual, so is the Euclidean image of compromise, i.e. the point of intersection of the axes corresponds to the center of gravity of points B_1, B_2, \dots, B_n . In this case, it is possible to interpret the position of the individuals on any axis.

Analogously to what was done in the PCA, the calculus is performed of the correlations of the principal component of the corresponding compromise to the k-th axis with the variables of each study.

For a centered and reduced variable $(x^j)^t$, the correlation between it and the k-th axis is equal to:

$$\langle \varepsilon_k, (x^j)^t \rangle_D = ((x^j)^t)' D \varepsilon_k \quad (3.116)$$

identical to expression (3.82).

The graph of correlation, where the variable $(x^j)^t$ is represented by a point whose coordinate over k-th axis is equal to $\langle \varepsilon_k, (x^j)^t \rangle_D$, allows us to visualize and interpret the compromise positions of individuals along the axes.

3.3.4 The intrastructure

The intrastructure highlights the differences between objects and, of these, with compromise. To explain these differences at individual level $d_{HS}^2(W_t, W_{t'})$ is decomposed in the sum of:

$$\sum_{i=1}^n p_i \sum_{j=1}^n p_j [(W_t)_{i,j} - (W_{t'})_{i,j}]^2 \quad (3.117)$$

In contributions of individuals, element by element:

$$\frac{p_i \sum_{j=1}^n p_j [(W_t)_{i,j} - (W_{t'})_{i,j}]^2}{d_{HS}^2(W_t, W_{t'})} \quad (3.118)$$

This decomposition results in a matrix of $n \times \left[\frac{1}{2}T(T-1) \right]$ dimension that can detect that individuals which most contribute to the opposition between pairs of objects, as described Lavit *et al.* (1994).

Another possible decomposition is the sum of squares of distances between all pairs of objects in the individuals' contributions:

$$\frac{p_i \sum_{t=1}^T \sum_{t'=1}^T \sum_{j=1}^n p_j [(W_t)_{i,j} - (W_{t'})_{i,j}]^2}{\sum_{t=1}^T \sum_{t'=1}^T d_{HS}^2(W_t, W_{t'})} \quad (3.119)$$

originating a vector of $n \times 1$ dimension.

Another way to highlight the individuals which most contribute to the differences between objects is the representation of trajectories, which takes place in the Euclidean image of the compromise and consists of representing in this image T subjects cloud, the t -th cloud defined by variables of X_t the table. This way we obtain a representation of nT points with n trajectories, each with T points. The technique implemented for this representation is similar to the one of the supplementary points; the different positions of an individual define its trajectory.

The coordinates of the B_1, B_2, \dots, B_n points of compromise over the k -th axis are the vector components of size n :

$$\sqrt{\mu_k} \varepsilon_k = \frac{1}{\sqrt{\mu_k}} W D \varepsilon_k \quad (3.120)$$

Considering each object as a supplementary element, the coordinates of points $B_1^t, B_2^t, \dots, B_n^t$ over the k -th axis are:

$$\frac{1}{\sqrt{\mu_k}} W_t D \varepsilon_k \quad (3.121)$$

for $t = 1, \dots, T$

It should be noted that none of these points intervened in the construction of the Euclidian image of compromise, but they can all be represented in it.

Property 3.2: The B_i compromise point is the center of gravity of $B_i^1, B_i^2, \dots, B_i^T$ points, weighted by coefficients $\alpha_1, \alpha_2, \dots, \alpha_T$; this property maintained in projection.

The trajectories allow us to identify which are the individuals responsible for deviations between tables X_t and $X_{t'}$. In fact, the distance between the B_i^t and $B_i^{t'}$ points over the k -

th axis is directly proportional to the distance between $W_t, \|W_t - W_{t'}\|_{HS}$ objects, and the expansion coefficient of $1/\sqrt{\mu_k}$.

These expansion coefficients, arranged in increasing order of k , increasingly deform the projections of individuals on the axes. This is why the analysis of the trajectories is limited to the first two axes, or if there is no large difference, between μ_2 and μ_3 , to the first three axes.

The trajectories are interpreted according to the evolution of a fictitious individual whose values are the averages of the variables under analysis.

We can distinguish two cases regarding the way that the direction of the trajectories can take:

- A narrow trajectory that remains around its compromise position corresponds to an individual with an evolution very close to that of the average: in other words, for each variable, the difference between the value of this variable for this individual and the fictitious individual average is regular from one study to another;
- A very broad trajectory reflects a change of structure of the individual over the studies, which differs from the average trend.

If the points tend to group by variable it is convenient to graph the correlations with the axes of the compromise, in order to study in more detail the trajectories of individuals. This makes it possible to explain the axes of the compromise according to the variables and interpret the trajectories.

If, however, the correlations between the variables of the study are strong, the points on the graph correlations are grouped more by study than by variable. It is therefore not possible to describe the axes by the variables, or to interpret the trajectories.

3.4 MFA

Multiple factor analysis (MFA) was initially studied by Escofier and Pagès (1985) and allows the analysis of a population of individuals characterized by several groups of quantitative or qualitative variables.

Similarly to the STATIS method, there are T data tables, X_1, \dots, X_T collected at different occasions on the same individuals in which the variables may differ over these tables. The steps that constitute the MFA differ from those proposed by the STATIS method and are:

1. Determining the eigenvalues associated with each of the variable groups;
2. Determination of compromise, here designated by consensus, of the axes of the infrastructure and simultaneous representation of clouds (trajectories);
3. Interpretation of the axes of the infrastructure, according to the variables;
4. Study of interstructure and interpretation of the position of the data tables;
5. Interpretation of consensus positions and trajectories of individuals.

The first stage of the MFA is to perform T analysis in principal components in of each table, determining the eigenvalues and eigenvectors of matrices $W_t D$. Be λ_1^t the larger singular value of $W_t D$ matrix for $t = 1, \dots, T$. The weighting coefficients used in MFA are the inverse of these eigenvalues. This weighting aims to balance the preponderance of tables during analysis.

3.4.1 The infrastructure

The $\mathbb{R}^{\sum_{t=1}^T p_t}$ contains the representations of individuals and from it there are two types of graphic outputs:

1. Cloud representation of individuals, characterized by the set of variables (consensus);
2. Simultaneous representation of T clouds, characterized by each group of variables (trajectories of individuals).

Be $\mathcal{N}_I = \{\tilde{x}_i: i = 1, \dots, n\}$ the cloud of individuals defined by the various data tables $X_1, \dots, X_T \in \mathbb{R}^{\sum_{t=1}^T p_t}$ with:

$$\tilde{x}_i = \begin{pmatrix} (x_i)^1 \\ \vdots \\ (x_i)^T \end{pmatrix} = \begin{pmatrix} (x_i)^1 \\ \vdots \\ (x_1^{p_1})^1 \\ \vdots \\ (x_1^T)^T \\ \vdots \\ (x_1^{p_T})^1 \end{pmatrix} \quad (3.122)$$

vector of $\mathbb{R}^{\sum_{t=1}^T p_t}$ and be $\mathcal{N}_I^t = \{(x_i)^T: i = 1, \dots, n\}$ the cloud of the individuals defined by the X_t data table. To represent the $\mathcal{N}_I^t, t = 1, \dots, T$ clouds, in the space $\mathbb{R}^{\sum_{t=1}^T p_t}$, it is necessary to note that this space can be decomposed as a direct sum of T subspaces isomorphic to the \mathbb{R}^{p_t} space:

$$\mathbb{R}^{\sum_{t=1}^T p_t} = \bigoplus_{t=1}^T \mathbb{R}^{p_t} \quad (3.123)$$

The coordinates of the points of cloud \mathcal{N}_I^t in space $\mathbb{R}^{\sum_{t=1}^T p_t}$ are contained in \tilde{X}_t table of $n \times \sum_{t=1}^T p_t$ dimension, in which is X_t is completed by zeros (Figure 3.16).

Figure 3.16 – Table of data \tilde{X}_t

		$1, \dots, p_t$	
1	0	X_t	0
⋮			
n			

Source: Adapted from Carvalhido (2005)

Be $\mathcal{N}_i^t = \{(x_i)^t : t = 1, \dots, T\}$ the cloud of space $\mathbb{R}^{\sum_{t=1}^T p_t}$ that provides the T images of the i -th individual and $\mathcal{N}_i^* = \{x_i^* : i = 1, \dots, n\}$ the cloud of the centers of gravity of \mathcal{N}_i^t clouds with $x_i^* = \frac{1}{T} \sum_{t=1}^T (x_i)^t$. Indeed, taking into account the relationship (3.123), the \mathcal{N}_i^* cloud it deduces from the \mathcal{N}_i cloud by a ratio of homothety $1/T$. Thus the \mathcal{N}_i^* and \mathcal{N}_i clouds are classified as homothetic.

3.4.2 Representation of Individuals

MFA has the property of balancing the influence of different sets of variables by attributing a weight to each variable which must be the same for all the variables of the same frame in order not to modify the shape of the \mathcal{N}_i^t clouds. The weight given to each variable of a X_t frame is equal to the inverse of the inertia of the first principal component associated with this table. Instead of using Q_t metric, the $\left(\frac{1}{\lambda_1^t}\right) \cdot Q_t$ metric is used, which gives a weighting for the different T groups of variables and normalizes the various clouds, since the inertia of the first principal component of each group of variables is equal to 1.

The representation of individuals is obtained by projecting the \mathcal{N}_i cloud or cloud \mathcal{N}_i^* (since these are homothetic) onto a lower dimensional space, such that its projection is as similar as possible to the cloud \mathcal{N}_i . This requires that a PCA is executed on the data table formed by the juxtaposition of tables X_1, \dots, X_T , noted by χ , using the metric ζ defined by:

$$\zeta = \begin{pmatrix} \frac{1}{\lambda_1^1} \cdot Q_1 & 1 & 0 & \dots & 0 \\ \frac{1}{\lambda_1^2} \cdot Q_2 & \dots & \dots & \dots & 0 \\ 0 & \frac{1}{\lambda_1^1} & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ 0 & 0 & \dots & \frac{1}{\lambda_1^T} \cdot Q_T & \dots \end{pmatrix} \quad (3.124)$$

of size $\sum_{t=1}^T p_t \times \sum_{t=1}^T p_t$, or, on an equivalent manner, to apply a PCA to the juxtaposition of tables.

$$\frac{X_1}{\sqrt{\lambda_1^1}}, \dots, \frac{X_T}{\sqrt{\lambda_1^T}} \quad (3.125)$$

from the metric

$$\zeta = \begin{pmatrix} Q_1 & 0 & \dots & 0 \\ 0 & Q_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_T \end{pmatrix} \quad (3.126)$$

From this weighted PCA it is possible to obtain the Euclidean image of the consensus, which summarizes the information contained in T tables in just one. By performing a weighted PCA on the \mathcal{N}_I^* cloud, the Euclidean image is identical, since the \mathcal{N}_I and \mathcal{N}_I^* clouds are homothetic. The general expression of the consensus is:

$$W = \chi \zeta \chi' = \sum_{t=1}^T \frac{W_t}{\lambda_1^t} \quad (3.127)$$

To interpret the consensus positions, it is necessary to previously determine the correlations of the variables with the axes of the consensus, as in the STATIS method.

3.4.3 Simultaneous representation of the T clouds

To perform the simultaneous representation of the T clouds so as to construct individuals'

trajectories it is necessary to project them in the subspace of $\mathbb{R}^{\sum_{t=1}^T p_t} = \bigoplus_{t=1}^T \mathbb{R}^{p_t}$,

allowing simultaneous comparison of the same subject in different $\mathcal{N}_I^1, \dots, \mathcal{N}_I^T$ clouds. For this to happen, the choice of this subspace must meet two essential conditions:

1. Every cloud \mathcal{N}_I^t ($t = 1, \dots, T$) should be well represented;
2. The representations of clouds \mathcal{N}_I^t ($t = 1, \dots, T$) should be similar.

With regard to condition 1, an orthogonal projection should be done of the respective clouds in the subspace, whose quality of representation is measured by the sum of the inertia of the projected clouds. The aim is to maximize inertia of union for the $\mathcal{N}_I^t: \bigcup_{t=1}^T \mathcal{N}_I^t$ clouds. Be x^* the center of gravity $\bigcup_{t=1}^T \mathcal{N}_I^t$ such that:

$$x^* = \frac{1}{n \cdot T} \sum_{i=1}^n \sum_{t=1}^T (x_i)^t \quad (3.128)$$

It is not possible to compare the position of an individual in the various clouds if its representations on them are very different. To guarantee condition 2 is necessary that the points representing the same subject in the various tables, $(x_i)^1, \dots, (x_i)^T$, are close to each other. In other words, it is necessary to minimize the inertia of the \mathcal{N}_I^T clouds around the respective x^* centers of gravity.

Conditions 1 and 2 are incompatible, since the quality of representation of clouds and the similarity between these representations cannot be optimized simultaneously.

According to the Huygens theorem, the total inertia of $\bigcup_{t=1}^T \mathcal{N}_I^t$ decomposes in the sum of \mathcal{N}_I^* Cloud inertia (inter inertia) with the inertia of \mathcal{N}_I^T clouds (intra inertia). The Huygens theorem suggests minimizing intra inertia and maximizing the total inertia, making the inter inertia maximum.

The subspace of $\mathbb{R}^{\sum_{t=1}^T p_t}$ on which the projection $\bigcup_{t=1}^T \mathcal{N}_I^t$ has the maximum inter inertia is formed by the first axes of inertia from \mathcal{N}_I^* cloud or from \mathcal{N}_I cloud, since these clouds are homothetic. Thus, the sought subspace will be obtained by performing a PCA to the χ table. The coordinates of the \mathcal{N}_I^T points are contained in \tilde{X}_t table thus considering each \tilde{X}_t as a supplementary element, the simultaneous representation is obtained of \mathcal{N}_I^T clouds,

and hence the trajectories of individuals are found in a manner similar to that used in the STATIS method.

3.4.4 Representation of variables

The MFA in $\mathbb{R}^{\sum_{t=1}^T p_t}$ allowed visualizing the average positions of each individual and the respective trajectories on the consensus plan. The simultaneous representation of the T groups of variables in \mathbb{R}^n space is obtained through an PCA to the χ table and allows the interpretation of the cloud representation of individuals and of the correlation between these variables. This representation is not more than the dual of the \mathcal{N}_I representation in $\mathbb{R}^{\sum_{t=1}^T p_t}$.

The projected inertia of each \mathcal{N}_j^t cloud formed by the group of X_t table variables can be interpreted as the contribution of this group to the construction of the axes. The weighting of these groups per $1/\lambda_1^t$ balances their influence in the sense that the contribution of each group to the construction of an axis is, at most, 1.

The comparison of groups of variables can be done through a systematic study of the correlations between the first principal components of each group. This study is based on a non-normated PCA of the principal components of all groups. Since the principal components of X_t table are projections of individuals on the orthonormal basis, the clouds of individuals defined by the PCA of the X_t table and by PCA of the table defined by the principal components of X_t , are identical.

To make a comparison of the principal components of the various groups, simply consider them as additional elements in the analysis of χ complete picture. Another possible alternative would be to make an PCA of the principal components, taking the variables as additional elements. By keeping only the first components of each group, thus having the advantage of reducing the tables dimension, as well as the duration of the calculations, similar results would be obtained in which the quality of the approximation depends on the maximum inertia of the principal component abandoned.

3.4.5 The interstructure

3.4.5.1 Representation of groups of variables

This step consists of the global comparison of the T groups of variables, through their representation on a plane. Be \mathcal{N}_j the cloud of groups of variables defined by X_1, \dots, X_T tables. In STATIS method, it was defined an object representative of each study denoted by $W_t = X_t Q X_t'$ and the used scalar product was the Hilbert-Schmidt product:

$$\langle W_t, W_{t'} \rangle_{HS} = tr (W_t D W_{t'} D) \quad (3.129)$$

wherein:

$$W_t = \sum_{j=1}^{p_t} q_j x^j (x^j)' \quad (3.130)$$

Instead of denoting by $(x^j)^t$ the x^j was adopted to simplify and not to burden the used notation.

Each W_t is of dimension $n \times n$, thus this object belongs to the dimension n^2 vector space noted by \mathbb{R}^{n^2} .

In MFA, the objects to be used will be weighted, in the W_t / λ_1^t form. The weighting by $1 / \lambda_1^t$ translates into \mathbb{R}^{n^2} by a homothety of the representative vector of this group. Hence the \mathcal{N}_j cloud greater resemblance to the cloud of the variables than to the cloud of individuals.

The square of the norm of the representative objects of each group of X_T variables in \mathbb{R}^{n^2} is:

$$\begin{aligned}
\eta_g^2(X_t) &= \left\| \frac{W_t}{\lambda_1^t} \right\|_{HS}^2 \\
&= \left\langle \frac{W_t}{\lambda_1^t}, \frac{W_t}{\lambda_1^t} \right\rangle_{HS} \\
&= \text{tr} \left(\frac{W_t}{\lambda_1^t} D \right)^2 \\
&= \sum_{i=1}^n \frac{(\lambda_1^t)^2}{(\lambda_1^t)^2}
\end{aligned} \tag{3.131}$$

With λ_1^t i-th eigenvalue of $W_t D$. This measure is always greater than or equal to 1, increasing likewise with the number of eigenvalues of comparable importance to the first eigenvalue, therefore depending on the structure of each group of variables. One can then say that $\eta_g^2(X_t)$ is a dimensionality index of the X_T group of variables.

3.4.5.2 Interpretation of the scalar product between two groups

The scalar product between two groups can be interpreted as a bond measure between them.

1. Each of the two representative objects of each group comprises a single variable.

Let u and v be to two centered and reduced variables, each constituting of a group, affected of the respective weighting coefficients. The \mathbb{R}^{n^2} elements associated with these two groups have norm 1 and the scalar product between them is the square of the correlation coefficient between u and v variables:

$$\begin{aligned}
\langle uu', vv' \rangle_{HS} &= \text{tr} (uu' D vv' D) \\
&= \sum_{i=1}^n \sum_{l=1}^n p_i p_l u_i u_l v_i v_l
\end{aligned} \tag{3.132}$$

$$\begin{aligned}
 &= \sum_{i=1}^n p_i u_i v_i \sum_{l=1}^n p_l u_l v_l \\
 &= \left(\sum_{i=1}^n p_i u_i v_i \right)^2 \\
 &= [r(u, v)]^2
 \end{aligned}$$

= inertia of the v projection over u

2. One of the groups comprises one variable and the other comprises multiple variables.

Let u be a centered and reduced variable of the X_t group and x^j the set of variables, centered and reduced to q_j weight of X_t group with $j = 1, \dots, p_t$. We have that:

$$\begin{aligned}
 \langle W_t, W_t \rangle_{HS} &= \langle uu', \sum_{j=1}^{p_t} q_j x^j (x^j)' \rangle_{HS} \\
 &= \sum_{j=1}^{p_t} q_j \langle uu', x^j (x^j)' \rangle_{HS} \\
 &= \sum_{j=1}^{p_t} q_j [r(u, x^j)]^2
 \end{aligned} \tag{3.133}$$

$$= \sum_{j=1}^{p_t} \textit{inertia of the projection of } x^j \textit{ over } u$$

which is a bond measure between u and the X_t group of variable denoted by $\zeta_g(u, X_t)$ which was introduced by Carroll (1968) when approaching the Generalized Canonical Analysis.

3. The two groups comprise several variables.

Let x^j be the set of variables, centered and reduced to q_j weight of the X_t $j = 1, \dots, p_t$ group and x^k the set of variables, centered and reduced to q_k weight of the $X_{t'}$ ($k = 1, \dots, p_{t'}$) group. The scalar product between the representative objects of X_t and $X_{t'}$ is:

$$\begin{aligned} \langle W_{t'}, W_t \rangle_{HS} &= \sum_{j=1}^{p_t} q_j \sum_{k=1}^{p_{t'}} q_k \langle x^j (x^j)', x^k (x^k)' \rangle_{HS} \\ &= \sum_{j=1}^{p_t} q_j \zeta_g(x^j, X_t) \\ &= \sum_{k=1}^{p_{t'}} q_k \zeta_g(x^k, X_t) \end{aligned} \quad (3.134)$$

This measurement can be made explicit in function of two groups of variables (affected the respective weightings) as:

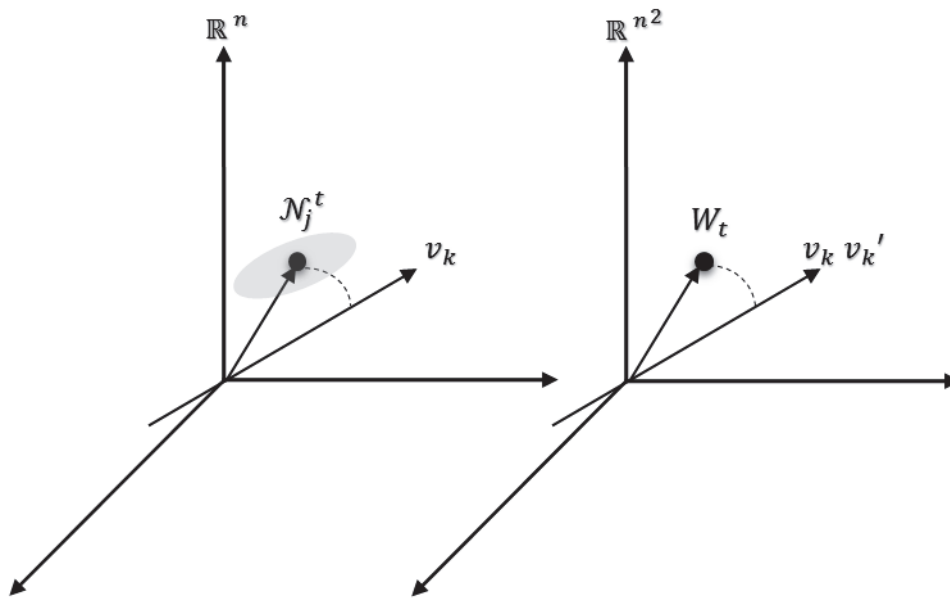
$$\begin{aligned} \zeta_g(X_t, X_{t'}) &= \left\langle \frac{W_t}{\lambda_1^t}, \frac{W_{t'}}{\lambda_1^{t'}} \right\rangle_{HS} \\ &= \sum_{j=1}^{p_t} q_j \zeta_g(x^j, X_{t'}) \\ &= \sum_{j=1}^{p_t} q_j [r(x^j, x^k)]^2 \end{aligned} \quad (3.135)$$

This term is a measure of binding between the two groups of variables, and it increases with the strengthening of the connection of each of the variables of the group to the other and the larger the number of common directions of maximum inertia directions of each group. It should be noted that the group formed by the centered and reduced variable defined by the v_k axis of \mathbb{R}^n is associated with the variable's representative object in \mathbb{R}^{n^2} :

$$W_{v_k} = v_k v_k' \tag{3.136}$$

These objects must be of order 1 in the sense that each of them is associated with a single direction of \mathbb{R}^n . The projected inertia of X_t group variables over v_k is no more than the scalar product (Hilbert-Schmidt) between W_t and $v_k v_k'$, i.e. the projection W_t over $v_k v_k'$ (Figure 3.17). So if two vectors are D-orthogonal in \mathbb{R}^n then the associated vectors in \mathbb{R}^{n^2} are also D-orthogonal. Thus, the graphical representation of the groups of variables can be interpreted as a projection of the cloud \mathcal{N}_j over a set of orthogonal axes. In order to make the overall comparison of groups, the proximities of the projections of W_t objects in a r -dimensional space, well inferior to the one of \mathbb{R}^{n^2} , should be analyzed.

Figure 3.17 – Representation of variable groups in \mathbb{R}^n and \mathbb{R}^{n^2}



Source: adapted from Carvalhido (2005)

The angles between objects should be well represented and it is not convenient to center the \mathcal{N}_j cloud. The projection of this cloud on the main axis is analogous to the representation of the cloud of variables in the PCA. The only drawback of this type of analysis will be to provide a frame comprising axes which is difficult to interpret since each of these axes is not explicitly expressed in function of the data. It is for this reason that it is imposed that the elements $v_k v_k'$ ($k = 1, \dots, r$) are D-symmetrical of order 1.

These elements are associated only with v_k and interpreted from the correlations of this vector with the initial variables.

In the PCA, the criterion of least squares was used, according to which it maximized the sum of squares of the projections of the vectors of the clouds. Because of the type of axis used in MFA (D-symmetrical elements of order 1), the maximized amount is in terms of the sum of the projections. As the coordinates of W_t objects over the elements of type $v_k v'_k$ are all positive, the sum of the projections of W_t over $v_k v'_k$ is:

$$\sum_{t=1}^T \langle W_t, v_k v'_k \rangle_{HS} \tag{3.137}$$

and is the inertia of the variables (of all groups) projected on v_k . The orthonormal group of D-symmetrical elements of order 1 that maximize this sum is associated with the principal components of the χ frame. Therefore, the calculations necessary for analysis in \mathbb{R}^{n^2} are deducted directly of the PCA results applied to χ . Vectors v_k are the normed principal components of χ and the coordinate W_t over $v_k v'_k$ is the X_t group inertia contribution of the principal component.

The representation of groups of variables can be seen as an additional aid in the interpretation of the other graphics. The coordinate of the W_t object (weighted) over the $v_k v'_k$ axis ranges between 0 and 1 and can be interpreted as:

- The inertia of the cloud projection of \mathcal{N}_j^t in \mathbb{R}^n over the principal component of v_k of the χ table (Figure 3.17);
- A binding measure between the principal component v_k and the X_t table variables, designated by $\zeta_g(v_k, X_t)$;
- The projection of X_t group in \mathbb{R}^{n^2} space.

When $\zeta_g(v_k, X_t) = 1$ the v_k is the first principal component of X_t . On the other hand, if $\zeta_g(v_k, X_t) = 0$, the variable v_k is not correlated with any X_t variables and is it is close to 0 it indicates a low inertia direction for \mathcal{N}_j^t , but that may raise some ambiguity. Then the

correlation coefficient should be calculated between the principal components associated and the χ complete table and with the X_t table:

$$F_{u_k} = \chi \varsigma_{u_k} = \sqrt{\lambda_k} u_k \quad (3.138)$$

and

$$F_{u_k}^t = \tilde{X}_t \varsigma_{u_k} \quad (3.139)$$

with $u_k = \frac{1}{\lambda_k} \chi' D F_{u_k}^t$, respectively. These components are connected by the following relationship:

$$\begin{aligned} F_{u_k}^t &= \frac{1}{\lambda_k} \tilde{X}_t \varsigma \chi' D F_{u_k} \\ &= \frac{1}{\lambda_k} W_t D F_{u_k} \\ &= \frac{1}{\lambda_k} W_t D v_k \end{aligned} \quad (3.140)$$

with λ_k eigenvalue of WD of order k , where $W = \sum_{t=1}^T W_t$. The correlation coefficient $r(F_{u_k}, F_{u_k}^t)$ is an indicator of the presence of F_{u_k} in the X_t group.

Then we have that the consensus image of the i -th individual is the centroid of the its projections in the \mathcal{N}_i^t clouds:

$$F_{u_k}(i) = \frac{1}{T} \sum_{t=1}^T F_{u_k}^t(i) \quad (3.141)$$

The quality of representation of the W_t objects can be evaluated by the ratio of the projected inertia and the total inertia, which is lower and rarely equal to 1, even with an increase of the number of axes to n . This is due to the way that the axes (D-symmetrical and order 1) were defined.

3.5 TUCKER3

TUCKER3 was initially studied by Tucker (1966) and allows the analysis of a population of individuals characterized by several groups of quantitative or qualitative variables.

In a manner analogous to that described for the STATIS and MFA methods, there are K data tables, X_1, \dots, X_k collected at different occasions or situations on the same individuals. The steps that constitute the TUCKER3 differ from those proposed by the STATIS and MFA methods and generally are:

1. Unfolding of the original data array;
2. Data preprocessing;
3. Estimation of model parameters;
4. Analysis of model fit;
5. Representation and interpretation of the solution.

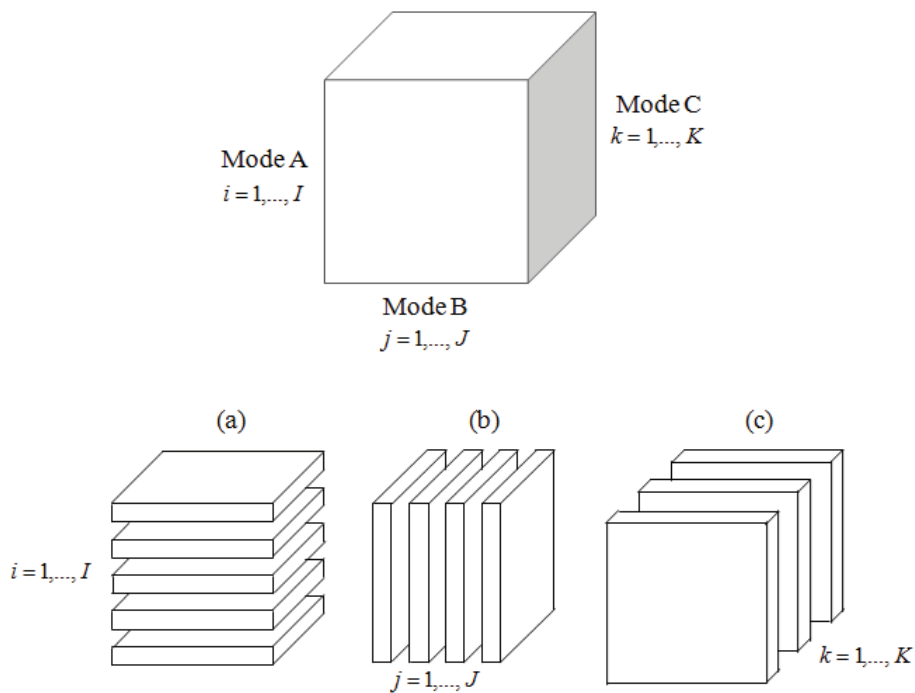
3.5.1 Unfolding the original data array

The unfolding procedure starts from a three-way data array, where the I index indicates the number of subjects, the J index represents the variables, and the K index represents the occasions or situations on which the measurements were taken. The high dimensionality of the modes requires the use of a procedure called unfolding, which is based on the transformation or rearrangement of three-mode data in matrices. In the case of data cubes, there are three possible unfoldings, in three orthonormal matrices: an A matrix of dimension $I \times P$ containing the coefficients for the individual's components, a B matrix of dimension $J \times Q$ containing the coefficients for the variable's components, a C matrix of dimension $K \times R$ containing the coefficients for the components of the occasions

mode, and a G central core of dimension P×Q×R. The main objective is to estimate the model parameters A, B, C, and G.

In Figure 3.18 a three-way array is depicted. The entities along the vertical axis are indicated by the first index (*i*), those along the horizontal axis by the second index (*j*) and those along the depth axis by the third index (*k*).

Figure 3.18 – Three-way array, cut into (a) horizontal, (b) lateral, and (c) frontal slices



Source: own elaboration

The TUCKER3 method is a generalization of the two-way component model to the three-way component model. The generalization can be represented as:

$$x_{ijk} = \hat{x}_{ijk} + e_{ijk}; \quad i = 1, \dots, I; j = 1, \dots, J; k = 1, \dots, K \quad (3.142)$$

In its general expression, taking summations, the TUCKER3 model is represented as:

$$x_{ijk} = \sum_{p=1}^P \sum_{q=1}^Q \sum_{r=1}^R g_{pqr} a_{ip} b_{jq} c_{kr} + e_{ijk}; i = 1, \dots, I; j = 1, \dots, J; k = 1, \dots, K \quad (3.143)$$

where a_{ip} , b_{jq} and c_{kr} denote elements of the component matrices A, B and C of orders $I \times P$, $J \times Q$ and $K \times R$ respectively, g_{pqr} denotes the \underline{G} center array elements, g_{pqr} denotes the element (p, q, r) of the $P \times Q \times R$ core array \underline{G} , and e_{ijk} denotes the error term for element x_{ijk} and is an element of the $I \times J \times K$ array \underline{E} .

To convert the TUCKER3 model to matrix notation it is important to clarify the concept of the Kronecker product. Here a_{ij} or $a_{i,j}$ is used to denote element (i, j) of a matrix A, and a_l denotes the l th column of A and a prime denotes transposition of a matrix or vector (Kiers, 2000). The Kronecker product is denoted by the symbol \otimes and is defined according to $(U \otimes V)_{ik,jl} = u_{ij} v_{kl}$. On a matrix representation we have:

$$U \otimes V = \begin{pmatrix} u_{11}V & \cdots & u_{1J}V \\ \vdots & \cdots & \vdots \\ u_{I1}V & \cdots & u_{IJ}V \end{pmatrix} \quad (3.144)$$

The matrix notation is expressed as:

$$X_a = A G_a \cdot (C' \otimes B') + E_a \quad (3.145)$$

where X_a is a matrix of dimension $I \times JK$ obtained by unfolding the A mode under \underline{X} , G_a is a matrix of dimension $P \times QR$ obtained by unfolding by the central core of the mode A and E_a is the matrix of residuals of dimension $I \times JK$ (Tucker, 1966).

Similarly, using the unfolding of mode B, the model has the form:

$$x_{ijk} = \sum_{q=1}^Q b_{jq} \sum_{p=1}^P \sum_{r=1}^R g_{pr} (a_{ip} c_{kr}) + e_{ijk} \quad (3.146)$$

and in matrix notation:

$$X_b = BG_B \cdot (C' \otimes A') + E_b \quad (3.147)$$

where X_b is a matrix of dimension $J \times IK$ obtained by unfolding of the mode B of the central core array \underline{X} , G_b is a matrix of dimension $Q \times PR$ obtained by unfolding by the B mode of the central core and E_b is the matrix of residuals of dimension $J \times IK$.

Using unfolding of the C mode, the model is:

$$x_{ijk} = \sum_{r=1}^R c_{kr} \sum_{p=1}^P \sum_{q=1}^Q g_{pq} (a_{ip} c_{jq}) + e_{ijk} \quad (3.148)$$

and in matrix notation:

$$X_c = CG_c \cdot (B' \otimes A') + E_c \quad (3.149)$$

where X_c is a matrix of dimension $K \times IJ$ obtained by unfolding the C mode under X , G_c is a matrix of dimension $R \times PQ$ obtained by unfolding by the C mode of the central arrangement and E_c is the matrix of residuals of dimension $K \times IJ$.

The TUCKER3 model is a higher-order singular value decomposition (HSVD), which in order to construct the structural image of \underline{X} , needs $P \times Q \times R$ terms of the form $(a_p \otimes b_q \otimes c_r)$, each one weighted by its own g_{pqr} . Using the notation of rank one, TUCKER3 model has the form:

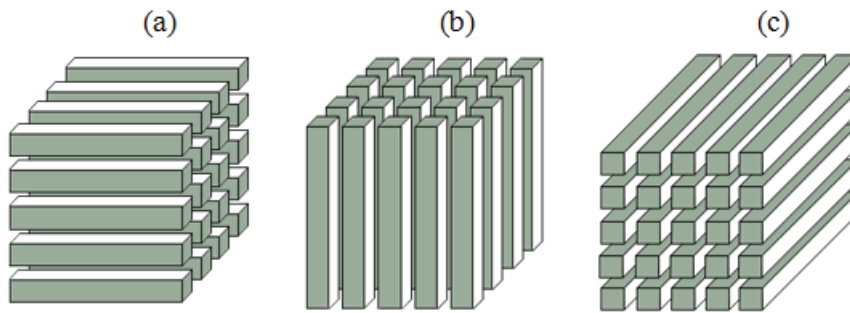
$$\underline{X} = \sum_{p=1}^P \sum_{q=1}^Q \sum_{r=1}^R g_{pqr} (a_p \otimes b_q \otimes c_r) + E \quad (3.150)$$

where a_p , b_q and c_r are the left singular vectors obtained from the singular value decomposition.

The term $(a_p b_q c_r)$ is called arrangement of range one and g_{pqr} range corresponds to the weights for the combinations of components (Kroonenberg, 2008).

According to Kiers (2000), a three-way array is usually considered in terms of a set of matrices and these matrices form the horizontal, lateral and frontal slices of the three-way array, as visualized in Figure 3.18. Specifically, the I horizontal slices pertain to the entities $i = 1, \dots, I$ of mode A, the J lateral slices pertain to the entities $j = 1, \dots, J$ of mode B and the K frontal slices pertain to the entities $k = 1, \dots, K$ of mode C. If we consider a three-way array as a set of vectors, then three sets of such vectors (*fibers*) can be distinguished as vertical, horizontal and depth fibers, which run over the mode A entities, mode B entities and mode C respectively, as visualized in Figure 3.19. For a three-way array the mode A space is the subspace spanned by all vertical fibers, the mode B space is the subspace spanned by all horizontal fibers and the mode C space is the subspace spanned by all depth fibers (Figure 3.19). These spaces are subspaces of \mathbb{R}^I , \mathbb{R}^J and \mathbb{R}^K respectively (Kiers, 2000).

Figure 3.19 – Three-way array, cut into (a) horizontal, (b) vertical and (c) depth fibers



Source: adapted from Kiers (2000)

3.5.2 Data preprocessing

Before performing the multiway analysis, it is often necessary to preprocess the data. For three-way data one may similarly wish to eliminate unwanted differences by column centering and normalization in a similar fashion as for the two-way data. Nonetheless in this three-way standardization it is important to appropriately select how each of the

modes should be treated. Centering through the subjects and scaling between variables are the most common procedures, which when applied together provide the standardization of data (Amaya, 2002). The centering is done through the columns of the matrix, while the scaling applies to complete slices or submatrices of the array. According to Harshman and Lundy (1984) and Ten Berge (1989) the type of centering is indicated by specifying across which modes the data are centered. The meaning of centering across mode A is to first average the data over the entities of mode A and then to subtract each thus obtained average from all data elements that contribute in it. In the same sense, to center across the combination of modes A and B implies first averaging the data over the entities of modes A and B and then subtracting each such average from the data that contribute in it. This centering across A, can be represented as:

$$\tilde{x}_{ijk} = x_{ijk} - \bar{x}_{.jk} \quad (3.151)$$

where the subscript dot is used to indicate the mean across $i = 1, \dots, I$. In the same context centering across the combination of modes A and B can be represented as:

$$\tilde{x}_{ijk} = x_{ijk} - \bar{x}_{.j..k} \quad (3.152)$$

The type normalization used is identified by specifying the entities within which the data are normalized. Therefore, normalization within mode A consists of, per entity, summing of the squares of all data elements associated with this particular entity and then dividing all these elements by the square root of this sum of squares. This normalization factor can be expressed by:

$$v_i = \sqrt{\sum_{j=1}^J \sum_{k=1}^K x_{ijk}^2} \quad (3.153)$$

and for the normalized data by:

$$\tilde{x}_{ijk} = \frac{x_{ijk}}{v_i} \quad (3.154)$$

Normalization within the combination of modes A and B consists of, for each combination of an entity of mode A and one of mode B, first computing the sum of squares over all other modes and then dividing all associated elements by the square root of this sum of squares.

Using across for centering and within for normalization is because in computing the centered data the subtracted average is best recognized by the modes across which the average is computed, whereas in normalization the scaling factor usually carries the subscript for the entities of the modes within which the normalization takes place (Kiers, 2000). The type of preprocessing to be used is dependent on the data and methodological approach adopted in the investigation.

3.5.3 Estimation of model parameters

Before the analysis, it is essential to determine the number of components through which the entities of each of the modes array will be summarized. Tucker (1966) proposed an algebraic method to estimate the TUCKER3 model parameters, corresponding to the A, B, and C matrixes and to the central core G. A process to determine the optimal dimensionality is to establish the percentages of model fit for each possible combination of the number of components in the three modes of the array. Then, the best combination of number of components is set to be the one which presents little variation in model fit when considering an additional component in any of the modes (Bro, 1998). The algorithm is as follows:

1. Initialize B and C
2. $[A, S, V] = svd(X_a(C \otimes B), P)$
3. $[B, S, V] = svd(X_b(C \otimes A), Q)$
4. $[C, S, V] = svd(X_c(B \otimes A), R)$
5. Go to step 1 until convergence
6. $G_a = A^T X_a (C \otimes B)$

In general, the expression $[U, S, V] = svd(X, P)$ means that a reduced SVD is performed on X retaining the first component P. The X matrix is decomposed as $X = USVT$, where the matrix U contains the first P left singular vectors, V the first P right singular vectors,

and where S is a diagonal matrix whose diagonal elements correspond to the square root of the first P values singular (Johnson and Wichern, 1998).

The estimate of the component matrix B is obtained by selecting the Q first left singular vectors of the SVD of X_b , and the initial estimate of the component matrix C corresponds to the R left singular vectors of the first SVD of X_c . Based on these initial estimates, the iterative procedure begins by applying SVD on X_a (CB), X_b (CA) and X_c (BA) matrix arrangements to obtain the estimates of the three component matrices by selecting respectively P , Q and R left singular vectors of the decomposition (Amaya, 2002). Additionally, based on the three components estimated matrices, the central arrangement in some form of unfolding, for example, deployment by mode A as shown in Step 5 of the algorithm (Bro, 1998) is obtained.

The Tucker algorithm requires the initialization mechanism of the components B and C matrices to obtain an estimate of the matrix A . The matrices B and C correspond to the initial left singular vectors of the singular value decomposition of the arrangement by unfolding of mode B ($X_{(J \times IK)}$) and the C mode ($X_{K \times IJ}$), retaining the first Q and left singular vectors R , respectively.

The number of singular vectors retained for P , Q and R are selected considering the minimum product solution rule (Kroonenberg, 2008), expressed as:

$$P \times Q \geq R, P \times R \geq Q, Q \times R \geq P \quad (3.155)$$

3.5.4 Analysis of model fit

To find the proper model and its dimensionality, there are different methods of selection. These methods are complementary and the researchers' decision is based on the conclusions drawn from them all.

3.5.4.1 Timmerman-Kiers DiffFit procedure

When selecting the dimensionality for Tucker models it is necessary to take in account that not all combinations of dimensions are feasible. For the Tucker3 model the minimum-product rule (Kroonenberg, 2008) as in eq. (3.155). This is the reason that in each step of the Tucker algorithm, the matrix for which the eigenvalues are calculated has the rank of the product of the numbers of components of the other modes and the algorithm will not work if that rank is lower than that of the current mode.

For TUCKER3 the core array is three-way orthogonal, and for that condition to be met the products of the component numbers must be larger or equal to the number of components in the remaining mode. In this sense, Timmerman and Kiers (2000), presented a dimensionality selection procedure that within the a class of TUCKER3 models with the same total number of components $S=P + Q + R$, the model to select is the one that has the highest proportion fitted sum of squares, V_S , or equivalently the smallest residual sum of squares or deviance. For comparing classes of different S , they compute $dif_S = V_S - V_{S-1}$ and only the dif_S that are sequentially highest are considered. Timmerman and Kiers also defined the *salience value*: $b_S = dif_S / dif_{S^*}$, where dif_{S^*} as the next highest value after dif_S . The model to select is the on for which b_S has the highest value. Timmerman and Kiers (2000) also defined a lower bound for dif_S below which models should not be considered. The dif_S should be greater than the average proportion explained variability taken over all feasible values $S(S_{min} = \min(I, JK) + \min(J, IK) + \min(K, IJ) - 3)$.

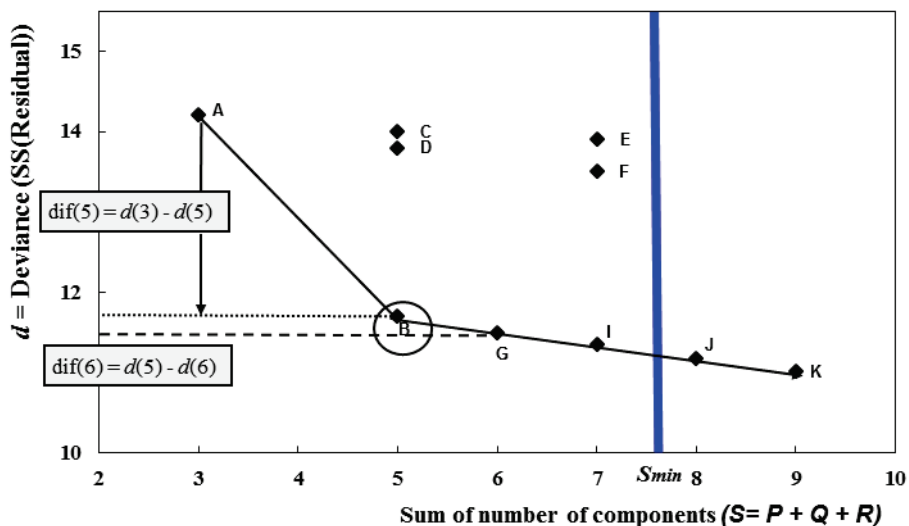
To visualize the procedure proposed by Timmerman and Kiers (2000), it is used a version of Cattell's scree plot, in which the residual sums of squares for each set of dimensionalities is plotted versus the sum of numbers of components S (Figure 3.20). The

DiffFit procedure defines the convex hull in the multiway scree plot, and models considered are those on the convex hull. Kroonenberg (2008) presented an example according to Fig. 3.20 with the models A, B, G, and K. Models I and J “*just miss the mark*”, but it is still possible to consider such models given the presence of random error. The lower bound, S_{min} has been represented in the multiway scree plot as a vertical line and all models to the right of that line do not comply with the minimum value. In the multiway scree plot it is possible to identify that:

- for $S = 5$ components: models C and D are much worse than B;
- for $S = 7$ components: models E and F are much worse than I;
- for of $S = 8$ and $S = 9$ all models fail to make the cut.

The increase of the total number of components from 3 to 5 gives an effective reduction in the residual sum of squares. It is not relevant to integrate additional components for the decrease of the residual is much slower. Therefore, the DiffFit criterion points to model B as having the correct dimensionalities.

Figure 3.20 – Multiway scree plot: Deviance versus sum of components

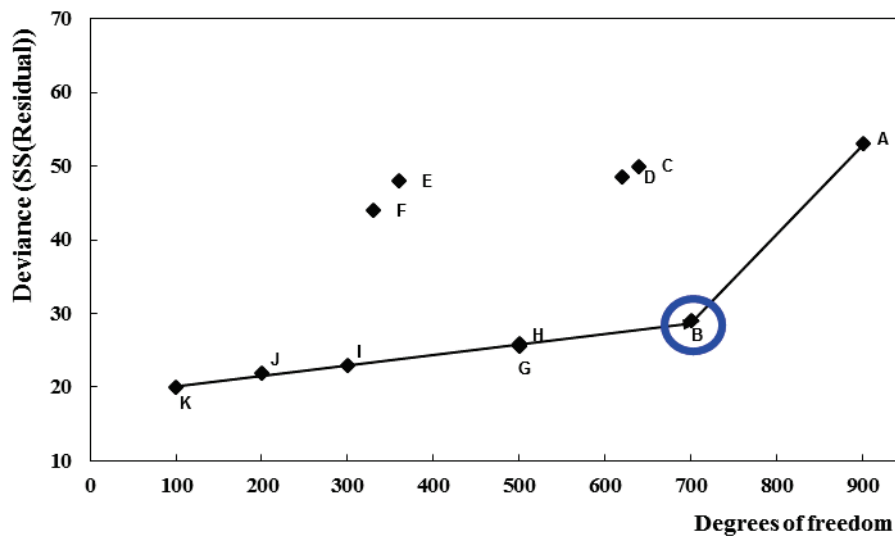


Source: Adapted from Kroonenberg, 2008

3.5.4.2 Deviance analysis

Kroonenberg and Van der Voort (1987) proposed the evaluation of the residual sum of squares or deviance together with the degrees of freedom. For its representation a deviance plot is constructed with the residual sum of squares d of each model plotted versus the degrees of freedom df . Kroonenberg (2008) presented an example according to Fig. 3.21, where a convex hull can be drawn to connect favored models; here A, B, G, and K. Models C, D, E, F within the hull are disfavored compared to the hull models E and F compared to I, who have similar or better fit and equal df , models C and D compared to A of equal fit with more df , or a combination of both. Nonetheless, the discarding of models H, I, and J close to the convex hull may not be sure procedure.

Figure 3.21 – Deviance plot: Deviance versus degrees of freedom



Source: Adapted from Kroonenberg, 2008

In deviance plots, model evaluation is facilitated by auxiliary lines between the loci of models with constant d/df . If the origin is included in such a plot, these lines can be seen to fan out from the origin with a slope of k . If $k = 1$, then each parameter added or degree of freedom lost leads to a gain of one unit of deviance. If two models on the convex hull are connected by a line having a steep slope (A and B), then, by adding extra parameters at the expense of degrees of freedom, it is possible to obtain a large decrease in deviance or a considerably better fitting model (Kroonenberg, 2008). This happens at the right-hand side of the plot. If the slope is very flat (from B to G to K), it is necessary to sacrifice many degrees of freedom or add many parameters to get a small decrease in deviance or an increase in fit (Kroonenberg, 2008). The principle of parsimony, which can be expressed here as a preference for models with good fit and few parameters or many degrees of freedom, suggests that models in bottom right of Figure 3.21 of the plot are the first to be considered for selection.

3.5.4.3 Ceulemans-Kiers st-criterion or numerical convex-hull

Ceulemans and Kiers (2006) proposed the numerical convex-hull to select models in deviance plots, where the selection criterion is the st-criterion, based on the deviance and degrees of freedom rather than sum of squares fit and number of free parameters and is defined as:

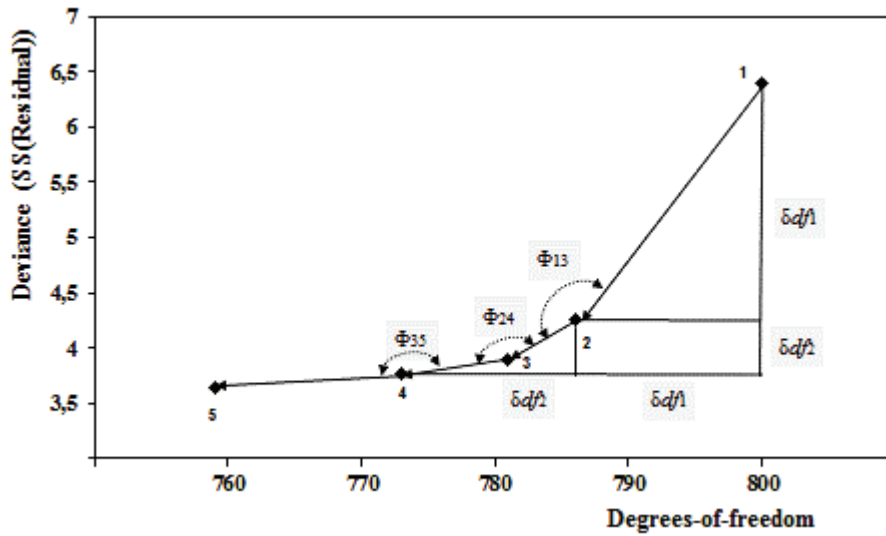
$$st_i = \frac{d_{i-1} - d_i}{df_{i-1} - df_i} / \frac{d_i - d_{i+1}}{df_i - df_{i+1}} \quad (3.150)$$

where d_i , is the deviance and df_i the degrees of freedom of model i . The heuristic is formulated as follows (Ceulemans and Kiers, 2006):

1. Determine the df and d values of all component solutions from which one wishes to choose.
2. For each of the n observed df values, retain only the best-fitting solution.
3. Sort the n solutions by their df values and denote them by $s_i = (i = 1, \dots, n)$.
4. Exclude all solutions s_i for which a solution s_j ($j < i$) exists such that $d_j < d_i$.
5. Consecutively consider all triplets of adjacent solutions. Exclude the middle solution if its point is above or on the line connecting its neighbors in the deviance plot.
6. Repeat Step 5 until no solution can be excluded.
7. Determine the st values of the convex hull solutions obtained.
8. Select the solution with the smallest st value.

The steps 1-6 determine which models lie on the convex hull, and steps 7 and 8 find smallest angle $\phi_{i-1,i+1}$ between the lines connecting a model i with its previous $i - 1$ and subsequent $i + 1$ models, as is shown in Figure 3.22 of Kroonenbergs' (2008) example. In this figure, the hull shows two places where the convex hull has a sharp angle with $\phi_{13} < \phi_{35}$ and a near 180 degree angle for ϕ_{24} . In such a case, it might be advantageous in practice to select the less parsimonious Model 4, which provides more detailed information about the data. Model 3 is not to be preferred because its angle is larger than both Model 2 and Model 4.

Figure 3.22 – Celeumans-Kiers heuristic for selecting a model in the deviance plot



Source: Adapted from Kroonenberg, 2008

3.5.5 Representation and interpretation of the solution

For three-ways arrays, there are seven possible graphic representations, three of which correspond to the planes that display the configuration of entities in a particular way in relation to the entities of the other two modes. The other three are identified as the entities' configurations of two particular modes by trajectories in relation to the entities of the remaining mode, and the latest version shows the more reduced data generated based on the values of the central array. For example, for obtaining configurations of the entities of the B mode and project on the same plane the entities of modes A and C, $\hat{X}_b = B((C \otimes A)G_b^T)^T$ is obtained, an orthonormal basis is found for the orthogonal matrix column $F \equiv B(C \otimes A)G_b^T$ by dividing each of its columns by the square roots of the singular values obtained from the decomposition, and this scaling is compensated in B by multiplying its columns by the same values. The thus calculated matrices, \tilde{B} and \tilde{F} , are used respectively to obtain the plane of the variables J and the projections of the original IK axes. Moreover, for the configuration of the modes of A and C entities with the projection of the B mode entities, we use: 1) the rows of the orthogonal matrix F

connecting the points belonging to the same individual in consecutive occasions, and 2) the B orthonormal matrix. Finally, the values of the \underline{G} center array are plotted by drawing trajectories through the occasions for each combination individual-variable (Kiers, 2000).

3.6 Comparison of methods

The STATIS, MFA, and TUCKER3 methodologies are applicable in quantitative, qualitative, and mixed data. In all methods the tables (matrices) cross individuals (rows) and variables (columns). In STATIS and MFA methods, the invariant individuals are crossed with variables that may differ over the tables. In the TUCKER3 method, the invariant individuals are combined with the variables and conditions depending on the unfolding of each mode A, B, and C.

The STATIS method provides a global visualization of the similarities between the conditions under analysis while the MFA enables a particular visualization of these similarities. On the other hand, the axes of the STATIS method interstructure are difficult to interpret, such that it is not possible to interpret the proximity between the groups on a given axis on the basis of initial data.

The normalization proposed by the STATIS method is such that the Hilbert-Schmidt norm of the representative objects is equal to 1. This normalization has the inconvenience of making the structural differences between the different conditions fade. On the other hand, the normalization proposed by MFA does not change the multiple structure of the various conditions, as the total inertia of each cloud of points does not intervene in this normalization and has the advantage of making the inertia of first component of each group equal to 1, while the remaining will be leveled by 1. It is through this weighing that no condition can influence preponderantly the first axis of the Euclidean image of consensus. It is for this reason that this image spaces are denominated as consensus for the MFA and compromise to the STATIS.

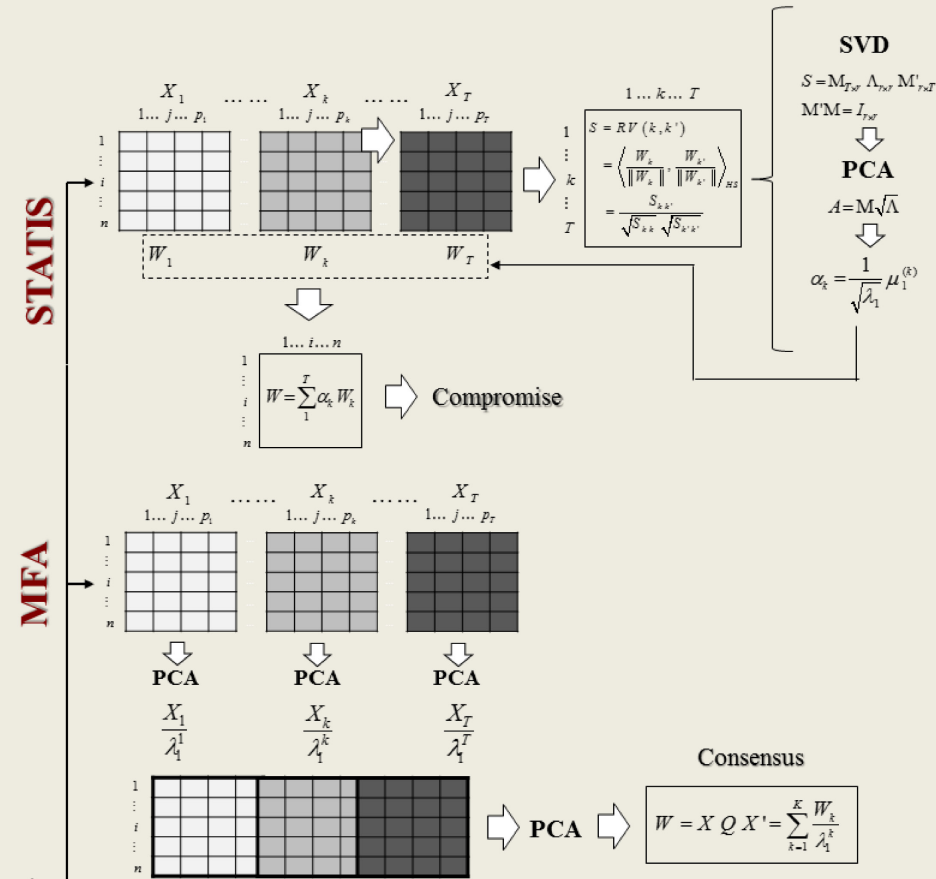
In turn, the TUCKER3 model estimates the central arrangement (G) to establish relations between the components extracted by the three modes and to identify three-dimensional interactions.

Both STATIS and MFA provide measurement of the relation between groups while TUCKER3 enables the interpretation of the interactions between components, thus obtaining valuable additional information for the analysis of three-way data structures.

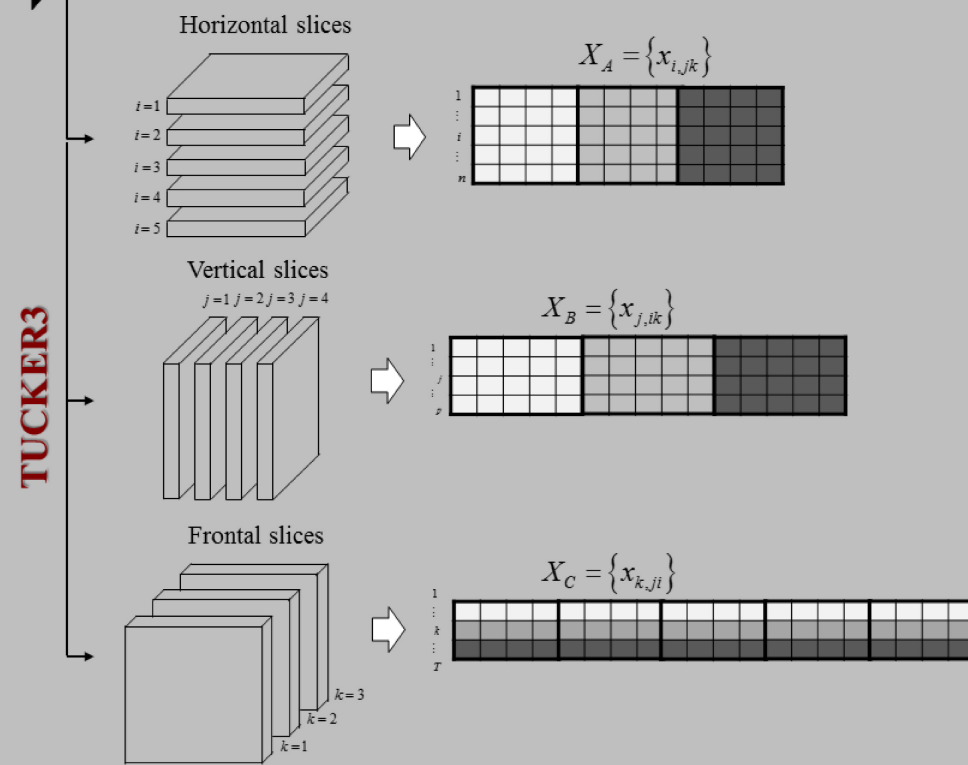
The diagram of decision for the three methods is presented on Figure 3.23.

Figure 3.23 – Decision diagram for STATIS, MFA and TUCKER3

Asymmetric methods



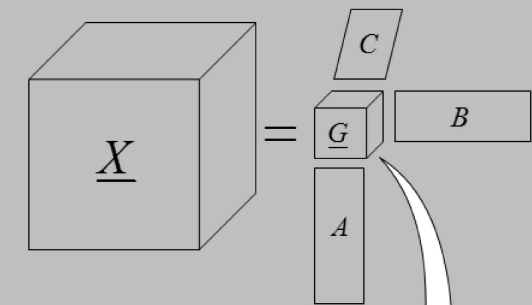
Symmetric method



Is a generalization of the PCA and the SVD and allows analyze data in three dimensions by reducing procedures that enable an easy interpretation and representation of factorial spaces with lower dimensions. The analysis has the objective of defining the structure of a data matrix summarizing the three sources of information in a core matrix that defines the fundamental structure of all data and especially the interactions between the three sources of variation.

$$x_{ijk} = \sum_{p=1}^P \sum_{q=1}^Q \sum_{r=1}^R a_{ip} b_{jq} c_{kr} g_{pqr} + e_{ijk}$$

$$X = A G (C' \otimes B') + E$$



$$G = A' X (C \otimes B)$$

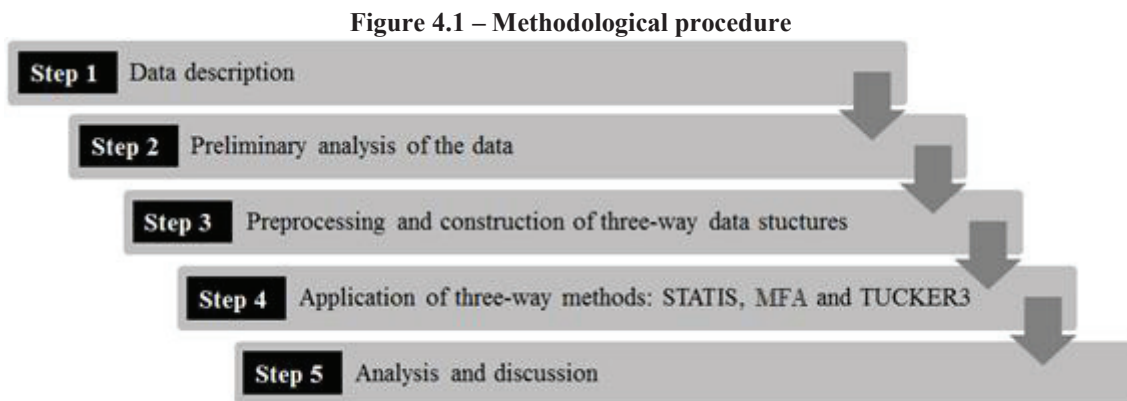
CHAPTER 4

RESULTS

4.1 Methodological procedure

This investigation intends to clarify the process that articulates growth with economic development in a context of comparable economies, and on a timeline that would not only allow evaluating periods of stability but also of change. It is used a set of disaggregated data on economic growth and development in 12 countries of the Eurozone, during the period 2005-2010 for analysis. The aim is to describe and to compare the countries in their economic dynamics in periods of stability and change, during the 3 years prior to the economic and financial crisis (2008) and 2 years later.

The results are presented in five steps according to the approach illustrated in Figure 4.1.



Source: own elaboration

The results were tested using *ade4 version 1.6-2* for STATIS, *FactoMineR version 1.28* for MFA, and *3WayPack Version 3.2* software for TUCKER3.

All factorial representations use the axis scales units to facilitate reading and interpretation of the relative position for the represented countries, years, or variables. All coordinates are presented in the respective tables to compare values and to confirm the component association.

4.2 Data Description

The data used in this research is of secondary databases provided by EUROSTAT and its identification and sources are presented in Table 4.1.

Table 4.1 – Identification of economic growth and development entities

CODE	Designation	Unit	Source
HNFCE	Household and NPISH Final Consumption Expenditure	Euro per inhabitant	Database: nama_gdp_c Accessed: 12/08/14 http://ec.europa.eu/eurostat/web/products-datasets/-/nama_gdp_c
GCF	Gross Capital Formation		
FCEGG	Final Consumption Expenditure of General Government		
EGS	Exports of Goods and Services		
IGS	Imports of Goods and Services		
R&D	Research & Development Expenditure		Database: rd_e_gerdtot Accessed: 12/08/14 http://ec.europa.eu/eurostat/web/products-datasets/-/rd_e_gerdtot
HDI	Human Development Index	Index (0 to 1)	Database: 137506 Accessed: 12/08/14 http://hdr.undp.org/sites/default/files/137506.xls
NP	Number of Patents	Count	Database: pat_ep_ntot Accessed: 12/08/14 http://ec.europa.eu/eurostat/web/products-datasets/-/pat_ep_ntot

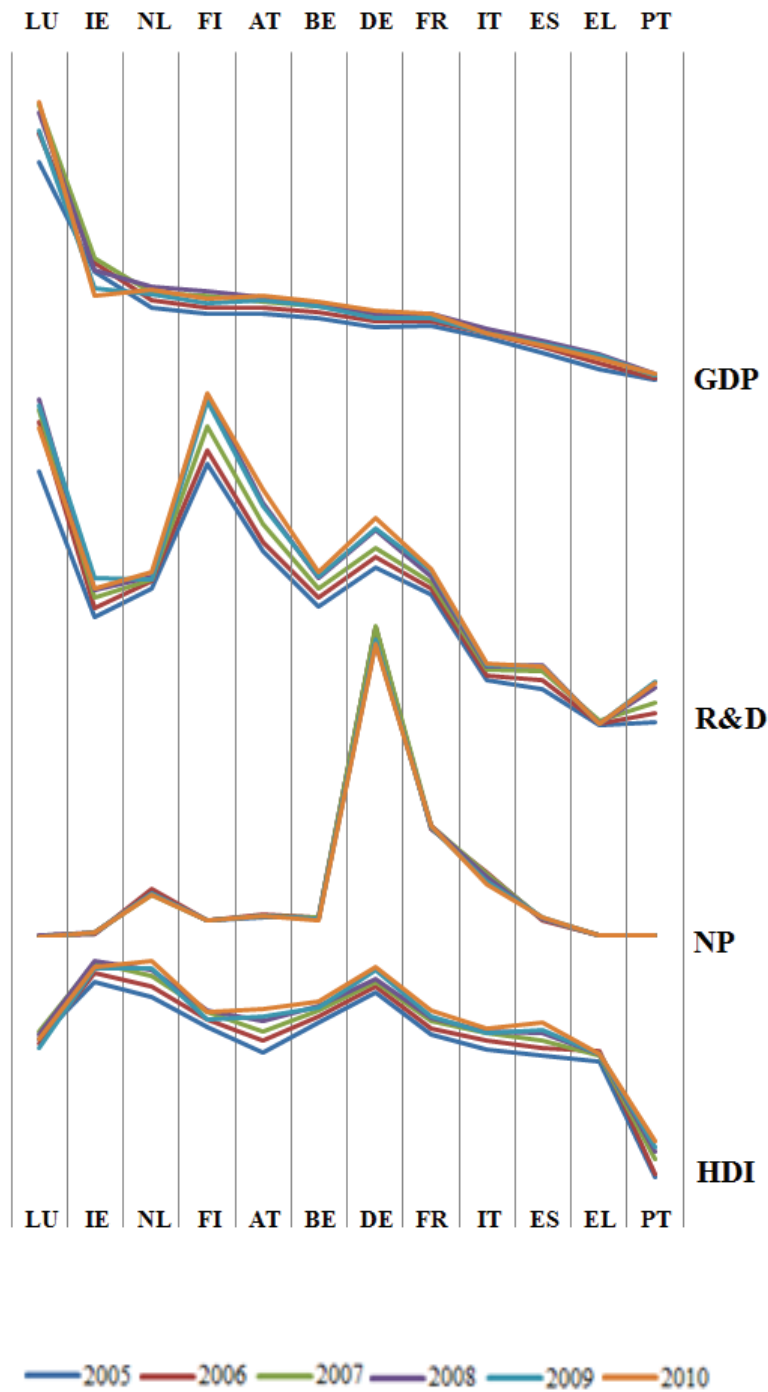
Source: own elaboration

4.3 Preliminary analysis of the data

The graphical representation of Figure 4.2 shows a profile analysis of the country indicators that can lead to a simplistic classification based on apparent similar relations between GDP, and the three entities of economic development which we considered (R&D, HDI, and NP).

This analysis of country indicators for the 2005-2010 period allows us, by an almost visual segmentation, to highlight the apparently more similar countries. However, the complete profile, integrating GDP and other non-GDP attributes, can result in such specific country profiles that it creates difficulties in group formation with a purely visual method. In a simplified manner and depending on the purpose of the analysis, this approach can be of relative usefulness.

Figure 4.2 – Profile comparison of GDP per capita, R&D expenditure, NP, and HDI



Source: own elaboration

By the analysis of GDP per capita performance it is possible to identify 4 groups of countries with different interpretations (Table 4.2) allowing a synthesis of the patterns described in Table 4.3.

Table 4.2 – Evolution of GDP

Group	GDP	2006	2007	2008	2009	2010
1	NL	↑	↑	↑	↓	↑
1	FI	↑	↑	↑	↓	↑
1	AT	↑	↑	↑	↓	↑
1	BE	↑	↑	↑	↓	↑
1	DE	↑	↑	↑	↓	↑
1	FR	↑	↑	↑	↓	↑
1	IT	↑	↑	↑	↓	↑
1	PT	↑	↑	↑	↓	↑
2	ES	↑	↑	↑	↓	↓
2	EL	↑	↑	↑	↓	↓
3	LU	↑	↑	↓	↓	↑
4	IE	↑	↑	↓	↓	↓

Source: own elaboration

Table 4.3 – Patterns of GDP

Group	Countries	Performance
1	NL, FI, AT, BE, DE, FR, IT and PT	increased in all years of this period except 2009, where there was a decrease
2	ES and EL	increased from 2005 till 2008 and decreased no 2009 and 2010
3	LU	increased in 2006, 2007 and 2010 and decreased in 2008 and 2009
4	IE	increased in 2006, 2007 and 2010 and decreased in 2008, 2009 and 2010

Source: own elaboration

Through the analysis of R&D performance it is also possible to describe 4 groups (Table 4.4) identifying the patterns featured in Table 4.5.

Table 4.4 – Evolution of R&D

Group	R&D	2006	2007	2008	2009	2010
1	BE	↑	↑	↑	↑	↑
1	DE	↑	↑	↑	↑	↑
1	FR	↑	↑	↑	↑	↑
1	IT	↑	↑	↑	↑	↑
2	NL	↑	↑	↑	↓	↑
2	FI	↑	↑	↑	↓	↑
2	AT	↑	↑	↑	↓	↑
3	ES	↑	↑	↑	↓	↓
3	LU	↑	↑	↑	↓	↓
3	EL	↑	↑	↑	↓	↓
4	PT	↑	↑	↑	↑	↓
4	IE	↑	↑	↑	↑	↓

Source: own elaboration

Table 4.5 – Patterns of R&D

Group	Countries	Performance
1	BE, DE, FR and IT	increased in all years of this period
2	NL, FI and AT	increased in all years, except for 2009 which had a decrease
3	ES, LU and EL	increased from 2006 until 2008 and decrease in 2009 and 2010
4	PT and IE	increased from 2006 and 2008 and decreased in 2009 and 2010

Source: own elaboration

The NP performance analysis allows the identification of 10 groups with different evolutions (Table 4.6) which lead to the patterns shown in the Table 4.7.

Table 4.6 – Evolution of NP

Group	NP	2006	2007	2008	2009	2010
1	NL	↑	↓	↑	↓	↓
1	LU	↑	↓	↑	↓	↓
2	FI	↑	↓	↓	↑	↓
2	AT	↑	↓	↓	↑	↓
3	IE	↑	↑	↑	↑	↑
4	ES	↓	↑	↑	↑	↑
5	DE	↑	↑	↓	↑	↓
6	BE	↑	↑	↓	↓	↓
7	FR	↑	↑	↑	↓	↑
8	IT	↑	↓	↓	↓	↓
9	PT	↓	↑	↓	↓	↓
10	EL	↓	↓	↓	↑	↓

Source: own elaboration

Table 4.7 – Patterns of NP

Group	Countries	Performance
1	NL and LU	increased in 2006 and 2008 and decreased in the remaining years
2	FI and AT	increased in 2006 and 2009 and decreased in the remaining years
3	IE	increased in all years
4	ES	increased only in 2006 and 2008 and decreased in the remaining years
5	DE	increased in 2006, 2007 and 2009 and decreased in the remaining years
6	BE	increased in 2006 and 2007 and decreased in the remaining years
7	FR	increased in all years except 2009 in which decreased
8	IT	decreased in all years except 2006 in which decreased
9	PT	decreased in all years except 2007 in which decreased
10	EL	decreased in all years except 2009 in which decreased

Source: own elaboration

Finally, by the analysis of HDI performance it's possible to identify 4 groups (Table 4.8) with different patterns (Table 4.9).

Table 4.8 – Evolution of HDI

Group	HDI	2006	2007	2008	2009	2010
1	NL	↑	↑	↑	↑	↑
1	AT	↑	↑	↑	↑	↑
1	ES	↑	↑	↑	↑	↑
1	DE	↑	↑	↑	↑	↑
1	FR	↑	↑	↑	↑	↑
1	PT	↑	↑	↑	↑	↑
2	FI	↑	↑	↑	↓	↑
2	IE	↑	↑	↑	↓	↑
2	BE	↑	↑	↑	↓	↑
2	IT	↑	↑	↑	↓	↑
3	LU	↓	↑	↓	↓	↑
4	EL	↑	↓	↑	→	→

Source: own elaboration

Table 4.9 – Patterns of HDI evolution

Group	Countries	Performance
1	NL, AT, ES, DE, FR and PT	increased in all years of this period
2	FI, IE, BE and IT	increased in all years, except for 2009 which had a decrease
3	LU	increased only in 2007 and 2010, and decreased in the remaining years
4	EL	increased in 2006 and 2008 and maintained the levels in 2009 and 2010

Source: own elaboration

4.4 Preprocessing and construction of three-way data structures

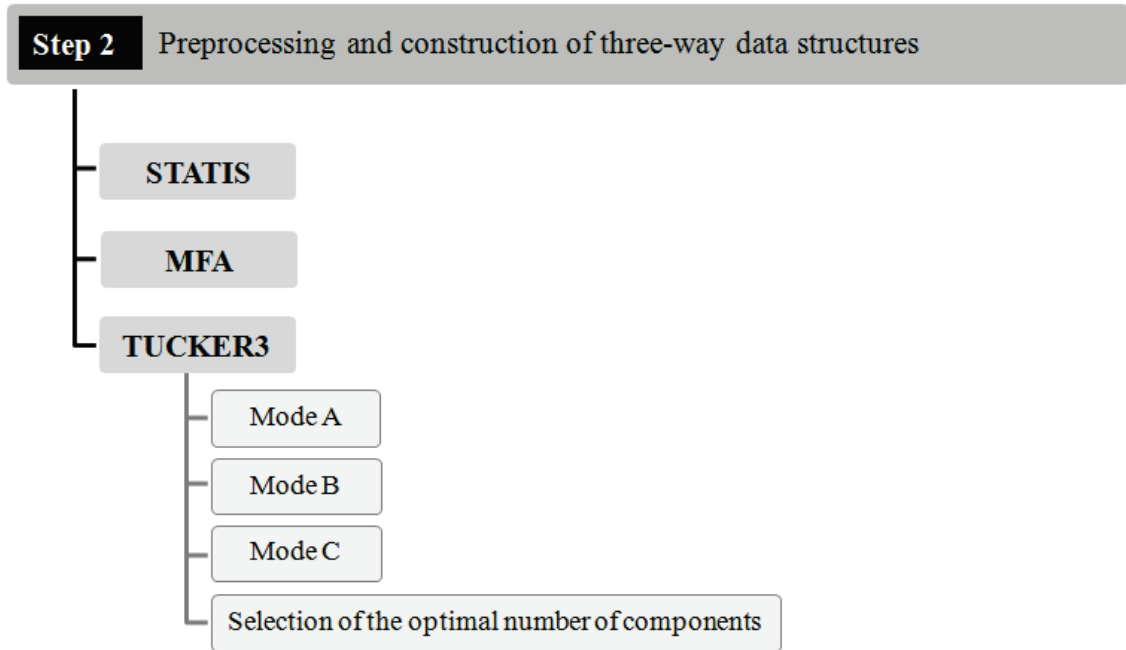
Data preprocessing and the construction of the three-way data arrays are steps previous to the analysis for the correct application of the methods.

The data preprocessing consists in the application of procedures to a multiway data set before a multiway model is fitted. This step takes an important place in data analysis (Bro and Smilde, 2003). Several types of data pretreatment are known, the most usual ones in two-way data analysis being centering and scaling. This research is performed using three methods on which there are various possible options of standardization, and to minimize the effects of different previous data transformation translating into different output interpretation or comparability reduction, we have chosen the standardization method by Z-scores for all the three techniques used here. It is done by transforming the scores into Z-scores expressed as standardized deviations from their means. Z-scores obtained from samples with different units are directly comparable because their values do not express the original unit of measurement. With the Z-scores standardization, the shape of the distribution is unchanged but Z-scores are centered on the value 0 and scaled such that its area is equal to 1.

The usual data layout is: I individuals (objects) in rows, the J variables in columns, and the K conditions in depth. However, for the analysis of our data will be considered: countries in the rows of each matrix, the different years in columns, and the variables in the depth axis, by the new interpretations this can provide.

So, the arrangement of the matrixes according to each method is presented in Figure 4.3.

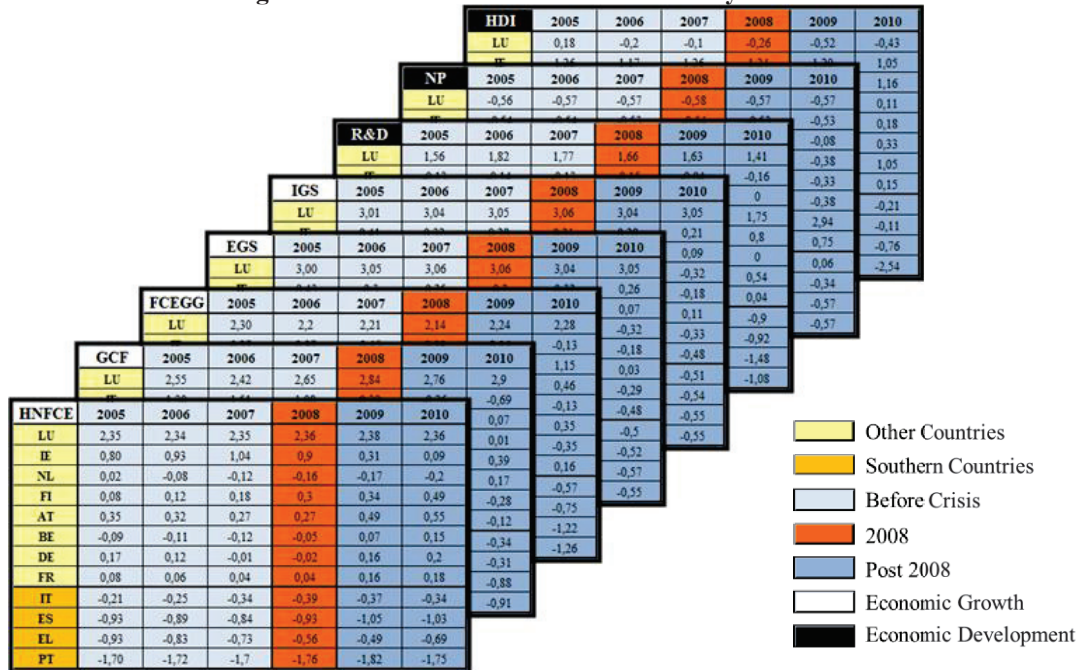
Figure 4.3 – Construction of three-way data structure



Source: own elaboration

The configuration of the STATIS initial data consists of 8 matrices X_t with 12 rows and 6 columns each as illustrated in Figure 4.4.

Figure 4.4 – Construction of the initial array for STATIS



Source: own elaboration

The configuration of the MFA initial data consists of a supermatrix with 12 rows and 48 columns created from the concatenation of the original matrices X_t as illustrated in Figure 4.5.

Figure 4.5 – Construction of the concatenated supermatrix for MFA

HNFCE	2005	2006	2007	2008	2009	2010	...	HDI	2005	2006	2007	2008	2009	2010
LU	2.35	2.34	2.35	2.36	2.38	2.36	...	LU	0.18	-0.2	-0.1	-0.26	-0.52	-0.43
IE	0.80	0.93	1.04	0.9	0.31	0.09	...	IE	1.26	1.17	1.26	1.24	1.20	1.05
NL	0.02	-0.08	-0.12	-0.16	-0.17	-0.2	...	NL	0.90	0.93	1.01	1.06	1.20	1.16
FI	0.08	0.12	0.18	0.3	0.34	0.49	...	FI	0.18	0.28	0.28	0.24	0.17	0.11
AT	0.35	0.32	0.27	0.27	0.49	0.55	...	AT	-0.18	-0.13	-0.1	0.03	0.17	0.18
BE	-0.09	-0.11	-0.12	-0.05	0.07	0.15	...	BE	0.18	0.35	0.32	0.35	0.17	0.33
DE	0.17	0.12	-0.01	-0.02	0.16	0.2	...	DE	0.90	0.93	0.87	0.88	0.86	1.05
FR	0.08	0.06	0.04	0.04	0.16	0.18	...	FR	0.18	0.11	0.11	0.1	0.17	0.15
IT	-0.21	-0.25	-0.34	-0.39	-0.37	-0.34	...	IT	-0.18	-0.13	-0.14	-0.18	-0.17	-0.21
ES	-0.93	-0.89	-0.84	-0.93	-1.05	-1.03	...	ES	-0.18	-0.26	-0.28	-0.22	-0.17	-0.11
EL	-0.93	-0.83	-0.73	-0.56	-0.49	-0.69	...	EL	-0.54	-0.33	-0.59	-0.65	-0.52	-0.76
PT	-1.70	-1.72	-1.7	-1.76	-1.82	-1.75	...	PT	-2.70	-2.73	-2.64	-2.61	-2.58	-2.54

Source: own elaboration

The configuration of the TUCKER3 initial data consists of the unfolding to Mode A, to Mode B and to Mode C as illustrated in Figures 4.6, 4.7, and 4.8, respectively.

Figure 4.6 – TUCKER3: Mode A (12x48 matrix)

	HNFCE						...	HDI						
	2005	2006	2007	2008	2009	2010	...	2005	2006	2007	2008	2009	2010	
LU	2.35	2.34	2.35	2.36	2.38	2.36	...	0.18	-0.2	-0.1	-0.26	-0.52	-0.43	
IE	0.80	0.93	1.04	0.9	0.31	0.09	...	1.26	1.17	1.26	1.24	1.20	1.05	
NL	0.02	-0.08	-0.12	-0.16	-0.17	-0.2	...	0.90	0.93	1.01	1.06	1.20	1.16	
FI	0.08	0.12	0.18	0.3	0.34	0.49	...	0.18	0.28	0.28	0.24	0.17	0.11	
AT	0.35	0.32	0.27	0.27	0.49	0.55	...	-0.18	-0.13	-0.1	0.03	0.17	0.18	
BE	-0.09	-0.11	-0.12	-0.05	0.07	0.15	...	0.18	0.35	0.32	0.35	0.17	0.33	
DE	0.17	0.12	-0.01	-0.02	0.16	0.2	...	0.90	0.93	0.87	0.88	0.86	1.05	
FR	0.08	0.06	0.04	0.04	0.16	0.18	...	0.18	0.11	0.11	0.1	0.17	0.15	
IT	-0.21	-0.25	-0.34	-0.39	-0.37	-0.34	...	-0.18	-0.13	-0.14	-0.18	-0.17	-0.21	
ES	-0.93	-0.89	-0.84	-0.93	-1.05	-1.03	...	-0.18	-0.26	-0.28	-0.22	-0.17	-0.11	
EL	-0.93	-0.83	-0.73	-0.56	-0.49	-0.69	...	-0.54	-0.33	-0.59	-0.65	-0.52	-0.76	
PT	-1.70	-1.72	-1.7	-1.76	-1.82	-1.75	...	-2.70	-2.73	-2.64	-2.61	-2.58	-2.54	

Source: own elaboration

Figure 4.7 – TUCKER3: Mode B (6x96 matrix)

	HNFCE																									
	LU	IE	NL	FI	AT	BE	DE	FR	IT	ES	EL	PT	...	HDI	LU	IE	NL	FI	AT	BE	DE	FR	IT	ES	EL	PT
2005	2.35	0.80	0.02	0.08	0.35	-0.09	0.17	0.08	-0.21	-0.93	-0.93	-1.70	...	0.18	1.26	0.90	0.18	-0.18	0.18	0.90	0.18	-0.18	-0.18	-0.54	-2.70	
2006	2.34	0.93	-0.08	0.12	0.32	-0.11	0.12	0.06	-0.25	-0.89	-0.83	-1.72	...	-0.2	1.17	0.93	0.28	-0.13	0.35	0.93	0.11	-0.13	-0.26	-0.33	-2.73	
2007	2.35	1.04	-0.12	0.18	0.27	-0.12	-0.01	0.04	-0.34	-0.84	-0.73	-1.7	...	-0.1	1.26	1.01	0.28	-0.1	0.32	0.87	0.11	-0.14	-0.28	-0.59	-2.64	
2008	2.36	0.9	-0.16	0.3	0.27	-0.05	-0.02	0.04	-0.39	-0.93	-0.56	-1.76	...	-0.26	1.24	1.06	0.24	0.03	0.35	0.88	0.1	-0.18	-0.22	-0.65	-2.61	
2009	2.38	0.31	-0.17	0.34	0.49	0.07	0.16	0.16	-0.37	-1.05	-0.49	-1.82	...	-0.52	1.20	1.20	0.17	0.17	0.17	0.86	0.17	-0.17	-0.17	-0.52	-2.58	
2010	2.36	0.09	-0.2	0.49	0.55	0.15	0.2	0.18	-0.34	-1.03	-0.69	-1.75	...	-0.43	1.05	1.16	0.11	0.18	0.33	1.05	0.15	-0.21	-0.11	-0.76	-2.54	

Source: own elaboration

Figure 4.8 – TUCKER3: Mode C (8x72 matrix)

	LU							PT					
	2005	2006	2007	2008	2009	2010	...	2005	2006	2007	2008	2009	2010
HNFCE	2,35	2,34	2,35	2,36	2,38	2,36	...	-1,70	-1,72	-1,7	-1,76	-1,82	-1,75
GCF	2,55	2,42	2,65	2,84	2,76	2,9	...	-1,04	-1,16	-1,22	-1,19	-1,23	-0,91
FCEGG	2,30	2,2	2,21	2,14	2,24	2,28	...	-1,29	-1,33	-1,36	-1,4	-1,36	-1,26
EGS	3,00	3,05	3,06	3,06	3,04	3,05	...	-0,61	-0,57	-0,55	-0,56	-0,56	-0,55
IGS	3,01	3,04	3,05	3,06	3,04	3,05	...	-0,59	-0,57	-0,56	-0,55	-0,56	-0,55
R&D	1,56	1,82	1,77	1,66	1,63	1,41	...	-1,33	-1,24	-1,19	-1,07	-1,06	-1,08
NP	-0,56	-0,57	-0,57	-0,58	-0,57	-0,57	...	-0,56	-0,57	-0,56	-0,57	-0,57	-0,57
HDI	0,18	-0,2	-0,1	-0,26	-0,52	-0,43	...	-2,70	-2,73	-2,64	-2,61	-2,58	-2,54

Source: own elaboration

In the TUCKER3 method, the computation of the mode A matrix produces the results presented in Table 4.10, where each row represents a country and each column is a component. The countries most associated with each component have the respective higher loading scores highlighted.

The unfolded X Mode A is a scalar product matrix, and therefore its eigendecomposition amounts to a PCA, and in Table 4.11 all the singular values and the total inertia are presented, which indicates the information retained in each axis. From this analysis we can explore the complete structure of these sets of observations.

Table 4.10 – TUCKER3: Mode A components

	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5	Axis 6	Axis 7	Axis 8	Axis 9
LU	0.800	-0.341	-0.334	-0.149	0.001	0.060	0.047	0.004	0.108
IE	0.138	0.056	0.471	-0.341	-0.408	-0.474	-0.372	0.003	-0.137
NL	0.080	0.155	0.305	-0.035	0.742	-0.109	0.071	-0.123	-0.258
FI	0.091	0.105	0.153	0.751	-0.147	-0.181	-0.029	-0.227	0.368
AT	0.035	0.004	0.070	0.325	-0.346	0.358	0.226	0.171	-0.653
BE	0.035	-0.023	0.158	0.012	0.240	0.124	0.130	-0.178	0.011
DE	-0.058	0.688	-0.551	-0.177	-0.114	-0.168	0.135	-0.200	-0.059
FR	-0.050	0.204	-0.069	0.084	0.144	0.266	-0.467	0.495	0.337
IT	-0.173	-0.001	0.013	-0.184	0.004	0.332	-0.215	0.225	-0.056
ES	-0.209	-0.119	0.124	-0.165	-0.072	-0.221	0.673	0.444	0.329
EL	-0.285	-0.208	0.091	-0.264	-0.171	0.412	0.013	-0.583	0.256
PT	-0.403	-0.521	-0.434	0.146	0.124	-0.394	-0.223	-0.015	-0.217

Source: own elaboration

Table 4.11 – TUCKER3: Mode A singular values and inertia

TUCKER3_Mode A	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5	Axis 6	Axis 7	Axis 8	Axis 9
Singular values	18.514	10.075	6.349	5.156	3.031	1.801	1.542	1.173	0.631
Inertia	0.649	0.192	0.076	0.050	0.017	0.006	0.005	0.003	0.001

Source: own elaboration

The computation of the mode B matrix produces the results presented in Table 4.12, where each row represents a year and each column is a component. The years most associated with each component have the respective higher loading scores highlighted. In Table 4.13 all the singular values and the total inertia are presented, which indicates the information retained in each axis. From this analysis we can explore the complete structure of these sets of observations.

Table 4.12 – TUCKER3: Mode B components

	Axis 1	Axis 2	Axis 3	Axis 4
2005	0.409	-0.367	0.636	0.121
2006	0.408	-0.443	0.044	-0.409
2007	0.410	-0.288	-0.356	0.239
2008	0.411	0.043	-0.540	0.403
2009	0.408	0.431	-0.162	-0.695
2010	0.404	0.631	0.386	0.340

Source: own elaboration

Table 4.13 – TUCKER3: Mode B singular values and inertia

TUCKER3 Mode B	Axis 1	Axis 2	Axis 3	Axis 4
Singular values	22.801	2.595	0.871	0.582
Inertia	0.985	0.013	0.001	0.001

Source: own elaboration

The mode C matrix produces Table 4.14, where each row represents a group of variables (economic growth and economic development) and each column is a component. The groups of variables most associated with each component have the respective higher loading scores highlighted. In Table 4.15 all the singular values and the total inertia are presented, which indicates the information retained in each axis. From this analysis we can explore the complete structure of these sets of observations.

Table 4.14 – TUCKER3: Mode C components

	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5	Axis 6	Axis 7
HNFCE	0.418	0.121	0.014	-0.058	-0.488	0.266	-0.706
GCF	0.416	-0.131	0.023	-0.164	-0.404	-0.752	0.234
FCEGG	0.412	0.063	0.161	0.115	0.736	-0.317	-0.380
EGS	0.405	-0.215	-0.247	-0.240	0.135	0.315	0.246
IGS	0.404	-0.226	-0.232	-0.250	0.142	0.301	0.229
R&D	0.349	0.227	-0.079	0.836	-0.116	0.126	0.304
NP	-0.047	0.666	-0.698	-0.185	0.068	-0.166	-0.022
HDI	0.177	0.610	0.604	-0.322	0.004	0.176	0.312

Source: own elaboration

Table 4.15 – TUCKER3: Mode C singular values and inertia

TUCKER3_Mode C	Axis 1	Axis 2	Axis 3	Axis 4	Axis 5	Axis 6	Axis 7
Singular values	18.573	10.086	6.258	5.086	3.028	1.984	1.760
Inertia	0.653	0.193	0.074	0.049	0.017	0.007	0.006

Source: own elaboration

The three-mode G supermatrix central array generated from the three component matrices before selecting the number of the A, B, and C modes of the array is presented in appendix I.

Finally, the percentage of fit was determined for each of the possible combinations of number of components in each of the modes of the array, as shown in Table 4.16.

For the selection of the optimal solution for the Tucker3 model it was used the *3WayPack Version 3.2* software (Appendix 2), using the data already standardized as prescribed for the STATIS and MFA methods.

In order to find the appropriate model and its dimensionality, the selection of number of components to retain in each mode (P in A, Q and R in B mode in C) considered five methods of decision whose results can be observed on the *3WayPack Version 3.2* software output (in Appendix 2). They are not exclusive but complementary methods and the decision will depend on the conclusions drawn from all of them:

1. **Sum of components.** A suitable model is one whose solution has a sum of components, less than or equal to 5. This criterion does not produce an effective decision for the model because the only solutions that verify are: 1x1x1 and 2x1x2

which consider, for some of the modes, a single component, which does not make sense (Table 4.16);

Table 4.16 – Results of the relevant models

Model Size	Sum of Components	Difference in Prop. Fit	Prop. SS (Fit)
1x1x1	3	0,06487	0,06487
2x1x2	5	0,1919	0,8405
2x2x2	6	0,0003	0,8408
3x1x3	7	0,0732	0,914
3x2x3	8	0,0012	0,9152
4x1x4	9	0,0469	0,9621
4x2x4	10	0,0022	0,9644
5x1x5	11	0,0133	0,9777
5x2x5	12	0,0035	0,9812
5x3x5	13	0,0006	0,9818
5x4x5	14	0,0002	0,9819
5x5x5	15	0,0001	0,982

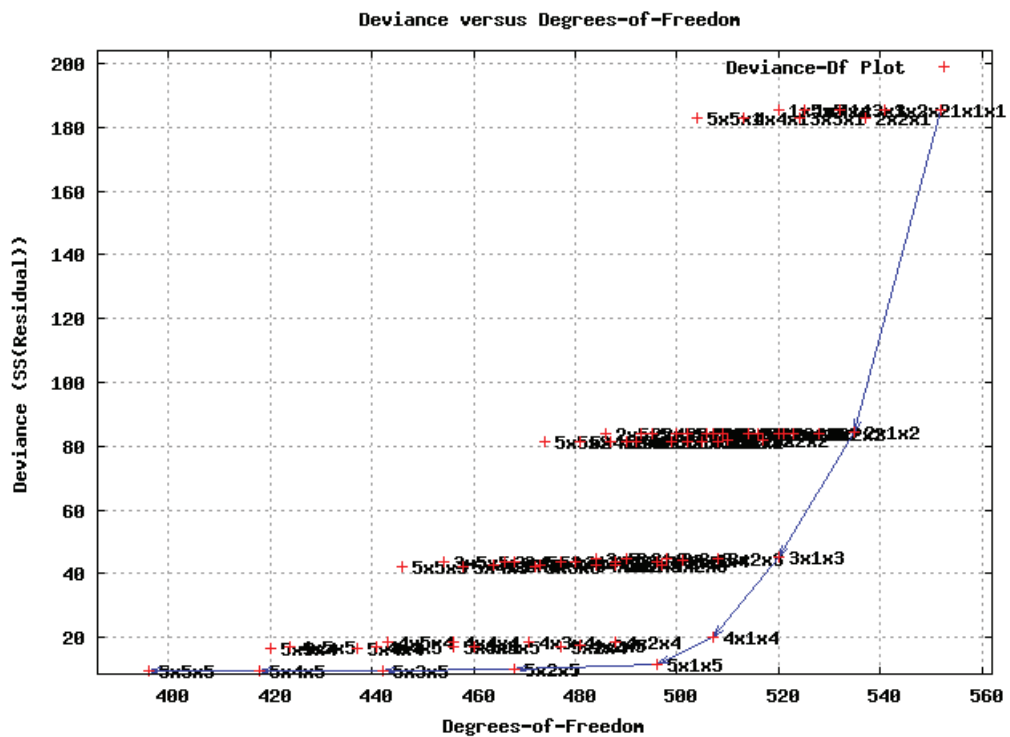
Source: own elaboration

- 2. Difference in Proportion of Fit.** According to this criterion, the model to consider is the one whose solution presents the highest difference the proportion between different adjustments. The solutions ordered by the difference (largest to smallest) are shown on the *3WayPack Version 3.2* software output (Appendix 2). Eliminating the solutions that retain only a single component, the combinations with the greatest difference are: 5x2x5, 4x2x4 and 3x2x3. Thus, this criterion prescribes a model, with the 5x2x5 solution as appropriate;
- 3. Goodness of model fitting.** Will search the model that has a "reasonable" amount of absorbed variance. In the Prop. SS (FIT) column of the *3WayPack Version 3.2* software output (Appendix 2) we see that, if we multiply the values by 100 one obtains the percentages of variance absorbed for each of the models and we can see the models with higher and lower variance. It becomes evident the

complementarity relation to other criteria. It can be seen how the models mentioned above are considered the best because they all absorb a high percentage of data (over 90%) which shows how any would be a suitable model;

4. **Three-mode deviance plot.** The best models are located on the blue line and, among them, those which form the nearest angle of 180 degrees (Figure 4.9). With this criterion, the best models (except those with a single component in some of its modes) are: 5x2x5, 5x3x5, 5x4x5 and 5x5x5. Therefore, of the three models previously analyzed, the 5x2x5 would be selected. However, according to this criterion, it can also be said that the 3x2x3 model is more appropriate for the 4x2x4 for its proximity to the blue line;

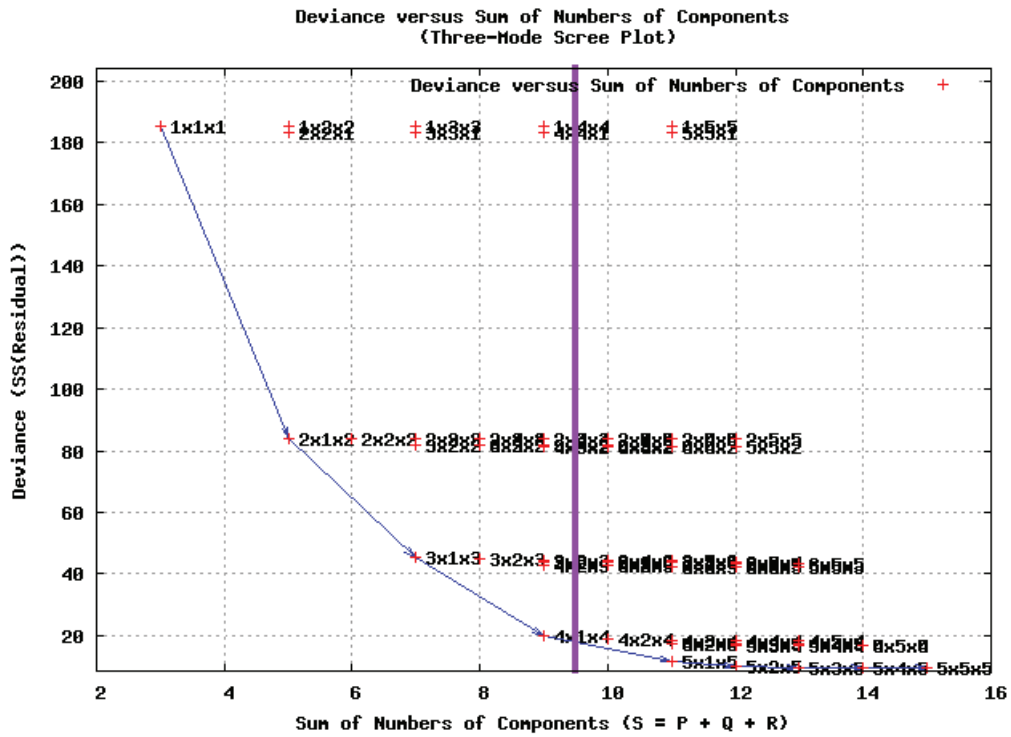
Figure 4.9 – Three-mode deviance plot



Source: 3WayPack Version 3.2 software output

5. **Three-mode scree plot.** This criterion is one of the most restrictive. Ensures that the models located on the right (above) the boundary line are not suitable (Figure 4.10).

Figure 4.10 – Three-mode scree plot



Source: 3WayPack Version 3.2 software output

The results of the three models selected by each test are summarized in Table 4.17. Thus, for previously selected models, the best would be 3x2x3, as 5x2x5 and 4x2x4 models lie to the right of the line.

Table 4.17 – TUCKER3: Summary of the three models selected by each dimensionality test

DiFit	% of Absorbed Variance	Deviance Plot	Scree Plot
5x2x5	5x2x5	5x2x5	3x2x3
4x2x4	4x2x4		
3x2x3	3x2x3		

Source: own elaboration

Finally, the decision is made according to all criteria. First, should be considered that the 3 detected models are highly representative, due to the variability absorbed. Although the 5x2x5 model is the best regarding the difference DiFit and Deviance Three-mode plot, the chosen model is 3x2x3 absorbing components, since this also collects a large proportion of information and is one of the best according to the criterion of the Three-Mode scree plot. Its interpretation will be easier because it contains a smaller number of components.

After the selection of the optimal number of components, we will seek the model compatible with this solution upon core matrix. The highlighted values indicate the highest percentage obtained for each adjustment increase in the sum of the components of the three modes based on which the optimal dimensionality for the model fit is determined. The difference in proportional fit is shown between parentheses on bold in Table 4.18. There were selected three components in mode A, two in mode B and three in mode C.

Thus, the optimal core matrix has the configuration presented in Table 4.18.

Table 4.18 – TUCKER3: Optimal core matrix

	C1 (R1)		C2(R2)		C3(R3)	
	B1 (Q1)	B2 (Q2)	B1 (Q1)	B2 (Q2)	B1 (Q1)	B2 (Q2)
A1 (P1)	18.504 (0.649)	-0.134 (0.000)	-0.004 (0.000)	-0.287 (0.000)	-0.025 (0.000)	-0.216 (0.000)
A2 (P2)	0.009 (0.000)	0.062 (0.000)	10.066 (0.192)	0.148 (0.000)	-0.046 (0.000)	0.088 (0.000)
A3 (P3)	0.015 (0.000)	-0.704 (0.000)	0.032 (0.000)	0.153 (0.000)	6.205 (0.073)	0.045 (0.000)

Source: own elaboration

Based on this core matrix, Table 4.19 shows the three components of the mode A and Table 4.20 shows their singular values and inertia.

Table 4.19 – TUCKER3: Mode A components

	Axis 1	Axis 2	Axis 3
LU	0.186	-0.107	-0.133
IE	0.032	0.018	0.187
NL	0.019	0.049	0.121
FI	0.021	0.033	0.061
AT	0.008	0.001	0.028
BE	0.008	-0.007	0.063
DE	-0.014	0.217	-0.218
FR	-0.012	0.064	-0.028
IT	-0.040	0.000	0.005
ES	-0.049	-0.038	0.049
EL	-0.066	-0.066	0.036
PT	-0.094	-0.164	-0.172

Source: own elaboration

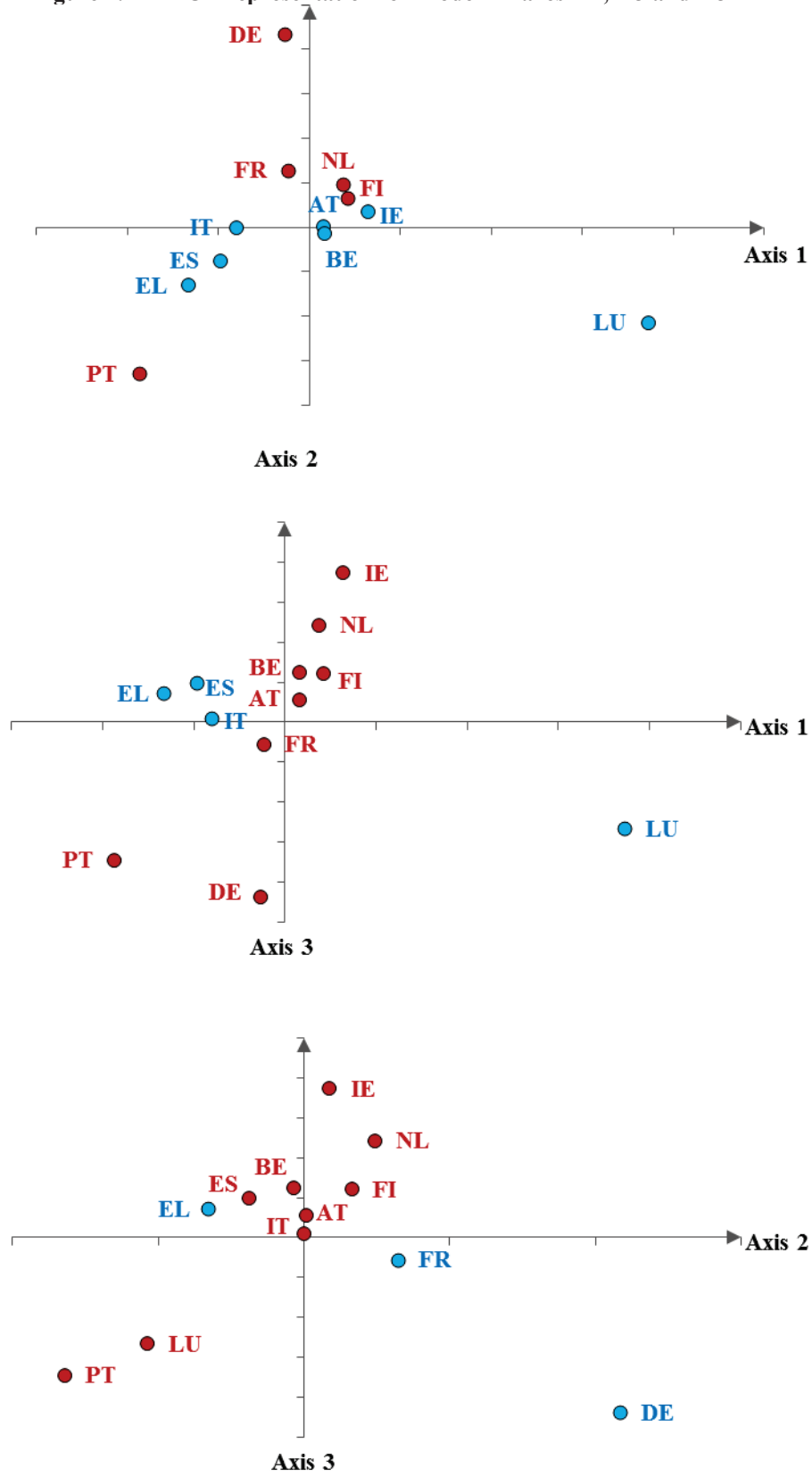
Table 4.20 – TUCKER3: Mode A singular values and inertia

TUCKER3_Mode A	Axis 1	Axis 2	Axis 3
Singular values	18.514	10.075	6.349
Inertia	0.649	0.192	0.076

Source: own elaboration

Figure 4.11 displays the three principal factorial planes of the mode A.

Figure 4.11 – PCA representation for Mode A – axes 1-2, 1-3 and 2-3



Source: own elaboration

Table 4.21 shows the two components of the mode B and Table 4.22 shows their singular values and inertia.

Table 4.21 – TUCKER3: Mode B components

	Axis 1	Axis 2
2005	0.086	-0.228
2006	0.085	-0.275
2007	0.086	-0.179
2008	0.086	0.027
2009	0.085	0.267
2010	0.085	0.392

Source: own elaboration

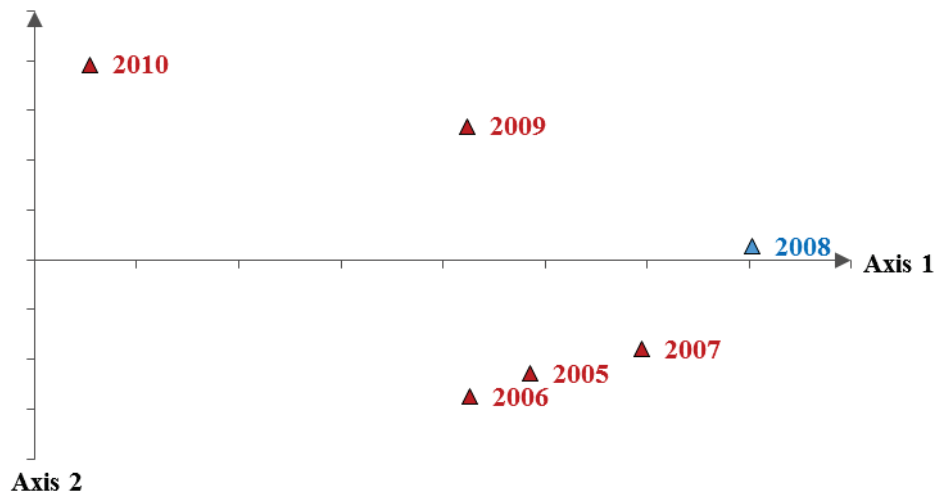
Table 4.22 – TUCKER3: Mode B singular values and inertia

TUCKER3_Mode B	Axis 1	Axis 2
Singular values	22.801	2.595
Inertia	0.985	0.013

Source: own elaboration

Figure 4.12 displays the principal factorial plan of the mode B.

Figure 4.12 – PCA representation for Mode B – axes 1-2



Source: own elaboration

Finally, Table 4.23 shows the three components of the mode C and Table 4.24 shows their singular values and inertia.

Table 4.23 – TUCKER3: Mode C components

	Axis 1	Axis 2	Axis 3
HNFCE	0.097	0.038	0.006
GCF	0.096	-0.041	0.009
FCEGG	0.096	0.020	0.064
EGS	0.094	-0.068	-0.099
IGS	0.094	-0.071	-0.093
R&D	0.081	0.071	-0.031
NP	-0.011	0.210	-0.279
HDI	0.041	0.192	0.241

Source: own elaboration

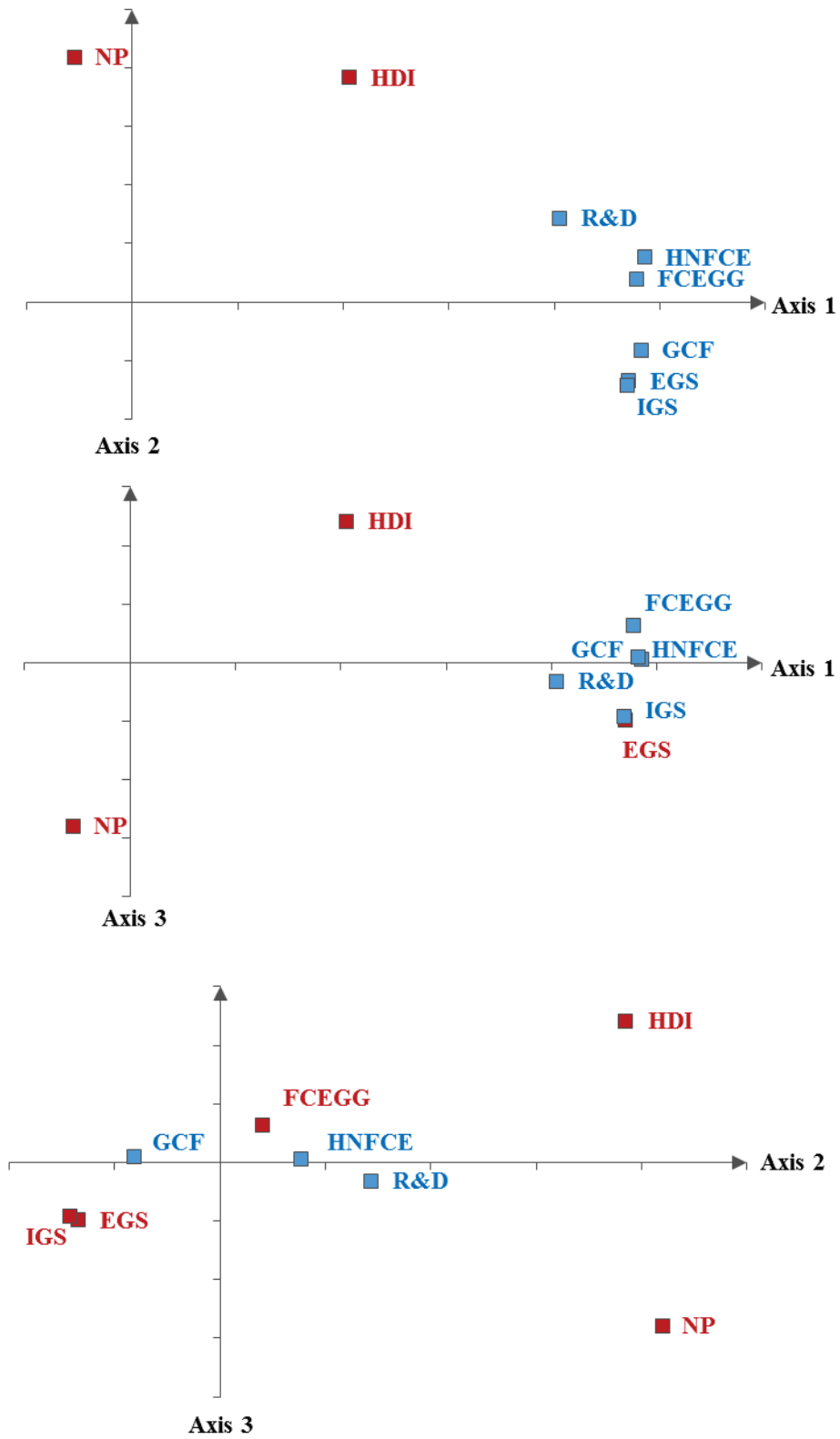
Table 4.24 – TUCKER3: Mode C singular values and inertia

TUCKER3_Mode C	Axis 1	Axis 2	Axis 3
Singular values	18.573	10.086	6.258
Inertia	0.653	0.193	0.074

Source: own elaboration

And Figure 4.13 displays the three principal factorial planes of the mode C.

Figure 4.13 – PCA representation for Mode C – axes 1-2, 1-3 and 2-3

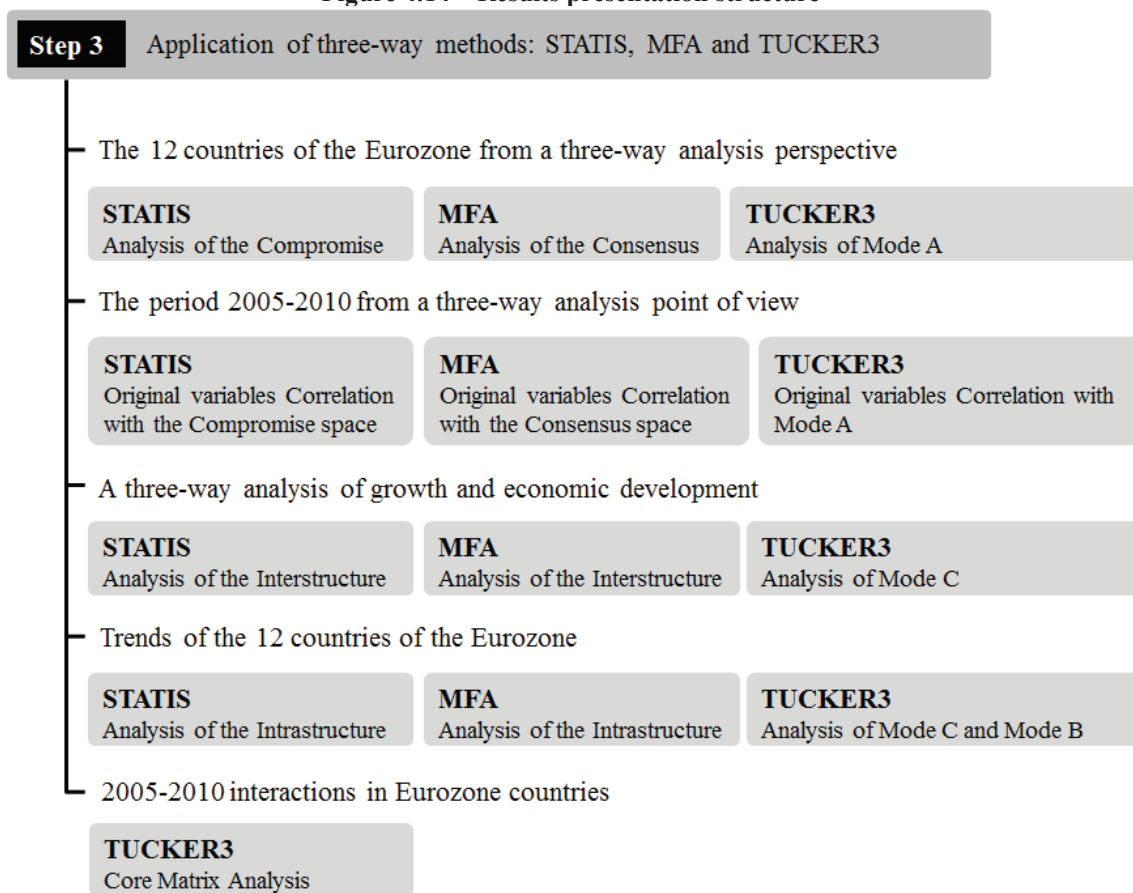


Source: own elaboration

4.5 Application of three-way methods: STATIS, MFA and TUCKER3

In order to allow a standard result presentation and the study of countries, the time period, economic growth, economic development, and paths of the 12 countries of the Eurozone, from the perspective of STATIS, MFA, and TUCKER3 methods, the structure and organization for the results is according to Figure 4.14.

Figure 4.14 – Results presentation structure



Source: own elaboration

4.5.1 The 12 countries of the Eurozone from a three-way analysis perspective

In the STATIS method, the computation of the compromise matrix produces Table 4.25 where each row represents a country and each column is a component. The compromise matrix is a scalar product matrix, and therefore its eigendecomposition amounts to a PCA. In Table 4.25 the Euclidian image coordinates of countries in compromise space is presented along axes 1-2. The countries more associated with the first or second axis have the respective higher loading scores highlighted. And in Table 4.26 the singular values and the inertia of the compromise space are presented, which indicates the information retained in each axis. From this analysis we can explore the structure of the set of observations from the STATIS point of view.

Table 4.25 – STATIS: Euclidian image of the compromise space in axes 1-2

Axis 1	Axis 2	
6.127	-0.963	LU
0.894	0.025	IE
0.441	0.438	NL
0.451	1.242	FI
0.184	0.483	AT
0.211	0.054	BE
-0.519	0.764	DE
-0.411	0.585	FR
-1.228	-0.176	IT
-1.502	-0.497	ES
-2.006	-0.720	EL
-2.636	-1.234	PT

Source: own elaboration

Table 4.26 – STATIS: Compromise space singular values and inertia

STATIS	Axis 1	Axis 2
Singular values	53.990	6.147
Inertia	0.984	0.013

Source: own elaboration

The first component of the compromise has an eigenvalue equal to 53.99, which corresponds to 98.4% of the inertia. The second component, with an eigenvalue of 6.15, retains only 1.3% of the inertia (information). The quality of representation is of 0.897.

On the other hand, the computation of the consensus matrix for the MFA method produces Table 4.27 where each row represents a country and each column is a component. The consensus matrix is also a scalar product matrix, and therefore its eigendecomposition amounts to a PCA. In Table 4.27 the Euclidian image coordinates of countries on consensus space is presented along trough axes 1-2. The countries more associated with the first or second axis have the respective high loading scores highlighted. And in Table 4.28 the singular values and the inertia of the consensus space are presented, which indicates the information retained in each axis. From this analysis we can explore the structure of the set of observations from the MFA point of view.

Table 4.27 – MFA: Euclidian image of the consensus space in axes 1-2

Axis 1	Axis 2	
6.384	-1.472	LU
1.105	0.251	IE
0.631	0.675	NL
0.719	0.455	FI
0.281	0.020	AT
0.275	-0.095	BE
-0.473	2.957	DE
-0.403	0.878	FR
-1.376	-0.003	IT
-1.652	-0.518	ES
-2.274	-0.894	EL
-3.214	-2.256	PT

Source: own elaboration

Table 4.28 – MFA: Consensus space singular values and inertia

MFA	Axis 1	Axis 2
Singular values	2.301	1.244
Inertia	0.652	0.190

Source: own elaboration

The first component of the consensus space has an eigenvalue equal to 2.301, which corresponds to 65.2% of the inertia. The second component, with an eigenvalue of 1.244, retains 19.0% of the inertia (information). The quality of representation is of 0.779.

Finally, in the TUCKER3 method, the computation of the mode A matrix produces Tables 4.29, where each row represents a country and each column is a component and Table 4.30, where the singular values and the inertia of the mode A matrix are presented. The inertia indicates the information retained in the respective axis. The countries more associated with the first or second component have the respective higher scores highlighted. With this analysis we can explore the structure of the set of observations, from the TUCKER3 point of view.

Table 4.29 – TUCKER3: Euclidian image of Mode A in axes 1-2

Axis 1	Axis 2	
0.186	-0.107	LU
0.032	0.018	IE
0.019	0.049	NL
0.021	0.033	FI
0.008	0.001	AT
0.008	-0.007	BE
-0.014	0.217	DE
-0.012	0.064	FR
-0.040	0.000	IT
-0.049	-0.038	ES
-0.066	-0.066	EL
-0.094	-0.164	PT

Source: own elaboration

Table 4.30 – TUCKER3: Mode A singular values and inertia

TUCKER3_Mode A	Axis 1	Axis 2
Singular values	18.514	10.075
Inertia	0.649	0.192

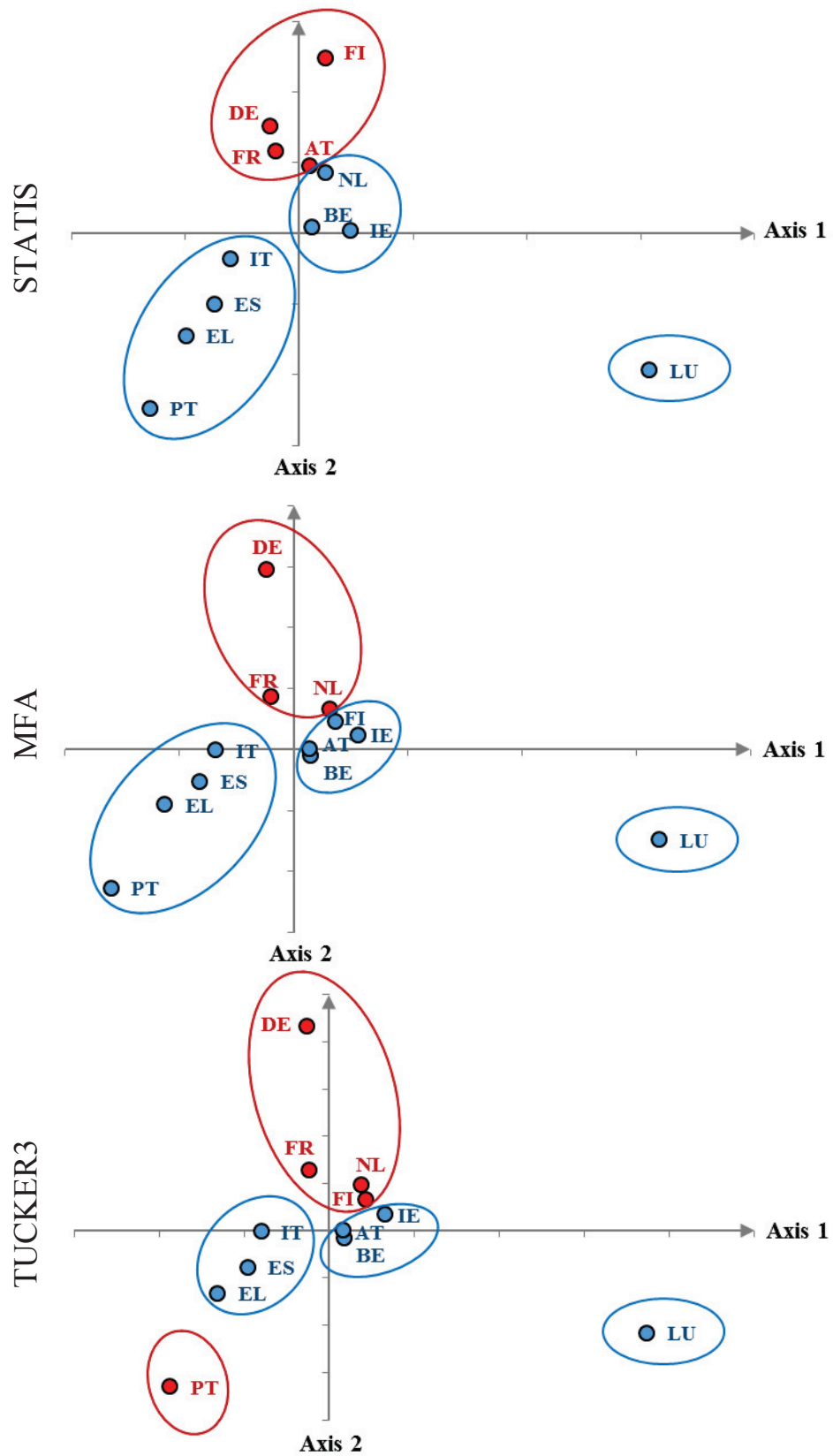
Source: own elaboration

The first component of the mode A matrix has an eigenvalue of 18.514, which corresponds to 64.9% of the inertia. The second component, with an eigenvalue of 10.075, retains 19.2% of the inertia (information).

Figure 4.15 displays the countries in the first two principal components space for STATIS Compromise, MFA Consensus, and TUCKER3 Mode A.

The countries highlighted in blue are predominantly associated with the first component and the ones in red are associated with the second component.

Figure 4.15 – Comparison of STATIS Compromise, MFA Consensus with TUCKER3 Mode A



Source: own elaboration

From the STATIS perspective, the first component is easily interpreted as a gradient of 8 countries (LU, NL, IE, BE, IT, ES, EL, and PT). On this gradient we can observe three behavior patterns of countries: one formed only by LU, a second created by IE, NL and BE and a third produced by IT, ES, EL, and PT. The first two are positively correlated with the first component and the third is negatively correlated, which indicates opposite performances among them. The other 4 countries (FI, AT, DE, and FR) are positively correlated with the second component.

From the MFA point of view, the first component is interpreted as a gradient of 9 countries (LU, IE, FI, AT, BE, IT, ES, EL, and PT), and the second component, with 19% of inertia, is interpreted as a gradient of 3 countries (NL, FR, and DE). On these two gradients we can observe four behavior patterns of countries, three of them associated with the first component and one related to the second component. On the first component we can examine LU alone and IE, FI, AT, and BE in a group, both of them positively correlated with this axis. On the same component we can also identify another group of countries with similar behavior and negatively correlated with this axis (IT, ES, EL, and PT). Lastly, on the second component NL, FR, and DE constitute a group, positively correlated with this axis and with opposition in terms of performance with the other two groups on the same axis.

On the TUCKER3 approach the first component is supported by a gradient of 7 countries (LU, IE, AT, BE, IT, ES, and EL). And the second component, with 19.2% of inertia, produces a gradient of 5 countries (PT, FI, NL, FR, and DE). On these two gradients we can observe five behavior patterns of countries, three of them associated with the first component and two related to the second component. On the first axis we can again examine LU alone, IE, AT, and BE in a group, both of them positively correlated with this component and in opposition, IT, ES, and EL, negatively correlated. On the second component, two groups of countries can be analyzed, PT alone and negatively correlated with this axis and in opposition, FI, NL, FR, and DE, positively correlated with the same axis. The gradients of countries are, over the first component on GDP per capita, from PT to LU, and over the second component (for MFA and TUCKER3) on number of patents, from PT to DE.

According to the Table 4.31 it is possible to synthesize the interpretation of the three techniques, regarding performance similarities of the countries. The positive sign indicates a positive correlation and a negative sign indicates a negative correlation.

Table 4.31 – Performance similarities of the countries in axes 1-2

	Axis 1	Axis 2
STATIS_Compromise	<ul style="list-style-type: none"> •LU (+) •IE, NL, and BE (+) •IT, ES, EL, and PT (-) 	<ul style="list-style-type: none"> •FI, AT, DE, and FR (+)
MFA_Consensus	<ul style="list-style-type: none"> •LU (+) •IE, FI, AT, and BE (+) •IT, ES, EL, and PT (-) 	<ul style="list-style-type: none"> •NL, FR, and DE (+)
TUCKER3_Mode A	<ul style="list-style-type: none"> •LU (+) •IE, AT, and BE (+) •IT, ES, and EL (-) 	<ul style="list-style-type: none"> •PT (-) •FI, NL, FR, and DE (+)

Source: own elaboration

4.5.2 The period 2005-2010 from a three-way analysis point of view

As in standard PCA, the variable loadings are the correlation between the original variables and the global factor scores. The global factor scores come from the compromise, from the consensus and from TUCKER3 (Mode A).

In this sense, Table 4.32 and Table 4.33 show, for the STATIS, the correlation coefficients of the original variables with both axes of the compromise space. It is noticeable that the correlation coefficients for the Axis 1 of the compromise space are substantially higher.

Table 4.32 – STATIS: Correlation of the original variables with axis 1 of compromise space

Axis 1	2005	2006	2007	2008	2009	2010
HNFCE	0.947	0.943	0.944	0.944	0.932	0.928
GCF	0.928	0.899	0.946	0.981	0.955	0.926
FCEGG	0.941	0.928	0.932	0.921	0.923	0.916
EGS	0.958	0.954	0.951	0.952	0.951	0.95
IGS	0.953	0.951	0.948	0.948	0.95	0.949
R&D	0.771	0.82	0.805	0.77	0.769	0.715

Source: own elaboration

Table 4.33 – STATIS: Correlation of the original variables with axis 2 of compromise space

Axis 2	2005	2006	2007	2008	2009	2010
HNFCE	.179	.166	.143	.157	.208	.245
GCF	-.132	-.135	-.124	-.109	-.039	-.107
FCEGG	.199	.211	.196	.225	.188	.190
EGS	-.276	-.290	-.296	-.290	-.298	-.298
IGS	-.295	-.301	-.311	-.306	-.300	-.301
R&D	.557	.474	.479	.494	.503	.565

Source: own elaboration

In the same way, Table 4.34 and Table 4.35 show, for the MFA, the correlation coefficients of the original variables with both axes of the consensus space. It is noticeable now that the correlation coefficients for the Axis 1 of the consensus space are substantially higher only for HNFCE, GCF, FCEGG, EGS, IGS, and R&D. In this case, the correlation coefficients for the Axis 2 of the consensus space are higher for NP, and HDI.

Table 4.34 – MFA: Correlation of the original variables with axis 1 of consensus space

Axis 1	2005	2006	2007	2008	2009	2010
HNFCE	0.958	0.954	0.954	0.954	0.942	0.938
GCF	0.923	0.897	0.942	0.973	0.949	0.91
FCEGG	0.951	0.94	0.945	0.936	0.935	0.926
EGS	0.938	0.931	0.928	0.929	0.928	0.927
IGS	0.932	0.928	0.924	0.924	0.927	0.926
R&D	0.798	0.842	0.827	0.792	0.792	0.742
NP	-.108	-.108	-.111	-.111	-.107	-.107
HDI	.499	.397	.442	.404	.321	.347

Source: own elaboration

Table 4.35 – MFA: Correlation of the original variables with axis 2 of consensus space

Axis 2	2005	2006	2007	2008	2009	2010
HNFCE	.178	.161	.123	.121	.163	.175
GCF	-.172	-.153	-.159	-.166	-.127	-.162
FCEGG	.084	.097	.080	.089	.065	.070
EGS	-.254	-.266	-.271	-.268	-.266	-.265
IGS	-.274	-.279	-.287	-.286	-.273	-.273
R&D	.331	.267	.256	.256	.270	.320
NP	0.825	0.828	0.824	0.828	0.829	0.828
HDI	0.714	0.751	0.738	0.753	0.771	0.798

Source: own elaboration

Tables 4.36 and 4.37 show, for TUCKER3, the correlation coefficients of the original variables with both axes of the Mode A space. Note also that the correlation coefficients for the Axis 1 of Mode A space are substantially higher, only for HNFCE, GCF, FCEGG, EGS, IGS, and R&D.

Table 4.36 – TUCKER3 – MODE A: Correlation of the original variables with axis 1 of Mode A

Axis 1	2005	2006	2007	2008	2009	2010
HNFCE	0.958	0.954	0.954	0.954	0.942	0.938
GCF	0.923	0.897	0.942	0.973	0.949	0.91
FCEGG	0.951	0.94	0.945	0.936	0.935	0.926
EGS	0.938	0.931	0.928	0.929	0.928	0.927
IGS	0.932	0.928	0.924	0.924	0.927	0.926
R&D	0.798	0.842	0.827	0.792	0.792	0.742
NP	-.108	-.108	-.111	-.111	-.107	-.107
HDI	.499	.397	.442	.404	.321	.347

Source: own elaboration

Table 4.37 – TUCKER3 – MODE A: Correlation of the original variables with axis 2 of Mode A

Axis 2	2005	2006	2007	2008	2009	2010
HNFCE	.178	.161	.123	.121	.163	.175
GCF	-.172	-.153	-.159	-.166	-.127	-.162
FCEGG	.084	.097	.080	.089	.065	.070
EGS	-.254	-.266	-.271	-.268	-.266	-.265
IGS	-.274	-.279	-.287	-.286	-.273	-.273
R&D	.331	.267	.256	.256	.270	.320
NP	0.825	0.828	0.824	0.828	0.829	0.828
HDI	0.714	0.751	0.738	0.753	0.771	0.798

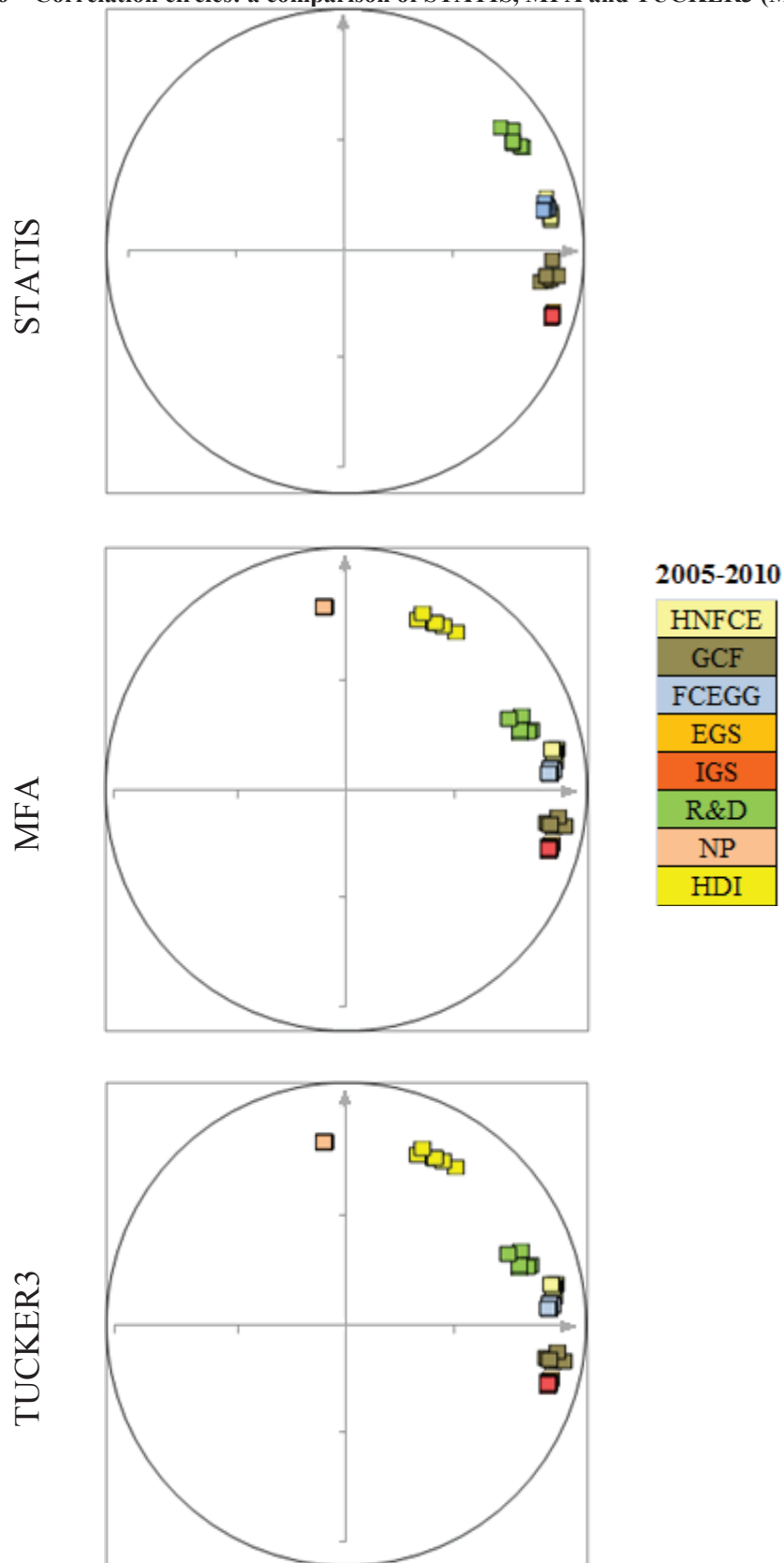
Source: own elaboration

It is important to clarify that the matrix used for this operation is the same for MFA and TUCKER3 (Mode A). In this sense, the correlation coefficients of the original variables with both axes of the Mode A space are the same as that described in Tables 4.34 and 4.35.

As in standard PCA, the variable loadings are the correlation between the original variables and the global factor scores. The global factor scores come from the compromise, from the consensus and from TUCKER3 (Mode A).

In Figure 4.16, the correlation coefficients between the original variables and the factorials axis of the compromise, consensus and TUCKER3 (only for the first two axes of Mode A for better comparability) are plotted, along with the circles of correlation.

Figure 4.16 – Correlation circles: a comparison of STATIS, MFA and TUCKER3 (Mode A)



Source: own elaboration

This circles of correlation approach is more common in STATIS and MFA, but it is possible to perform in TUCKER3 (Mode A), allowing the visualization of the correlation between the variables for the time period in analysis.

Figure 4.16 shows that for MFA and TUCKER3 the variables NP and HDI are responsible for the second component. The correlations of years are very similar for the various groups of variables. Therefore, the years aren't much different in each group of variables.

According to Table 4.38 it is possible to synthesize the interpretation of the three techniques, regarding performance similarities over 2005-2010 in growth and development entities. The positive sign indicates a positive correlation and a negative sign indicates a negative correlation.

Table 4.38 – Performance similarities of 2005-2010 over growth and development in axes 1-2

	Axis 1	Axis 2
STATIS _Original variables Correlation with the Compromise space	•HNFCE, FCEGG, R&D, GCF, EGS, and IGS (+)	
MFA _Original variables Correlation with the Consensus space	•HNFCE, FCEGG and R&D (+)	•NP and HDI (+) •GCF, EGS, and IGS (-)
TUCKER3 _Original variables Correlation with the Mode A	•HNFCE, FCEGG and R&D (+)	•NP and HDI (+) •GCF, EGS, and IGS (-)

Source: own elaboration

4.5.3 A three-way analysis of growth and economic development

For STATIS, the weights used for constructing the compromise space are obtained from the PCA of the cosine matrix. Because this matrix is not centered, the first eigenvector represents what is common to the different studies. Thus groups of variables with larger values on the first eigenvector are more similar and therefore will have a larger weight. The weights are obtained by rescaling the elements of the first eigenvector of the matrix so that their sum is equal to one.

In the STATIS method, the computation of the compromise matrix produces Table 4.39 where each row represents a group of variables and each column is a component. The groups of variables more associated with the first or second component have the respective higher loading scores highlighted.

In Table 4.40 the singular values and the inertia of compromise are presented, which indicates the information retained in each dimension of the eigendecomposition.

Table 4.39 – STATIS: Interstructure Euclidian image coordinates in axes 1-2

Axis 1	Axis 2	
0.900	0.140	HNFCE
0.950	-0.128	GCF
0.865	0.139	FCEGG
0.910	-0.279	EGS
0.906	-0.282	IGS
0.634	0.321	R&D
0.051	0.425	NP
0.191	0.831	HDI

Source: own elaboration

Table 4.40 – STATIS: Interstructure singular values and inertia

STATIS	Axis 1	Axis 2
Singular values	4.551	1.187
Inertia	0.872	0.059

Source: own elaboration

The first component of the STATIS interstructure has an eigenvalue equal to 4.551, which corresponds to 87.2% of the inertia. The second component, with an eigenvalue of 1.187, only retains 5.9% of the inertia, therefore making it irrelevant. The quality of representation is 0.793.

In the MFA method, the computation of the consensus matrix produces Table 4.41 where each row represents a group of variables and each column is a component. The groups of variables more associated with the first or second component have the respective higher loading scores highlighted.

On Table 4.42 the singular values and the inertia of consensus are presented, which indicates the information retained in each dimension of the eigendecomposition.

Table 4.41 – MFA: Interstructure Euclidian image coordinates in axes 1-2

Axis 1	Axis 2	
0.400	0.020	HNFCE
0.409	0.022	GCF
0.386	0.005	FCEGG
0.376	0.057	EGS
0.374	0.063	IGS
0.279	0.065	R&D
0.005	0.550	NP
0.072	0.463	HDI

Source: own elaboration

Table 4.42 – MFA: Interstructure singular values and inertia

MFA	Axis 1	Axis 2
Singular values	2.301	1.244
Inertia	0.652	0.190

Source: own elaboration

The first component of the MFA interstructure has an eigenvalue equal to 2.301, which corresponds to 65.2% of the inertia. The second component, with an eigenvalue of 1.244, retains 19.0% of the inertia (information). The quality of representation is 0.779.

In the TUCKER3 method, the computation of the Mode C matrix produces Table 4.43 where each row represents a group of variables and each column is a component. The

groups of variables more associated with the first or second component have the respective higher loading scores highlighted.

On Table 4.44 the singular values and the inertia of TUCKER3 Mode C are presented, which indicates the information retained in each dimension of the eigendecomposition.

Table 4.43 – TUCKER3: Mode C Euclidian image coordinates in axes 1-2

Axis 1	Axis 2	
0.097	0.038	HNFCF
0.096	-0.041	GCF
0.096	0.020	FCEGG
0.094	-0.068	EGS
0.094	-0.071	IGS
0.081	0.071	R&D
-0.011	0.210	NP
0.041	0.192	HDI

Source: own elaboration

Table 4.44 – TUCKER3: Mode C singular values and inertia

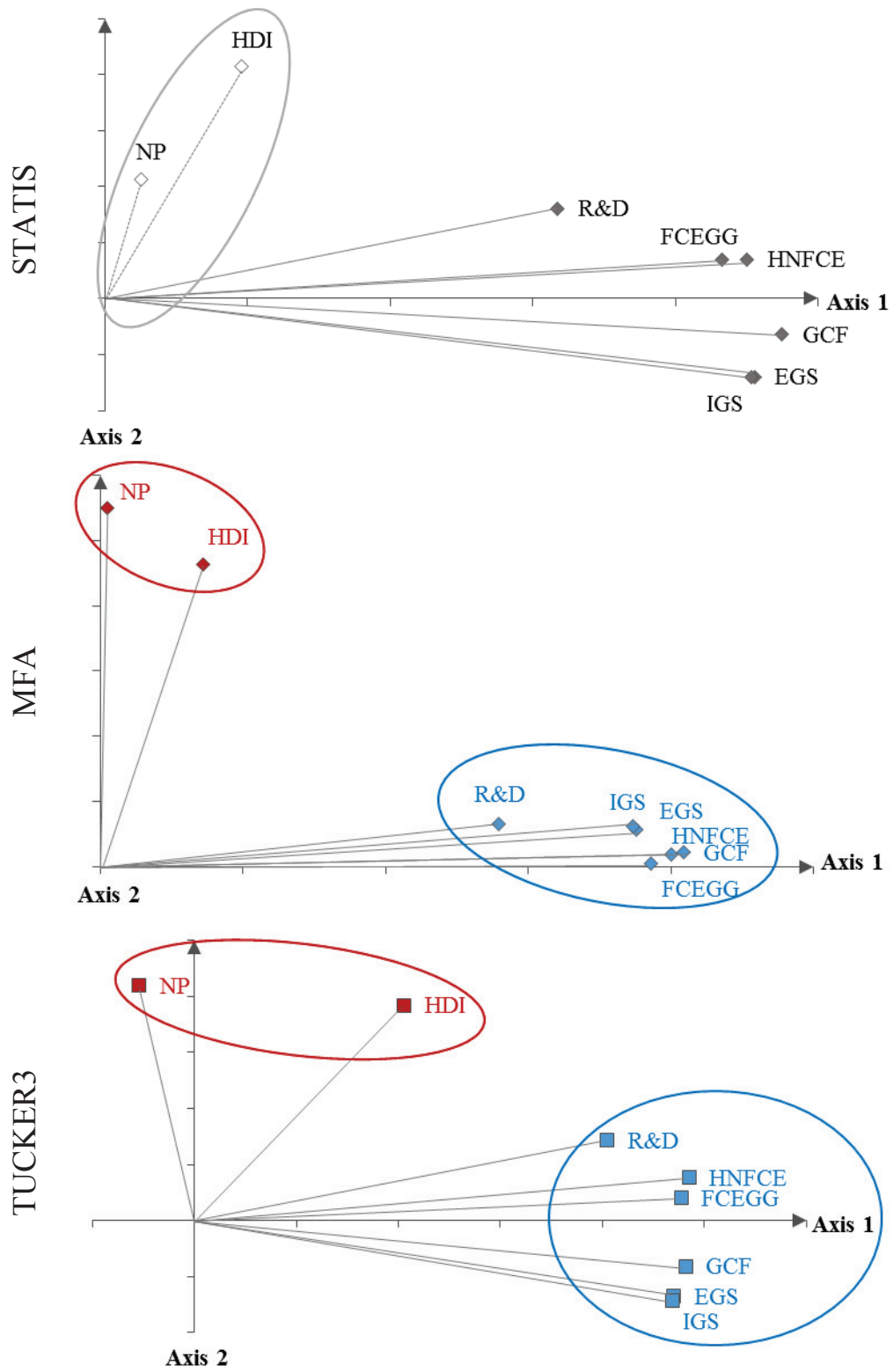
TUCKER3_Mode A	Axis 1	Axis 2
Singular values	18.573	10.086
Inertia	0.653	0.193

Source: own elaboration

The first component of the TUCKER3 Mode C has an eigenvalue equal to 18.573, which corresponds to 65.3% of the inertia. The second component, with an eigenvalue of 10.086, retains 19.3% of the inertia (information).

The comparison of the groups of variables in the space of the first two principal components for STATIS and MFA interstructures with TUCKER3 Mode C is displayed in Figure 4.17.

Figure 4.17 – Comparison of STATIS and MFA Interstructures with TUCKER3 Mode C



Source: own elaboration

In the STATIS results, the configuration of the interstructure denotes instability and this method, by default, is only interpretable over the first dimension. So, from the STATIS point of view, the first component is interpreted as a gradient of 6 groups of variables mostly related to economic growth (HNFCE, FCEGG, R&D, GCF, EGS, and IGS). On this gradient we can observe two performance patterns. One formed only by HNFCE, FCEGG, and R&D (first quadrant) and a second created by GCF, EGS, and IGS (fourth quadrant). These two behavior patterns reveal different behaviors regarding stability. The others 2 groups of variables (NP and HDI) cannot be interpreted on this factorial space because they are associated with the second component, which is irrelevant.

From the MFA perspective, the first component is easily interpreted as a gradient of 6 groups of variables (HNFCE, GCF, FCEGG, EGS, IGS, and R&D). On this gradient we can observe one performance pattern. This behavior pattern mostly related to economic growth shows great homogeneity in a stable performance along 2005-2010. The other two groups of variables (HDI and NP), related to the second component, are completely associated with economic development, with 19% of inertia.

In the TUCKER3 Mode C approach the first component is also supported by a gradient of 6 groups of variables, totally associated with economic growth and representative of two stable patterns of behavior. One formed only by HNFCE, FCEGG, and R&D (first quadrant) and a second created by GCF, EGS, and IGS (fourth quadrant). The second component (NP and HDI) is completely connected to economic development with 19.3% of inertia.

According to Table 4.45 it is possible to synthesize the interpretation of the three techniques, regarding performance similarities of growth and development entities over 2005-2010.

Table 4.45 – Performance similarities of growth and development over 2005-2010 in axes 1-2

	Axis 1	Axis 2
STATIS_Interstructure	<ul style="list-style-type: none"> ▪ HNFCE, FCEGG and R&D ▪ GCF, EGS and IGS 	
MFA_Interstructure	<ul style="list-style-type: none"> ▪ HNFCE, GCF, FCEGG, EGS, IGS and R&D 	<ul style="list-style-type: none"> ▪ NP and HDI
TUCKER3_Mode C	<ul style="list-style-type: none"> ▪ HNFCE, FCEGG and R&D ▪ GCF, EGS and IGS 	<ul style="list-style-type: none"> ▪ NP and HDI

Source: own elaboration

4.5.4 Trends of the 12 countries of the Eurozone

The analysis of the compromise and consensus spaces for STATIS and MFA respectively, reveals the structure of the country space common to the groups of variables. Hence, we can observe how each group of variables interprets this space. This is achieved by projecting the scalar product matrix of each group of variables onto the previously mentioned spaces. For STATIS and MFA, a line segment links the position for a given group of variables to the compromise or consensus position of the country. From this analysis we can explore the relations between the groups of variables and the period under study for each individual (country).

It is important to stress that TUCKER3 allows one useful feature, which is the possibility of relating/connecting, for each country, the relation with the groups of variables and with each year.

In Table 4.46 the coordinates of Luxembourg are presented for STATIS and MFA respectively.

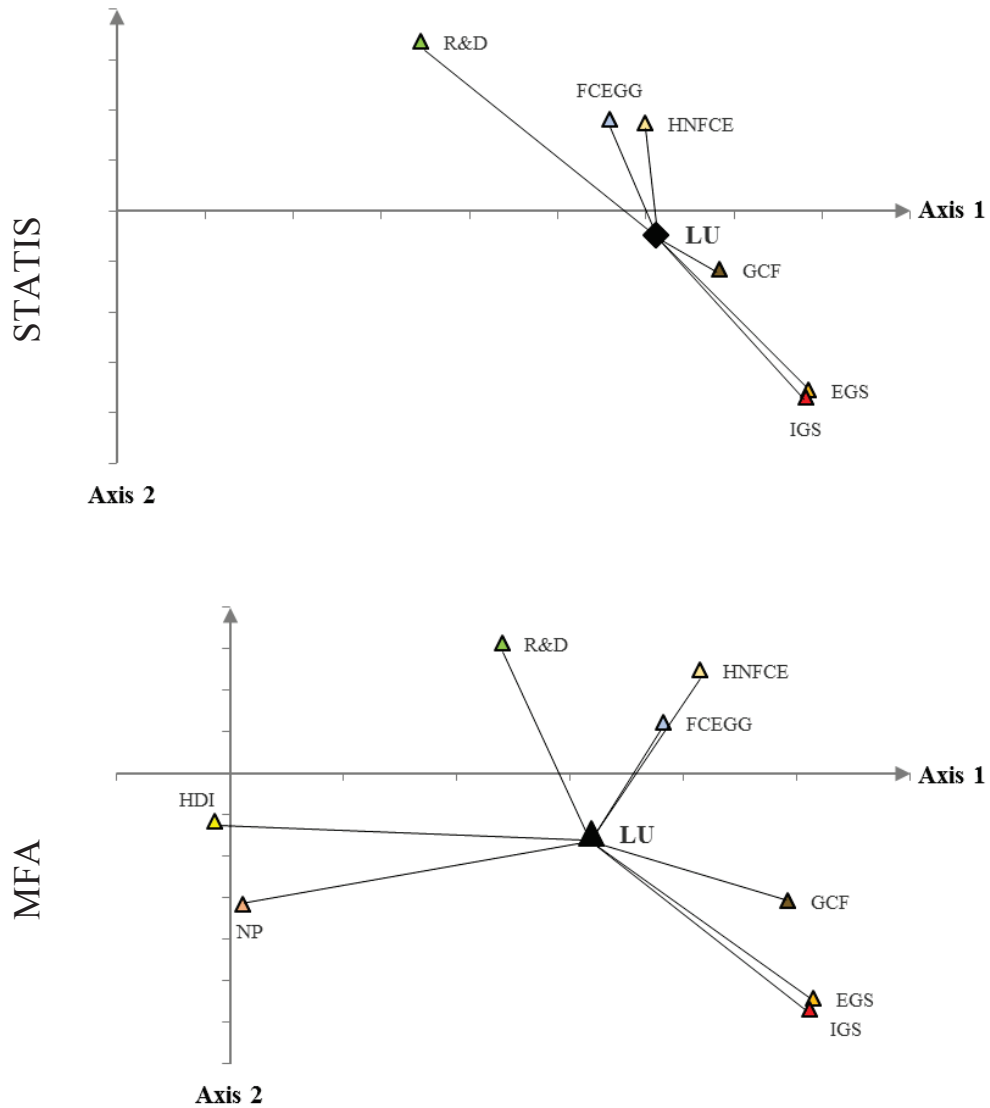
Table 4.46 – Compromise/consensus coordinates of economic growth and development for Luxembourg

	Axis 1	Axis 2	
S T A T I S	6.127	-0.963	LU
	5.995	3.462	HNFCE
	6.836	-2.306	GCF
	5.596	3.604	FCEGG
	7.848	-7.115	EGS
	7.826	-7.382	IGS
	3.457	6.713	R&D
M F A	6.384	-1.472	LU
	8.294	2.466	HNFCE
	9.851	-3.075	GCF
	7.652	1.205	FCEGG
	10.289	-5.433	EGS
	10.239	-5.711	IGS
	4.801	3.110	R&D
	0.225	-3.168	NP
-0.279	-1.172	HDI	

Source: own elaboration

Figure 4.18 graphically illustrates Tables 4.46.

Figure 4.18 – Comparison of STATIS and MFA: Projection of economic growth and development on Luxembourg’s factorial representation



Source: own elaboration

Table 4.47 synthesizes the interpretation of these two techniques, regarding Luxembourg’s performance.

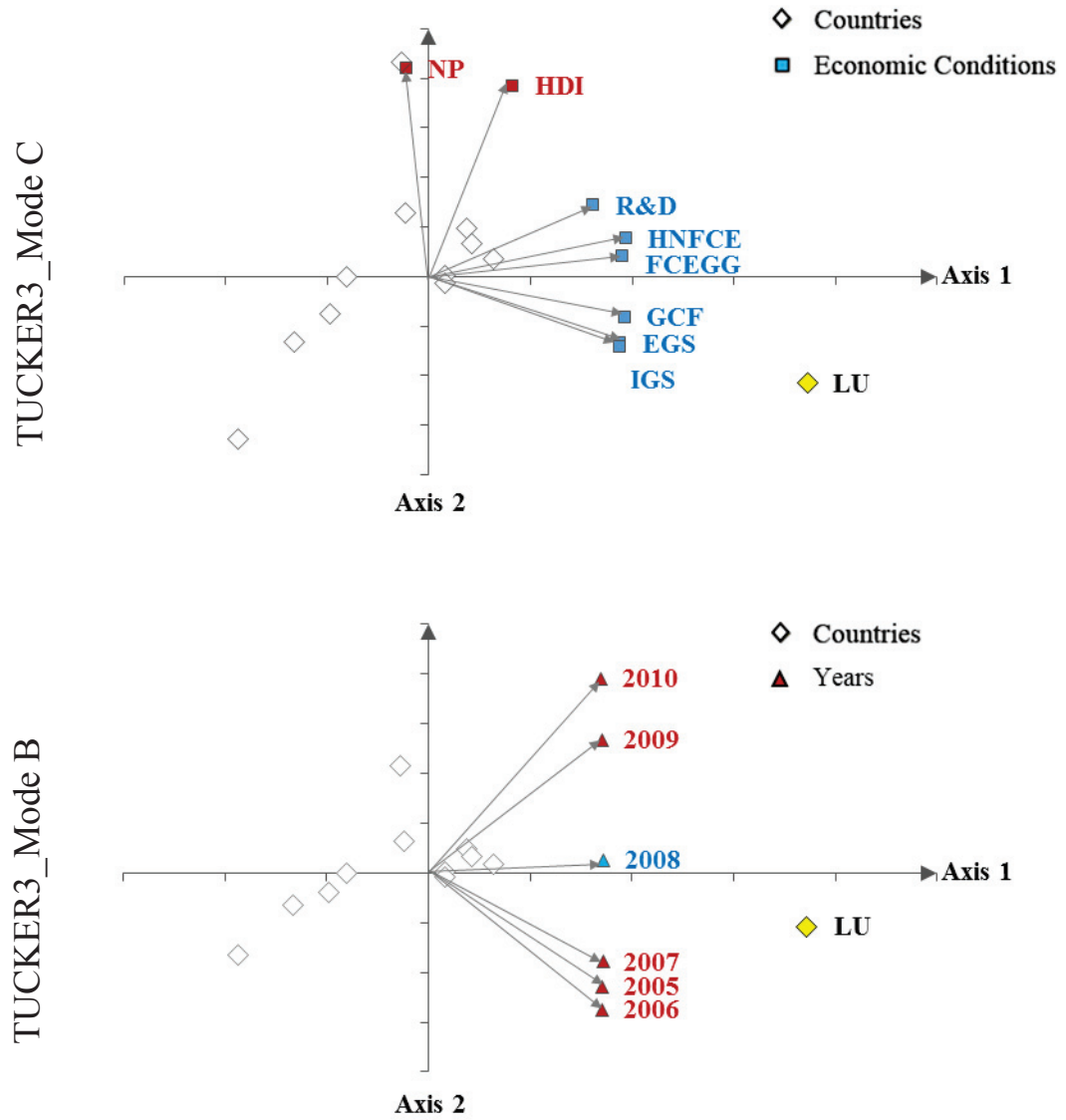
Table 4.47 – Luxembourg’s Performance from STATIS and MFA point of view

LUXEMBOURG	
STATIS	<ul style="list-style-type: none"> • the group of variable that most contributed to the average performance of Luxembourg over 2005-2010 was R&D and the one that contributed the least was GCF; • there are two patterns of groups of variables (R&D, FCEGG and HNFCE) and (GCF, EGS and IGS), with positive correlation within each pattern and negative correlation between patterns. • the contribution of the two groups were opposed for the construction of the centroid, and thus to Luxembourg’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group that most contributed to the average performance of Luxembourg over 2005-2010 was HDI and the one that contributed the least was FCEGG; • There are three patterns of groups of variables, (R&D, FCEGG and HNFCE), (GCF, EGS and IGS) and (NP and HDI), with positive correlation within each pattern and predominantly negative correlation between patterns, with the exceptions of R&D and (FCEGG and HNFCE) which have weak positive correlation with the (NP and HDI) group and (GCF, EGS and IGS) group respectively.

Source: own elaboration

Figure 4.19 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Luxembourg’s factorial representation.

Figure 4.19 –TUCKER3 Mode C and Mode B: Projection of economic growth and development on Luxembourg’s factorial representation



Source: own elaboration

And Table 4.48 synthesizes the interpretation of the Figure 4.18 regarding Luxembourg’s performance.

Table 4.48 – Luxembourg’s Performance from TUCKER3 point of view

LUXEMBOURG	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Luxembourg's performance was more positively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.49 the coordinates of Ireland are presented for STATIS and MFA respectively.

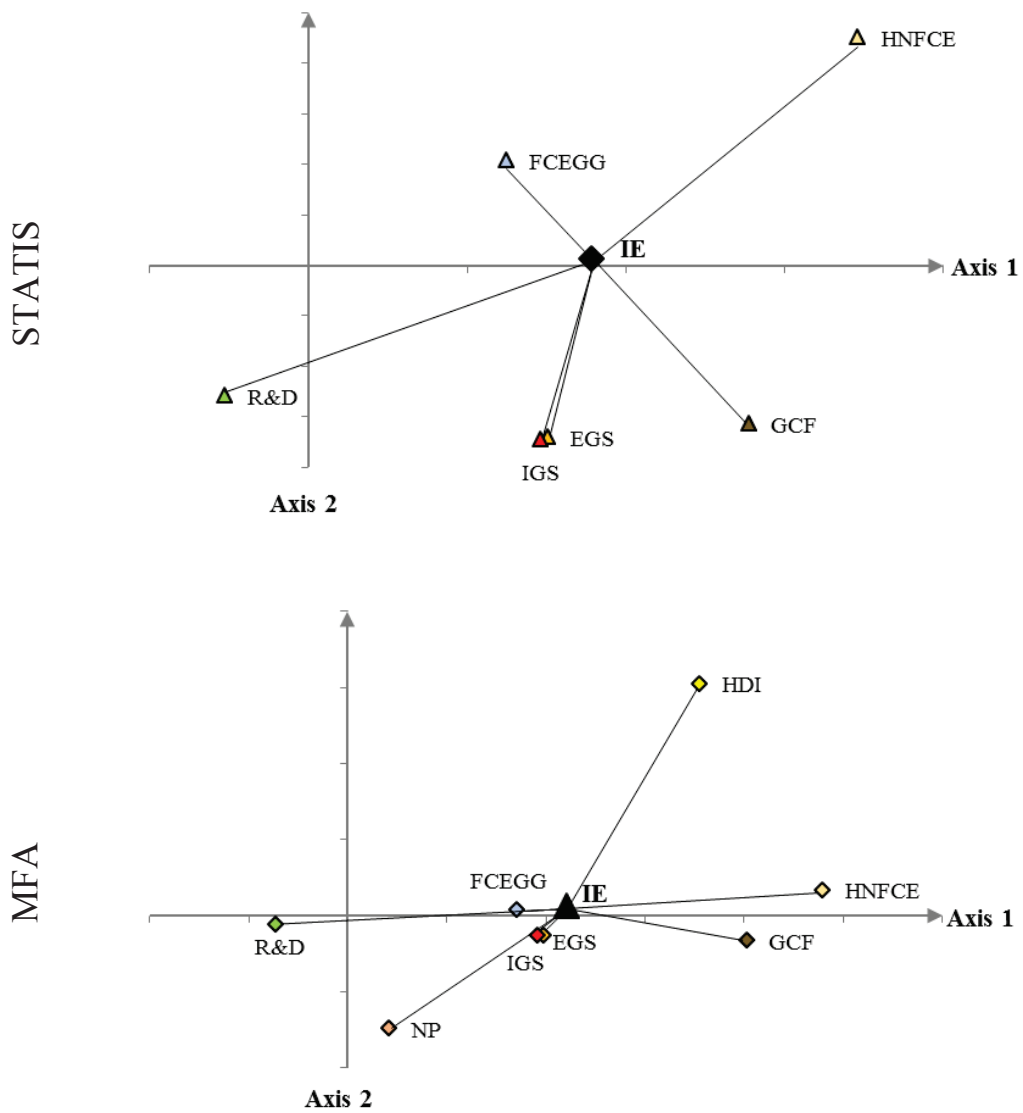
Table 4.49 – Compromise/consensus coordinates of economic growth and development for Ireland

	Axis 1	Axis 2	
S T A T I S	0.894	0.025	IE
	1.732	0.902	HNFCE
	1.390	-0.627	GCF
	0.626	0.414	FCEGG
	0.757	-0.683	EGS
	0.734	-0.690	IGS
	-0.262	-0.516	R&D
M F A	1.105	0.251	IE
	2.397	0.676	HNFCE
	2.015	-0.665	GCF
	0.857	0.145	FCEGG
	0.993	-0.521	EGS
	0.960	-0.534	IGS
	-0.364	-0.240	R&D
	0.211	-2.973	NP
1.776	6.118	HDI	

Source: own elaboration

Figure 4.20 graphically illustrate Tables 4.49.

Figure 4.20 – Comparison of STATIS and MFA: Projection of economic growth and development on Ireland’s factorial representation



Source: own elaboration

Table 4.50 synthesizes the interpretation of these two techniques, regarding Ireland’s performance.

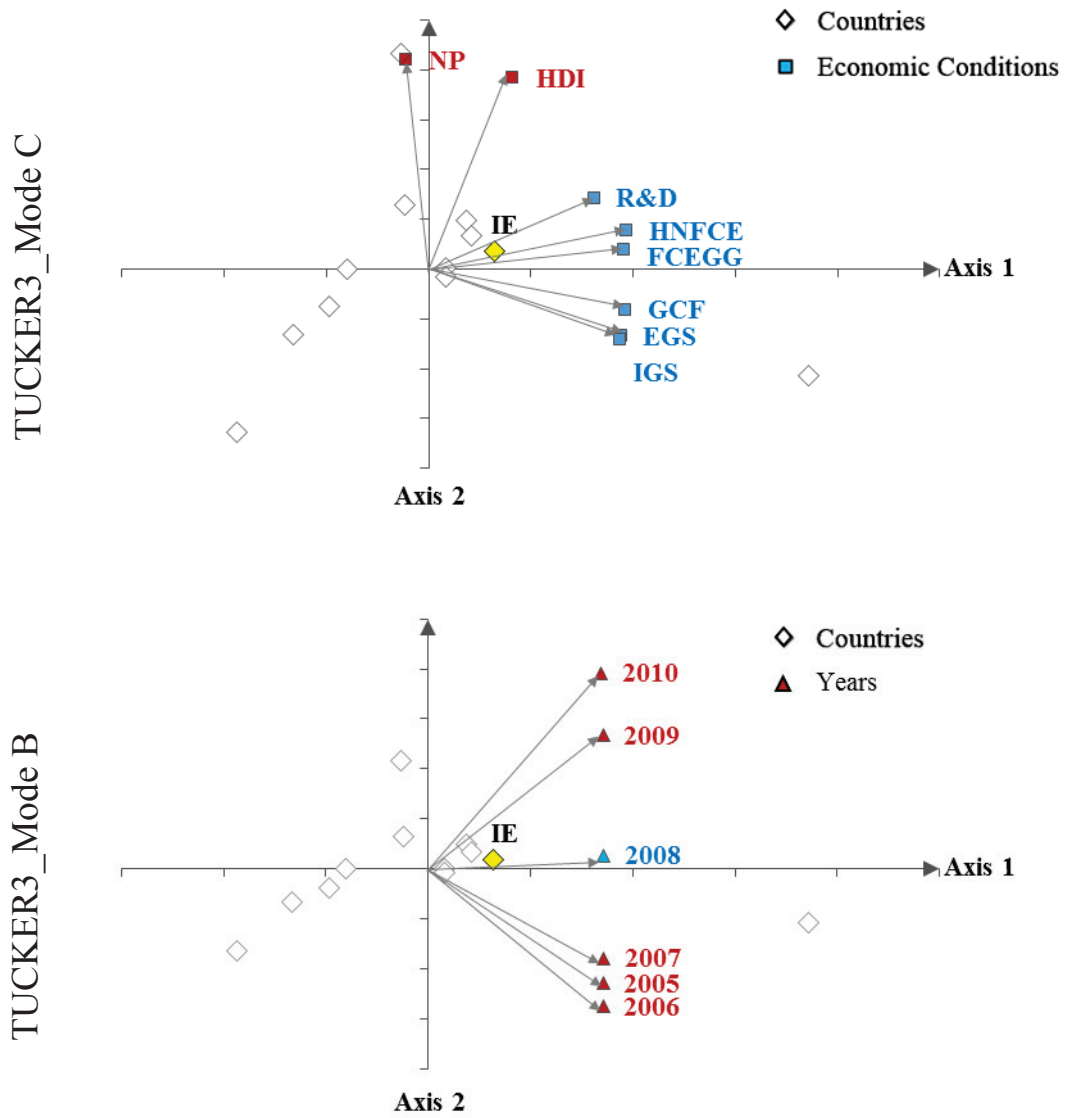
Table 4.50 – Ireland’s Performance from STATIS and MFA point of view

IRELAND	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was FCEGG. • there are three patterns of groups of variables (R&D), (FCEGG), (HNFCE) and (GCF, EGS and IGS) with positive correlation within each pattern and negative correlation between patterns, with the exception of R&D with (EGS and IGS) which have a weak positive correlation. • the contribution of the groups of variables were most notoriously opposed between FCEGG and GCF, for the construction of the centroid, and thus to Ireland’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was R&D and the one that contributed the least was IGS. • there are three patterns of groups of variables (GCF and HNFCE), (R&D, NP, FCEGG, EGS and IGS) and (HDI) with positive correlation within each pattern and predominantly with negative correlation between patterns, with the exceptions of (GCF and HNFCE) that has a weak positive correlation with the (HDI). • The contributions of the R&D group and HNFCE group were the most opposed for the construction of the centroid, and thus to Ireland’s centroid positioning.

Source: own elaboration

Figure 4.21 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Ireland’s factorial representation.

Figure 4.21 –TUCKER3 Mode C and Mode B: Projection of economic growth and development on Ireland’s factorial representation



Source: own elaboration

And Table 4.51 synthesizes the interpretation of the Figure 4.21 regarding Ireland’s performance.

Table 4.51 – Ireland’s Performance from TUCKER3 point of view

IRELAND	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Irelands' performance was more positively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.52 the coordinates of the Netherlands are presented for STATIS and MFA respectively.

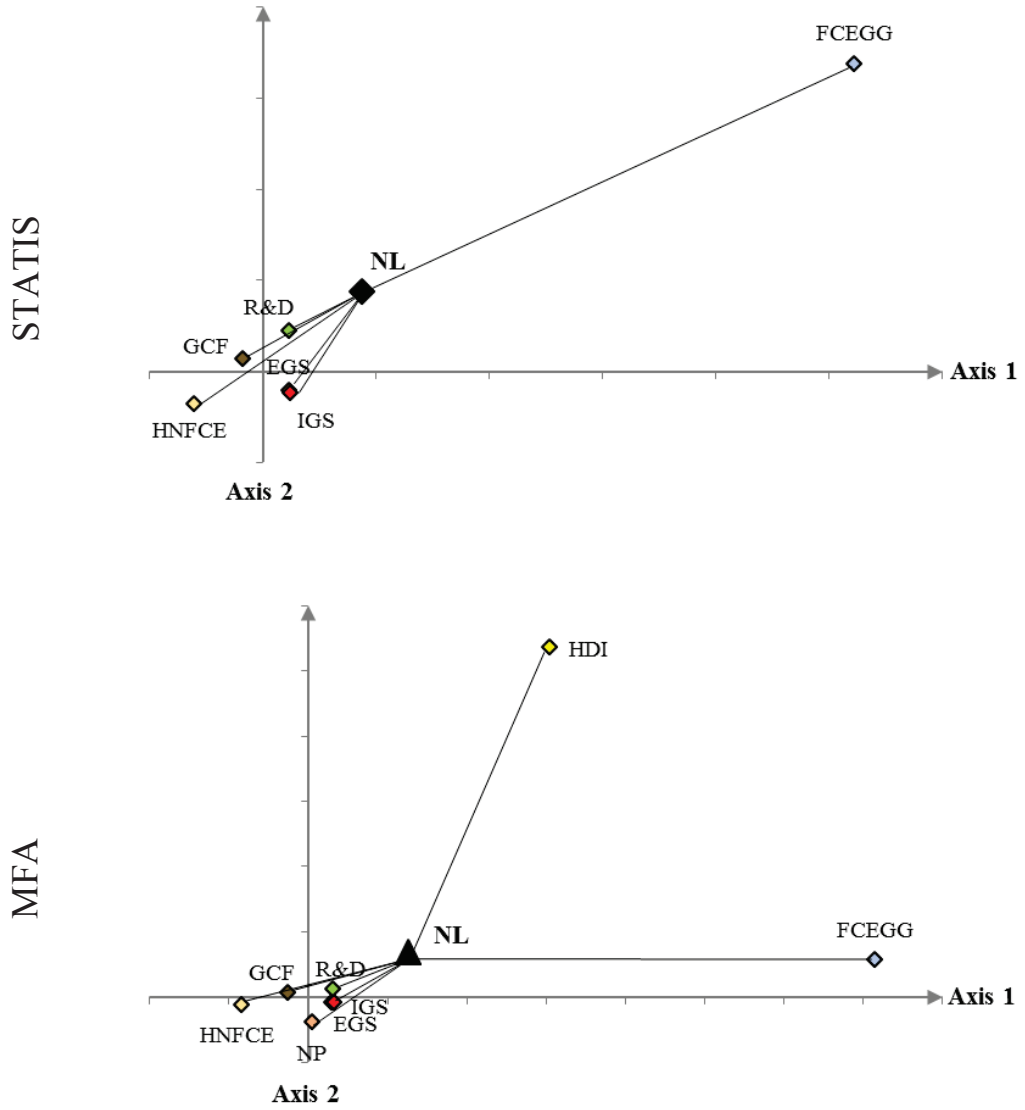
Table 4.52 – Compromise/consensus coordinates of economic growth and development for the Netherlands

	Axis 1	Axis 2	
S T A T I S	0.441	0.438	NL
	-0.300	-0.182	HNFCE
	-0.089	0.069	GCF
	2.613	1.685	FCEGG
	0.116	-0.105	EGS
	0.124	-0.117	IGS
	0.115	0.224	R&D
M F A	0.631	0.675	NL
	-0.415	-0.121	HNFCE
	-0.130	0.054	GCF
	3.574	0.559	FCEGG
	0.152	-0.080	EGS
	0.163	-0.090	IGS
	0.161	0.110	R&D
	0.028	-0.389	NP
1.519	5.360	HDI	

Source: own elaboration

Figure 4.22 graphically illustrate Tables 4.52.

Figure 4.22 – Comparison of STATIS and MFA: Projection of economic growth and development on the Netherlands’ factorial representation



Source: own elaboration

Table 4.53 synthesizes the interpretation of these two techniques, regarding the Netherlands’ performance.

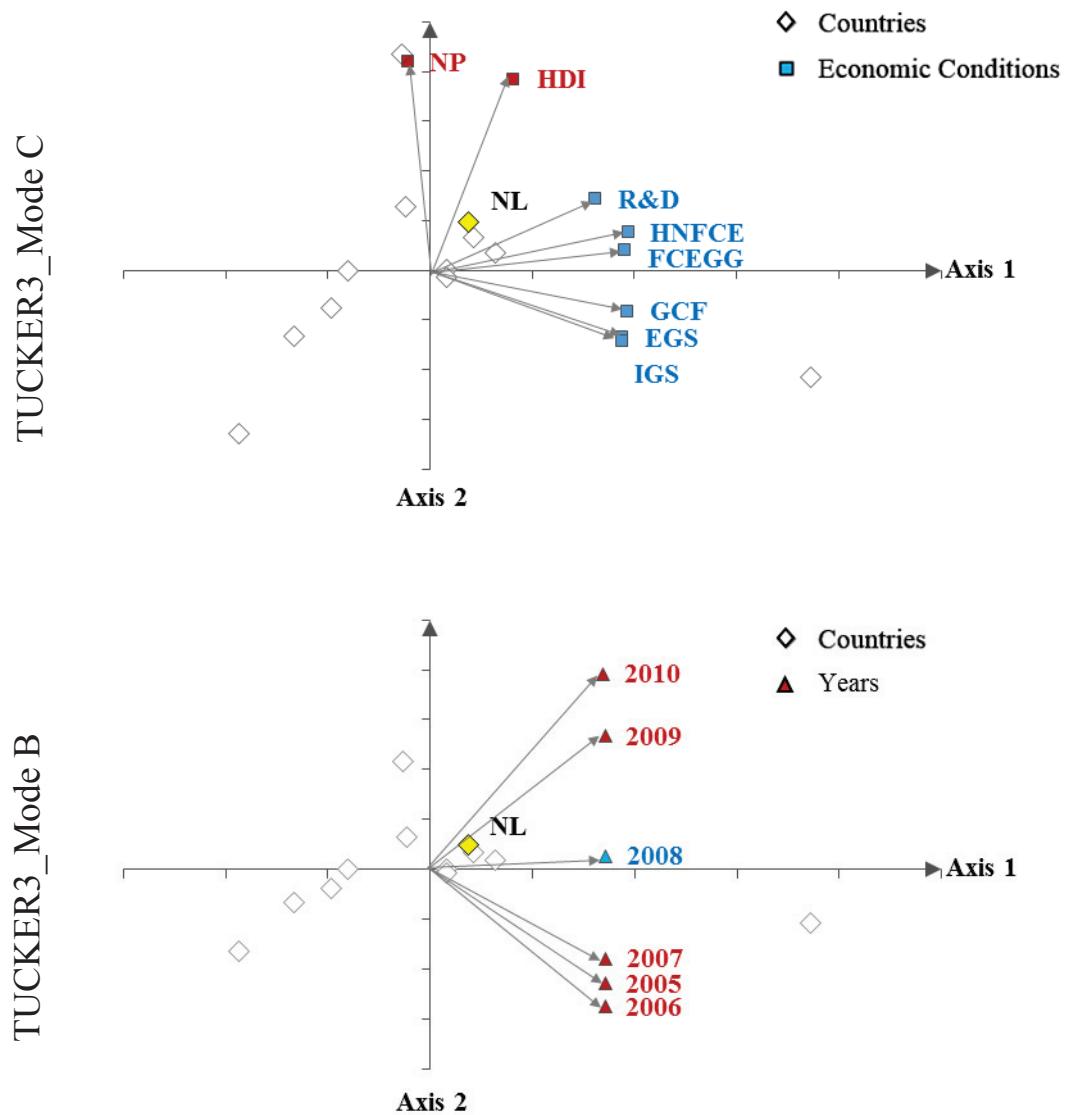
Table 4.53 – The Netherland’s Performance from STATIS and MFA point of view

NETHERLANDS	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was FCEGG, and the one that contributed the least was R&D. • there are two patterns of groups of variables (FCEGG), and (HNFCE, R&D, GCF, EGS and IGS) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the patterns was opposed between FCEGG and the other groups of variables, for the construction of the centroid, and thus to the Netherlands ’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was FCEGG follow closely by HDI, and the one that contributed the least was R&D. • there are three patterns of groups of variables (FCEGG), (HNFCE, R&D, GCF, EGS, IGS and NP) and HDI with positive correlation within each pattern and negative correlation between patterns with the exception of HDI that as an a weak positive correlation with FCEGG. • The contributions of FCEGG and HDI are similarly opposed to the one of the remaining group patterns for the construction of the centroid, and thus to the Netherlands’s centroid positioning.

Source: own elaboration

Figure 4.23 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Netherlands’ factorial representation.

Figure 4.23 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on the Netherlands’ factorial representation



Source: own elaboration

And Table 4.54 synthesizes the interpretation of the Figure 4.23 regarding Netherlands’ performance.

Table 4.54 – The Netherland's Performance from TUCKER3 point of view

THE NETHERLANDS	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • The Netherland's performance was more positively associated with years 2005-2007 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.55 the coordinates of Finland are presented for STATIS and MFA respectively.

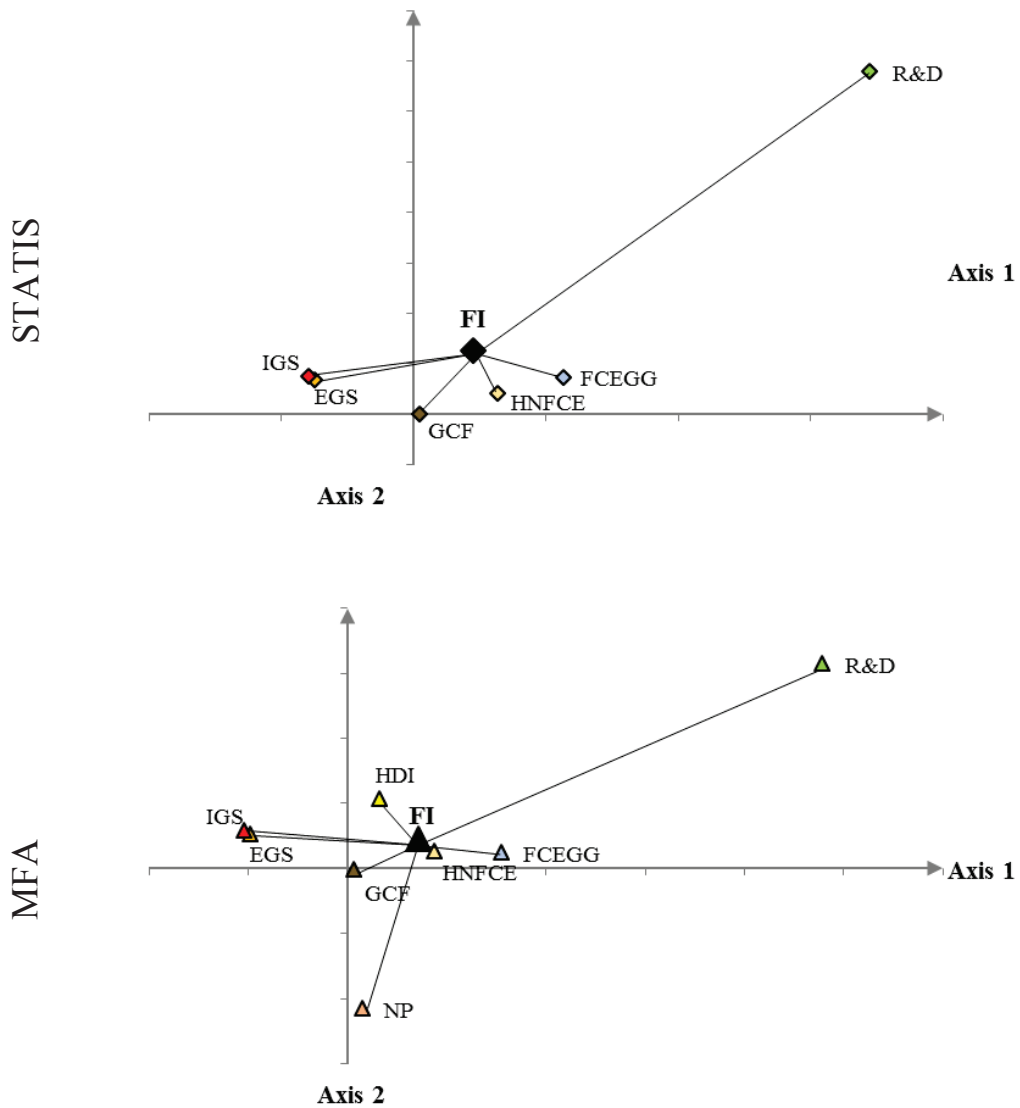
Table 4.55 – Compromise/consensus coordinates of economic growth and development for Finland

	Axis 1	Axis 2	
S T A T I S	0.451	1.242	FI
	0.638	0.398	HNFCE
	0.047	-0.006	GCF
	1.134	0.733	FCEGG
	-0.743	0.675	EGS
	-0.789	0.744	IGS
	3.450	6.782	R&D
M F A	0.719	0.455	FI
	0.882	0.265	HNFCE
	0.068	-0.019	GCF
	1.551	0.244	FCEGG
	-0.975	0.514	EGS
	-1.032	0.575	IGS
	4.792	3.146	R&D
	0.153	-2.158	NP
0.315	1.070	HDI	

Source: own elaboration

Figure 4.24 graphically illustrate Tables 4.55.

Figure 4.24 – Comparison of STATIS and MFA: Projection of economic growth and development on Finland’s factorial representation



Source: own elaboration

Table 4.56 synthesizes the interpretation of these two techniques, regarding Finland’s performance.

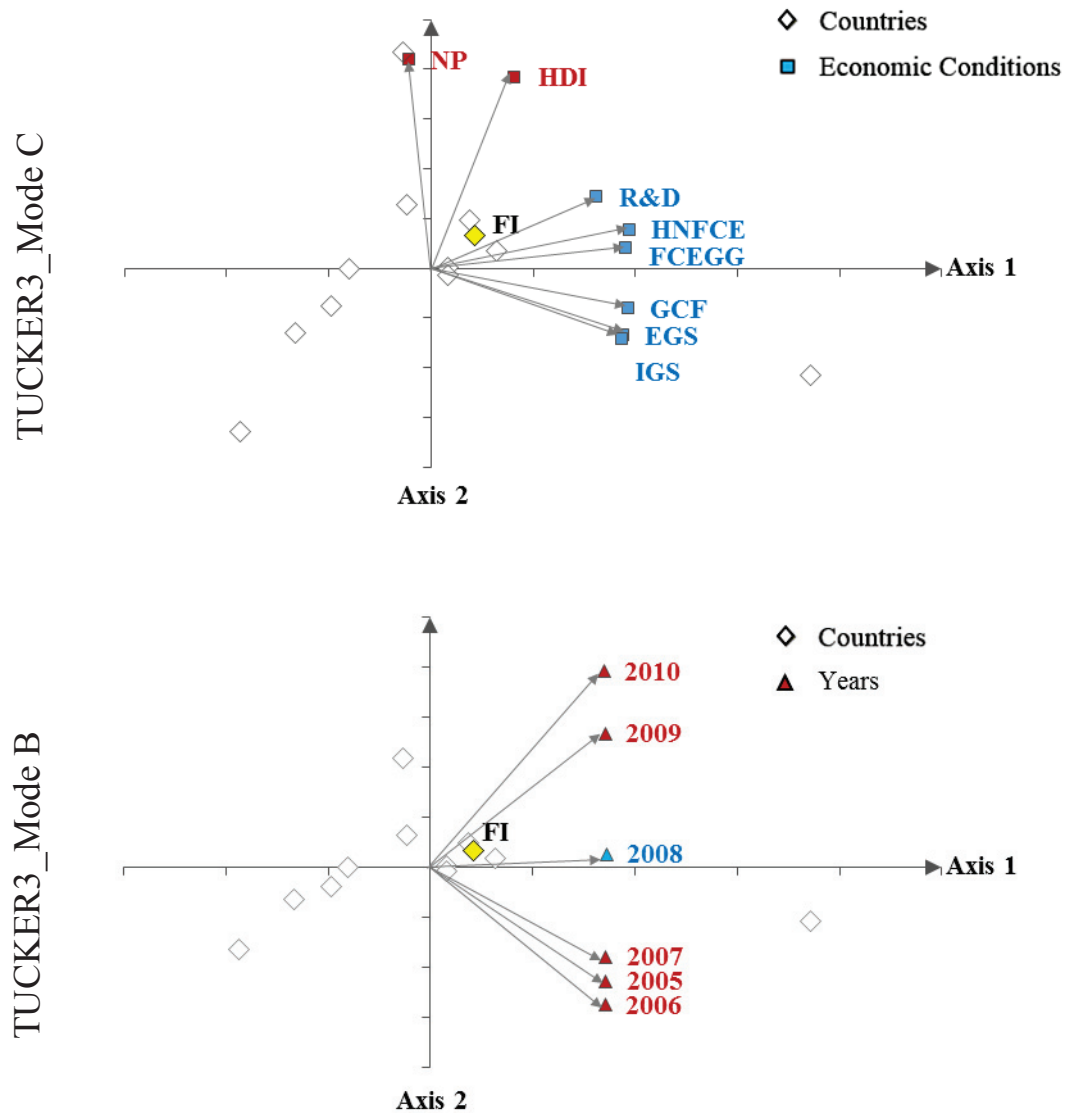
Table 4.56 – Finland’s Performance from STATIS and MFA point of view

FINLAND	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid were R&D, and the one that contributed the least was HNFCE. • there are three patterns of groups of variables (R&D), (FCEGG and HNFCE) and (GCF, EGS and IGS) with positive correlation within each pattern and negative correlation between patterns, with the exceptions of R&D with FCEGG, and HNFCE with GCF, which have weak positive correlation. • the contribution of the patterns was most notoriously opposed between R&D and GCF, for the construction of the centroid, and thus to Finland’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was R&D and the one that contributed the least was HNFCE. • there are three patterns of groups of variables (R&D, FCEGG and HNFCE), (EGS and IGS and HDI) and (NP and GCF) with positive correlation within each pattern and predominantly with negative correlation between patterns, with the exceptions which have weak positive correlation with the (NP and HDI) group and (GCF, EGS and IGS) group respectively. • The contribution of the R&D and GCF groups was the most opposed for the construction of the centroid, and thus to Finland’s centroid positioning.

Source: own elaboration

Figure 4.25 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Finland’s factorial representation.

Figure 4.25 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Finland’s factorial representation



Source: own elaboration

And Table 4.57 synthesizes the interpretation of the Figure 4.25 regarding Finland’s performance.

Table 4.57 – Finland’s Performance from TUCKER3 point of view

FINLAND	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Finland’s performance was more positively associated with years 2005-2007 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.58 the coordinates of Austria are presented for STATIS and MFA respectively.

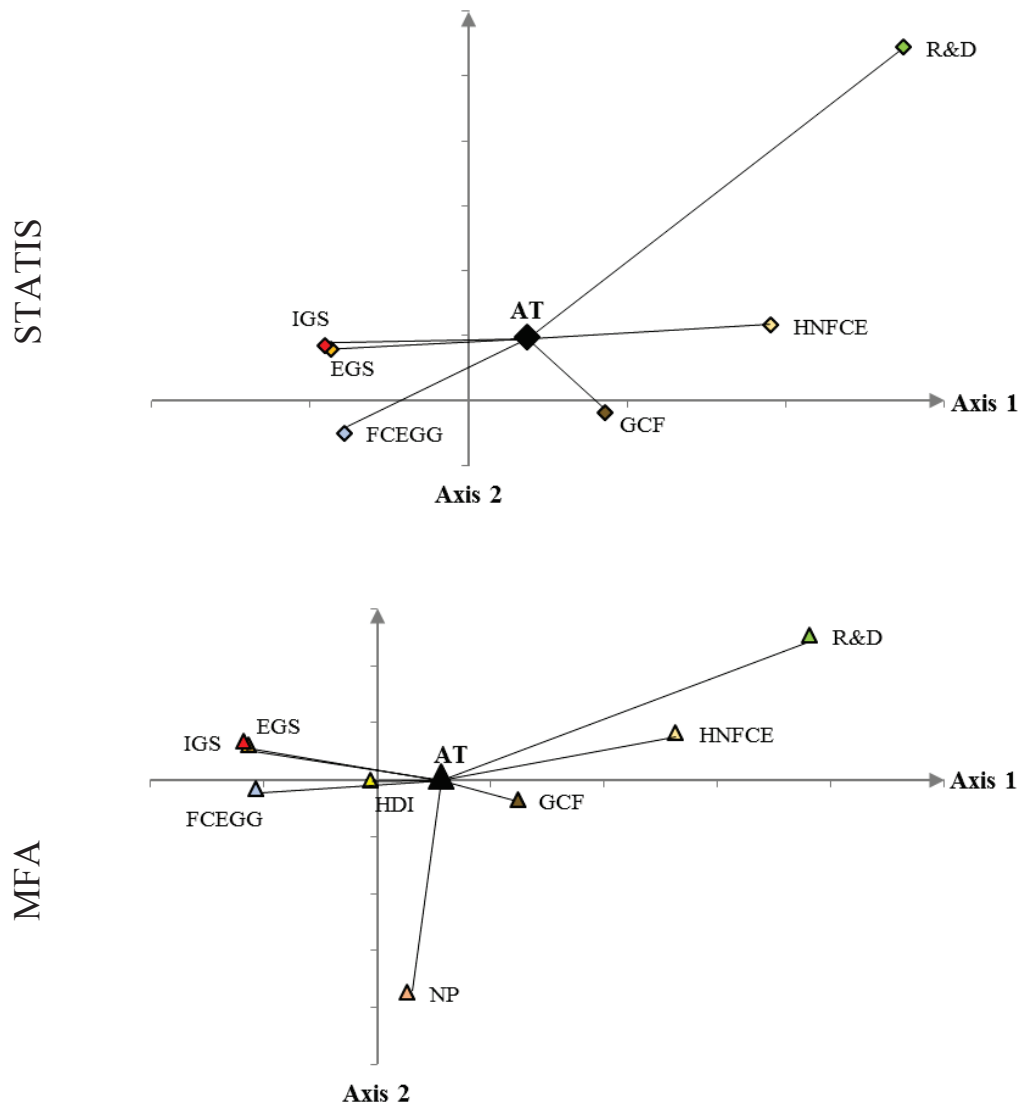
Table 4.58 – Compromise/consensus coordinates of economic growth and development for Austria

	Axis 1	Axis 2	
S T A T I S	0.184	0.483	AT
	0.952	0.579	HNFCE
	0.431	-0.099	GCF
	-0.389	-0.251	FCEGG
	-0.434	0.394	EGS
	-0.450	0.425	IGS
	1.371	2.716	R&D
M F A	0.281	0.020	AT
	1.317	0.404	HNFCE
	0.619	-0.179	GCF
	-0.532	-0.084	FCEGG
	-0.569	0.300	EGS
	-0.589	0.329	IGS
	1.906	1.260	R&D
	0.133	-1.871	NP
	-0.035	-0.003	HDI

Source: own elaboration

Figure 4.26 graphically illustrate Tables 4.58.

Figure 4.26 – Comparison of STATIS and MFA: Projection of economic growth and development on Austria’s factorial representation



Source: own elaboration

Table 4.59 synthesizes the interpretation of these two techniques, regarding Austria’s performance.

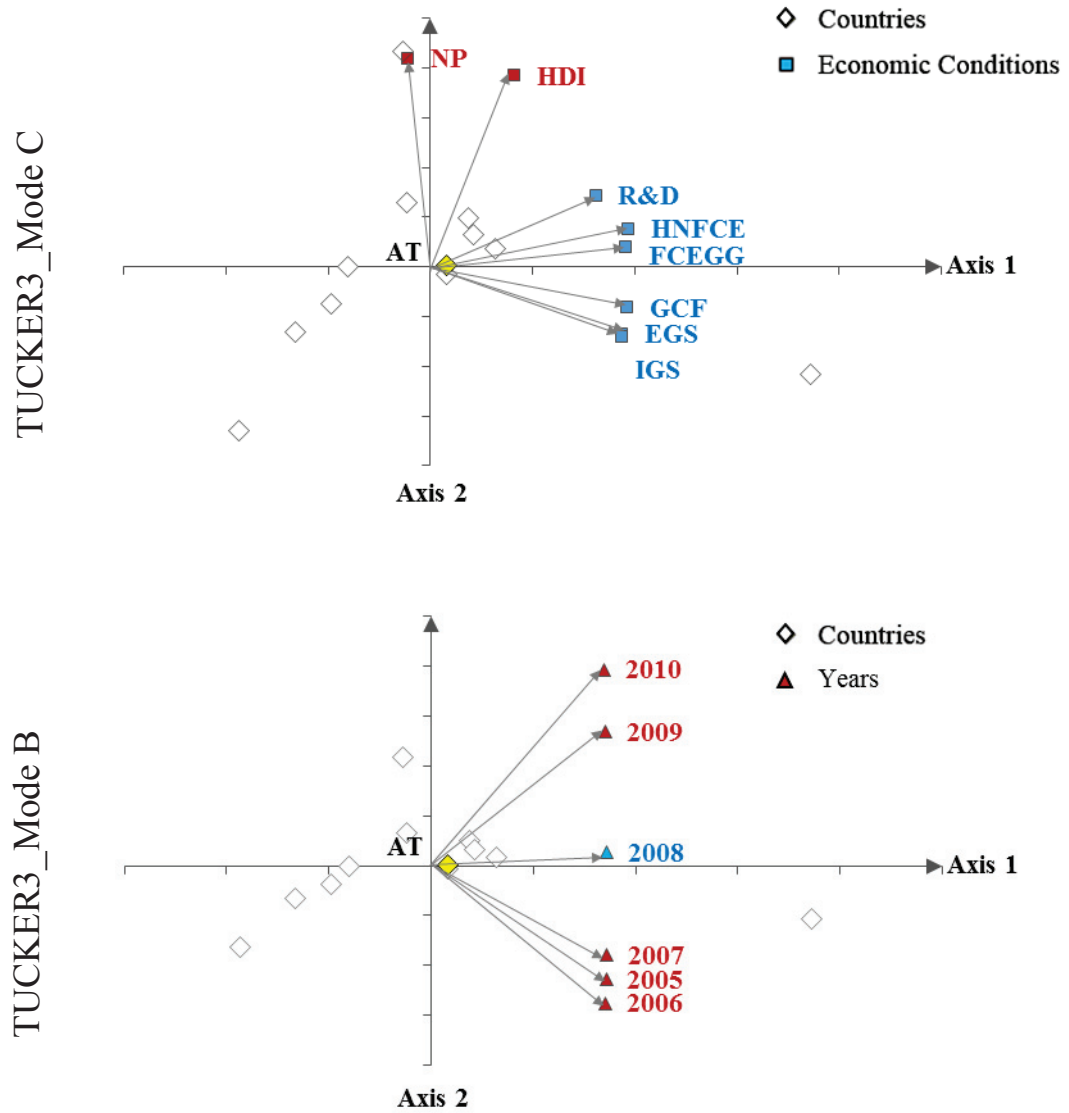
Table 4.59 – Austria’s Performance from STATIS and MFA point of view

AUSTRIA	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid were R&D, and the one that contributed the least was GCF. • there are three patterns of groups of variables (R&D), (HNFCE and GCF) and (FCEGG, EGS and IGS) with positive correlation within each pattern and negative correlation between patterns, with the exceptions of R&D with HNFCE, which have a weak positive correlation. • the contribution of the patterns was most notoriously opposed between R&D and FCEGG, and also between IGS-EGS and HNFCE, for the construction of the centroid, and thus to Austria’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was R&D and the one that contributed the least was HDI. • there are three patterns of groups of variables (GCF, HNFCE and R&D), (EGS, IGS FCEGG and HDI) and (NP) with positive correlation within each pattern and predominantly with negative correlation between patterns, with the exception of NP, that has a weak positive correlation with the HDI, FCEGG and GCF groups. • The contributions of the FCEGG and HDI groups in relation to R&D was the most opposed in the construction of the centroid, and thus to Austria’s centroid positioning.

Source: own elaboration

Figure 4.27 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Austria’s factorial representation.

Figure 4.27 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Austria’s factorial representation



Source: own elaboration

And Table 4.60 synthesizes the interpretation of the Figure 4.27 regarding Austria’s performance.

Table 4.60 – Austria's Performance from TUCKER3 point of view

AUSTRIA	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Austria's performance was more positively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.61 the coordinates of Belgium are presented for STATIS and MFA respectively.

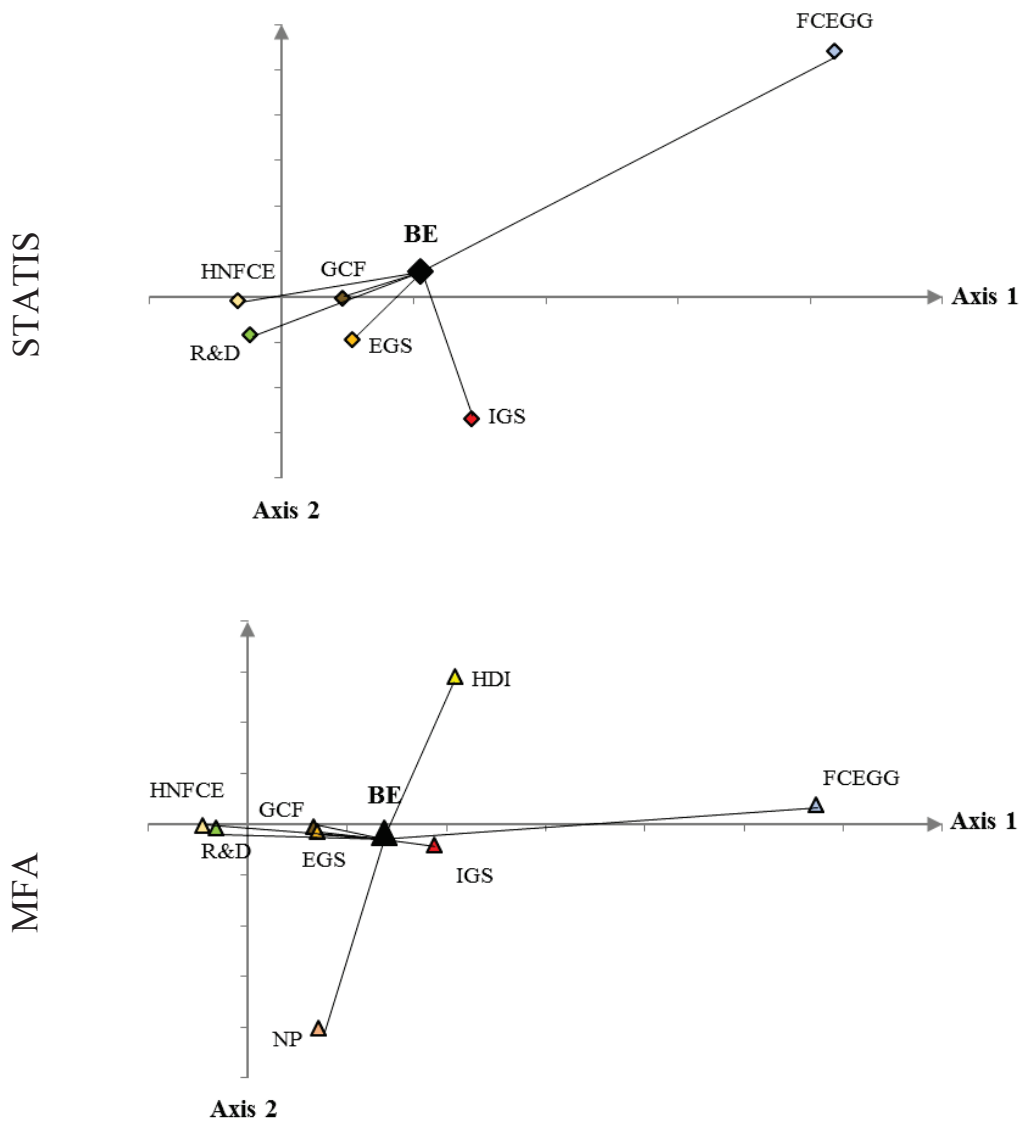
Table 4.61 – Compromise/consensus coordinates of economic growth and development for Belgium

	Axis 1	Axis 2	
S T A T I S	0.211	0.054	BE
	-0.066	-0.011	HNFCE
	0.093	-0.005	GCF
	0.837	0.540	FCEGG
	0.108	-0.096	EGS
	0.287	-0.270	IGS
	-0.046	-0.086	R&D
M F A	0.275	-0.095	BE
	-0.091	-0.019	HNFCE
	0.133	-0.035	GCF
	1.144	0.180	FCEGG
	0.141	-0.074	EGS
	0.376	-0.209	IGS
	-0.064	-0.039	R&D
	0.143	-2.019	NP
0.417	1.455	HDI	

Source: own elaboration

Figure 4.28 graphically illustrate Tables 4.61.

Figure 4.28 – Comparison of STATIS and MFA: Projection of economic growth and development on Belgian’s factorial representation



Source: own elaboration

Table 4.62 synthesizes the interpretation of these two techniques, regarding Belgium’s performance.

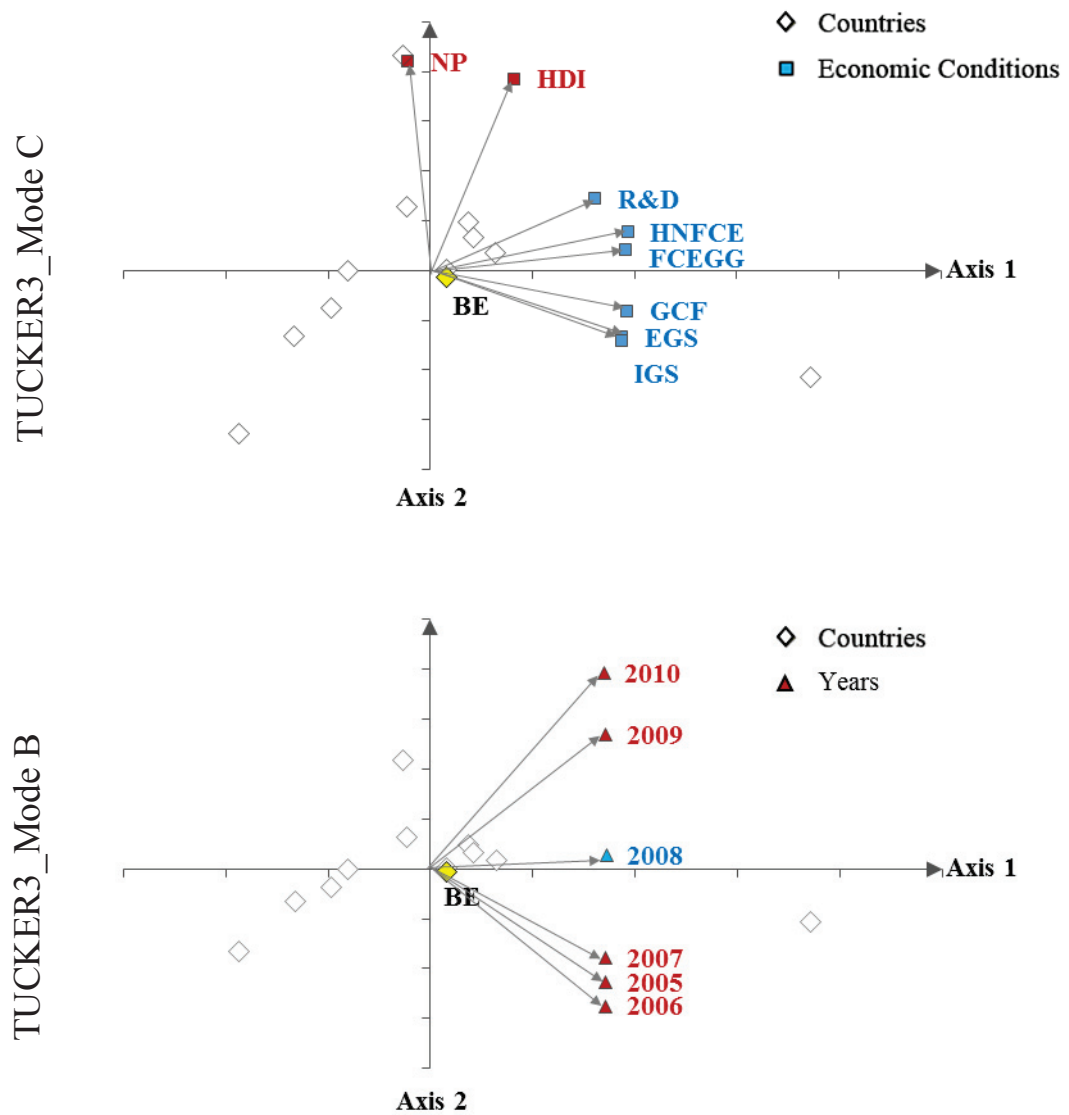
Table 4.62 – Belgium Performance from STATIS and MFA point of view

BELGIUM	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was FCEGG, and the one that contributed the least was GCF. • there are three patterns of groups of variables (FCEGG), and (HNFCE, R&D, GCF, and EGS) and (IGS) with positive correlation within each pattern and negative correlation between patterns. The only exception is the weak positive correlation between IGS and EGS. • the contribution of the patterns was most notoriously opposed between FCEGG and the R&D, for the construction of the centroid, and thus to Belgium's centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was FCEGG, and the one that contributed the least was IGS • there are four patterns of groups of variables (FCEGG and IGS), (HNFCE, R&D, GCF and EGS), (NP) and HDI with positive correlation within each pattern and negative correlation between patterns with the exception of HDI and NP that as an a weak positive correlation with FCEGG and the (HNFCE, R&D, GCF and EGS) pattern respectively. • The contributions of (FCEGG, IGS) and HDI are similarly opposed to (HNFCE, R&D, GCF and EGS) and NP respectively, for the construction of the centroid, and thus to the Netherlands's centroid positioning.

Source: own elaboration

Figure 4.29 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Belgian's factorial representation.

Figure 4.29 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Belgian’s factorial representation



Source: own elaboration

And Table 4.63 synthesizes the interpretation of the Figure 4.29 regarding Belgian’s performance.

Table 4.63 – Belgium Performance from TUCKER3 point of view

BELGIUM	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. Belgium's performance was more positively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.64 the coordinates of Germany are presented for STATIS and MFA respectively.

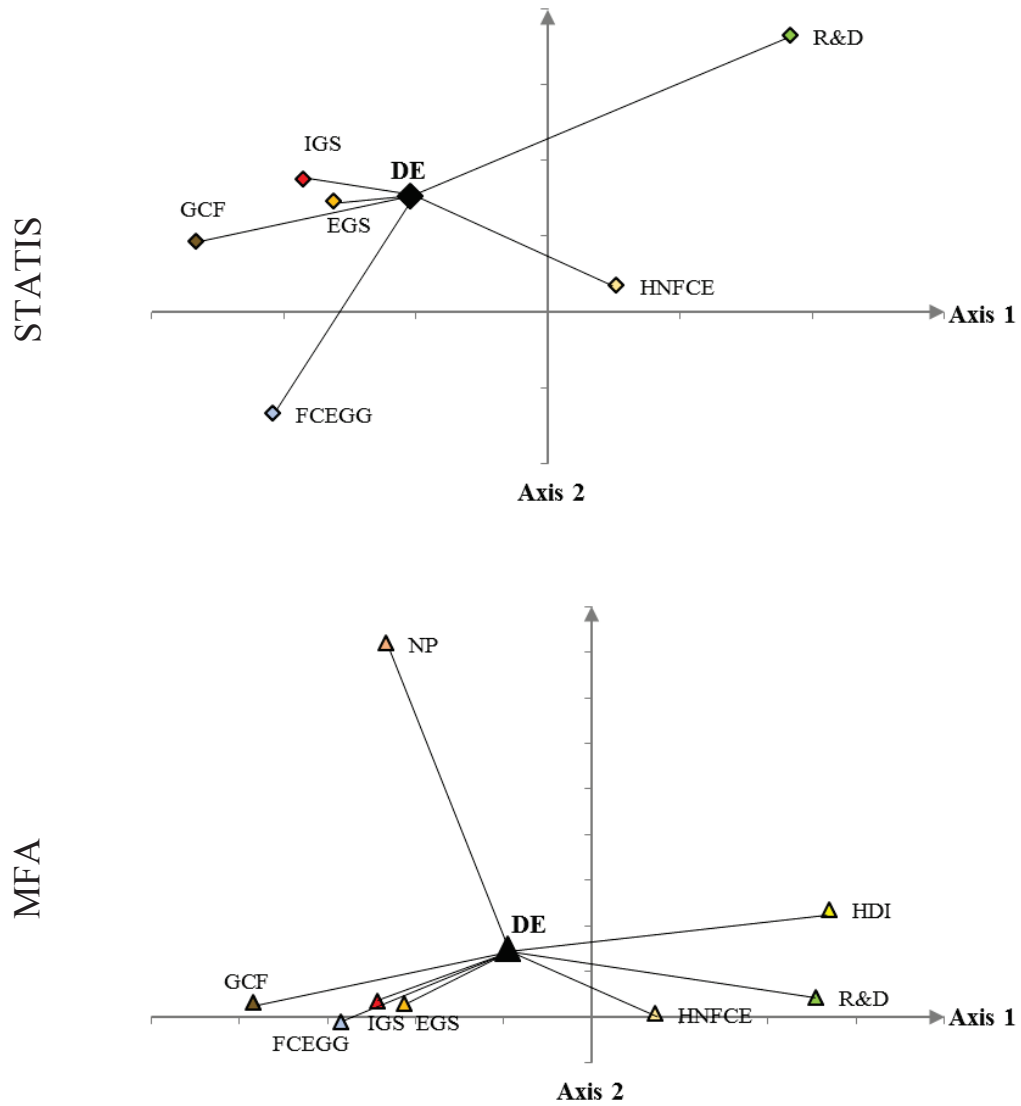
Table 4.64 – Compromise/consensus coordinates of economic growth and development for Germany

	Axis 1	Axis 2	
S T A T I S	-0.519	0.764	DE
	0.262	0.171	HNFCE
	-1.329	0.465	GCF
	-1.038	-0.671	FCEGG
	-0.808	0.732	EGS
	-0.926	0.874	IGS
	0.919	1.822	R&D
M F A	-0.473	2.957	DE
	0.362	0.122	HNFCE
	-1.918	0.600	GCF
	-1.419	-0.224	FCEGG
	-1.060	0.559	EGS
	-1.212	0.676	IGS
	1.277	0.847	R&D
	-1.163	16.386	NP
1.346	4.694	HDI	

Source: own elaboration

Figure 4.30 graphically illustrate Table 4.64.

Figure 4.30 – Comparison of STATIS and MFA: Projection of economic growth and development on Germany’s factorial representation



Source: own elaboration

Table 4.65 synthesizes the interpretation of these two techniques, regarding Germany’s performance.

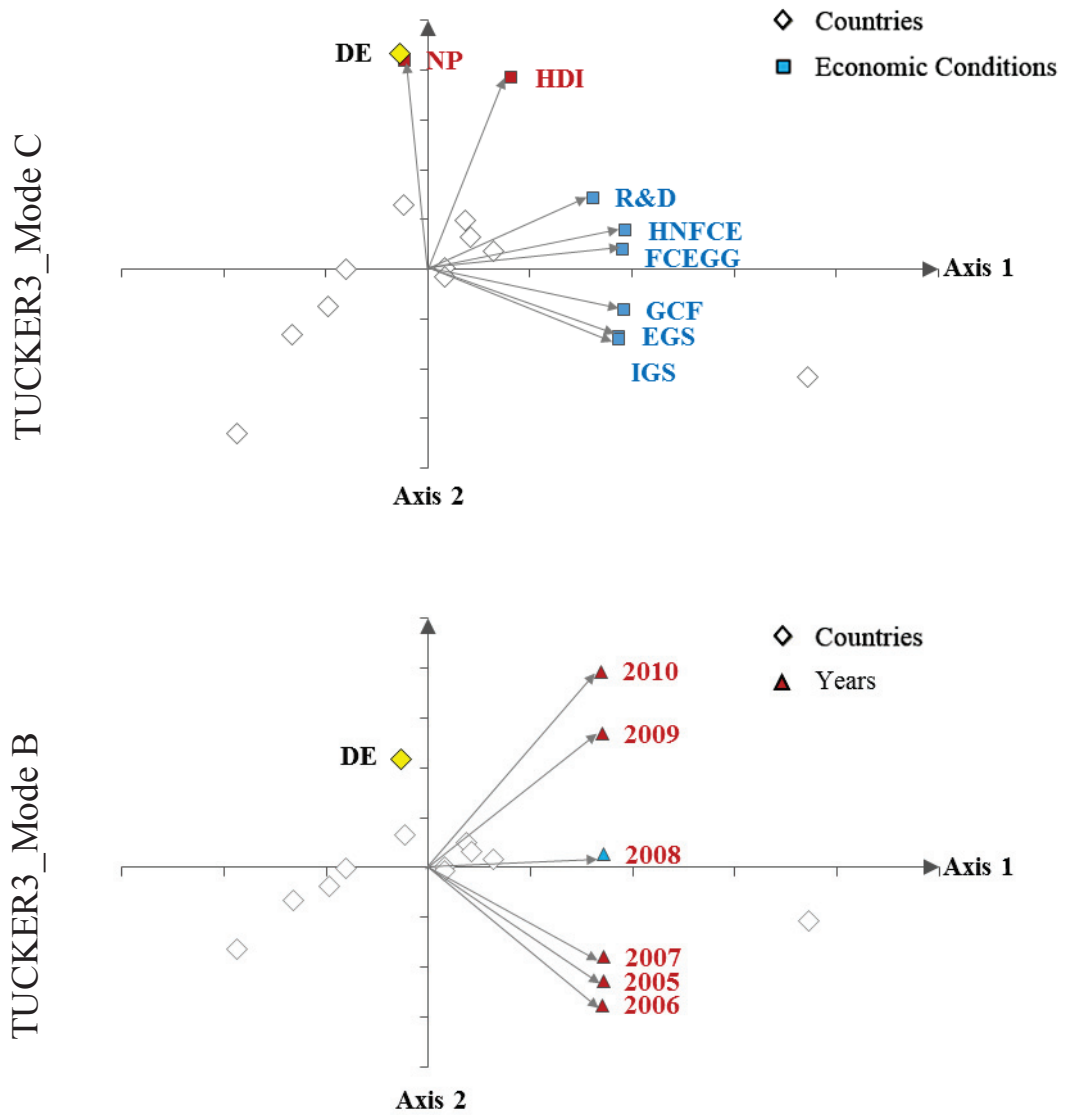
Table 4.65 – Germany’s Performance from STATIS and MFA point of view

GERMANY	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was EGS. • there are two patterns of groups of variables (FCEGG, GCF, IGS and EGS), and (HNFCE, R&D) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the two patterns of groups of variables was opposed, for the construction of the centroid, and thus to Germany’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was NP, and the one that contributed the least was EGS • there are three patterns of groups of variables (FCEGG, GCF, EGS and IGS), (HNFCE, R&D and HDI), and (NP) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to Germany’s centroid positioning.

Source: own elaboration

Figure 4.31 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Germany’s factorial representation.

Figure 4.31 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Germany’s factorial representation



Source: own elaboration

And Table 4.66 synthesizes the interpretation of the Figure 4.31 regarding Germany’s performance.

Table 4.66 – Germany’s Performance from TUCKER3 point of view

GERMANY	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Germany's performance was more negatively associated with years 2005-2007 and with HDI and NP.

Source: own elaboration

In Table 4.67 the coordinates of France are presented for STATIS and MFA, respectively.

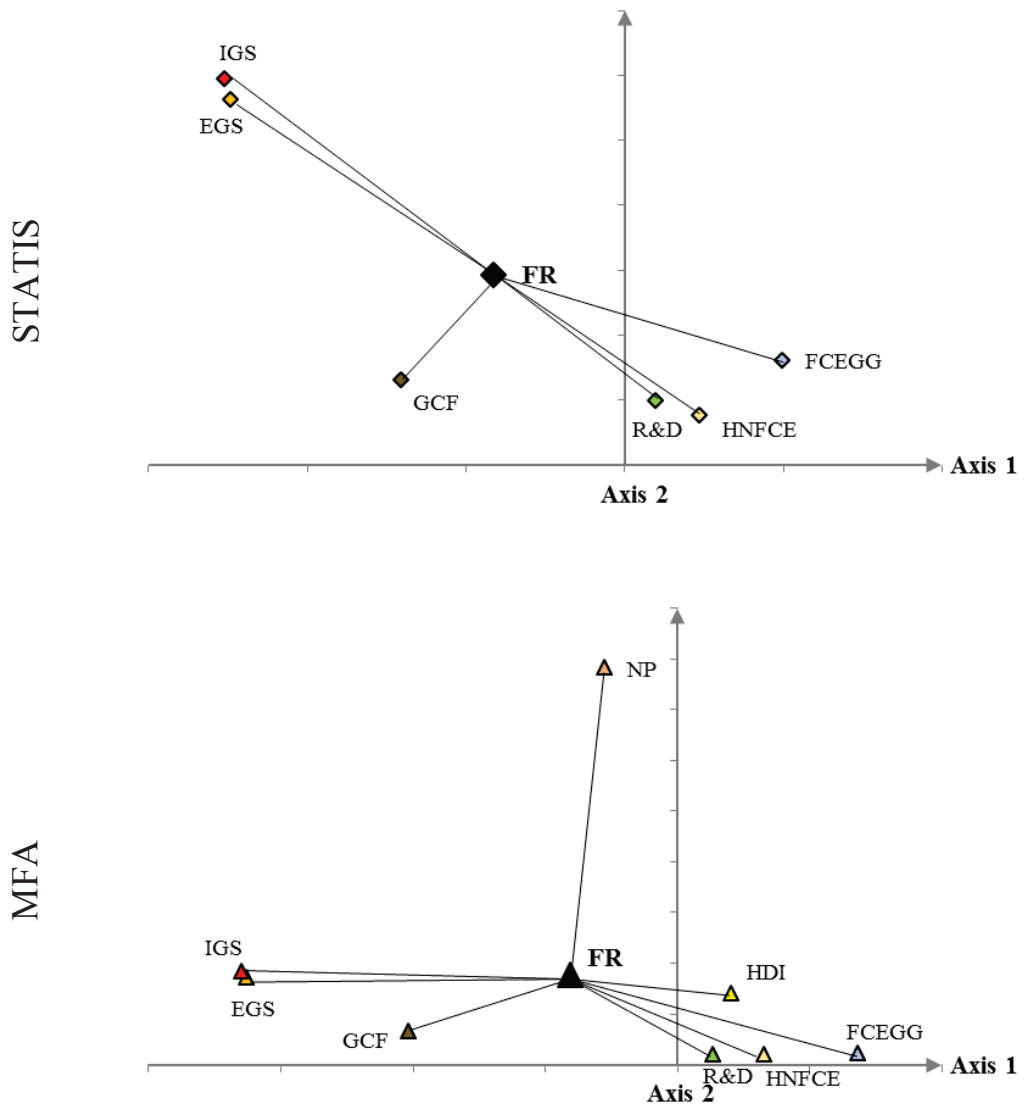
Table 4.67 – Compromise/consensus coordinates of economic growth and development for France

	Axis 1	Axis 2	
S T A T I S	-0.411	0.585	FR
	0.236	0.152	HNFCE
	-0.705	0.260	GCF
	0.499	0.322	FCEGG
	-1.242	1.126	EGS
	-1.261	1.189	IGS
	0.098	0.199	R&D
M F A	-0.403	0.878	FR
	0.327	0.104	HNFCE
	-1.018	0.323	GCF
	0.683	0.111	FCEGG
	-1.629	0.860	EGS
	-1.649	0.920	IGS
	0.137	0.096	R&D
	-0.277	3.909	NP
	0.202	0.700	HDI

Source: own elaboration

Figure 4.32 graphically illustrate Table 4.67.

Figure 4.32 – Comparison of STATIS and MFA: Projection of economic growth and development on France’s factorial representation



Source: own elaboration

Table 4.68 synthesizes the interpretation of these two techniques, regarding France’s performance.

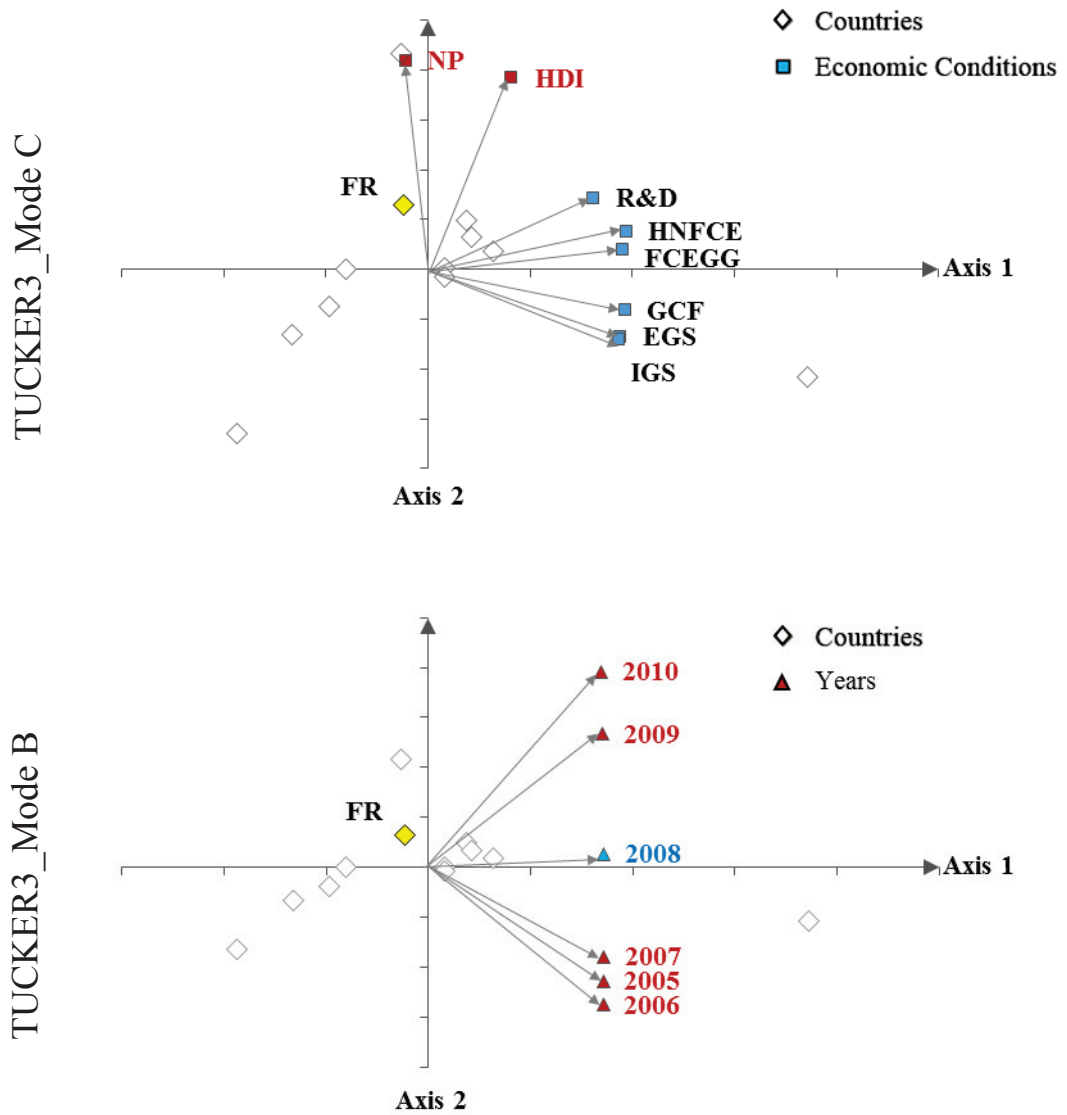
Table 4.68 – France’s Performance from STATIS and MFA point of view

FRANCE	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was IGS, and the one that contributed the least was GCF. • there are three patterns of groups of variables (IGS and EGS), and (HNFCE, FCEGG, R&D), and (GCF) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to France’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was NP, and the one that contributed the least was HDI. • there are three patterns of groups of variables (GCF, IGS and EGS), and (HNFCE, FCEGG, R&D and HDI), and (NP) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to France’s centroid positioning.

Source: own elaboration

Figure 4.33 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on France’s factorial representation.

Figure 4.33 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on France’s factorial representation



Source: own elaboration

And Table 4.69 synthesizes the interpretation of the Figure 4.33 regarding France’s performance.

Table 4.69 – France’s Performance from TUCKER3 point of view

FRANCE	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • France's performance was more negatively associated with years 2005-2007 and with HDI and NP.

Source: own elaboration

In Table 4.70 the coordinates of Italy are presented for STATIS and MFA, respectively.

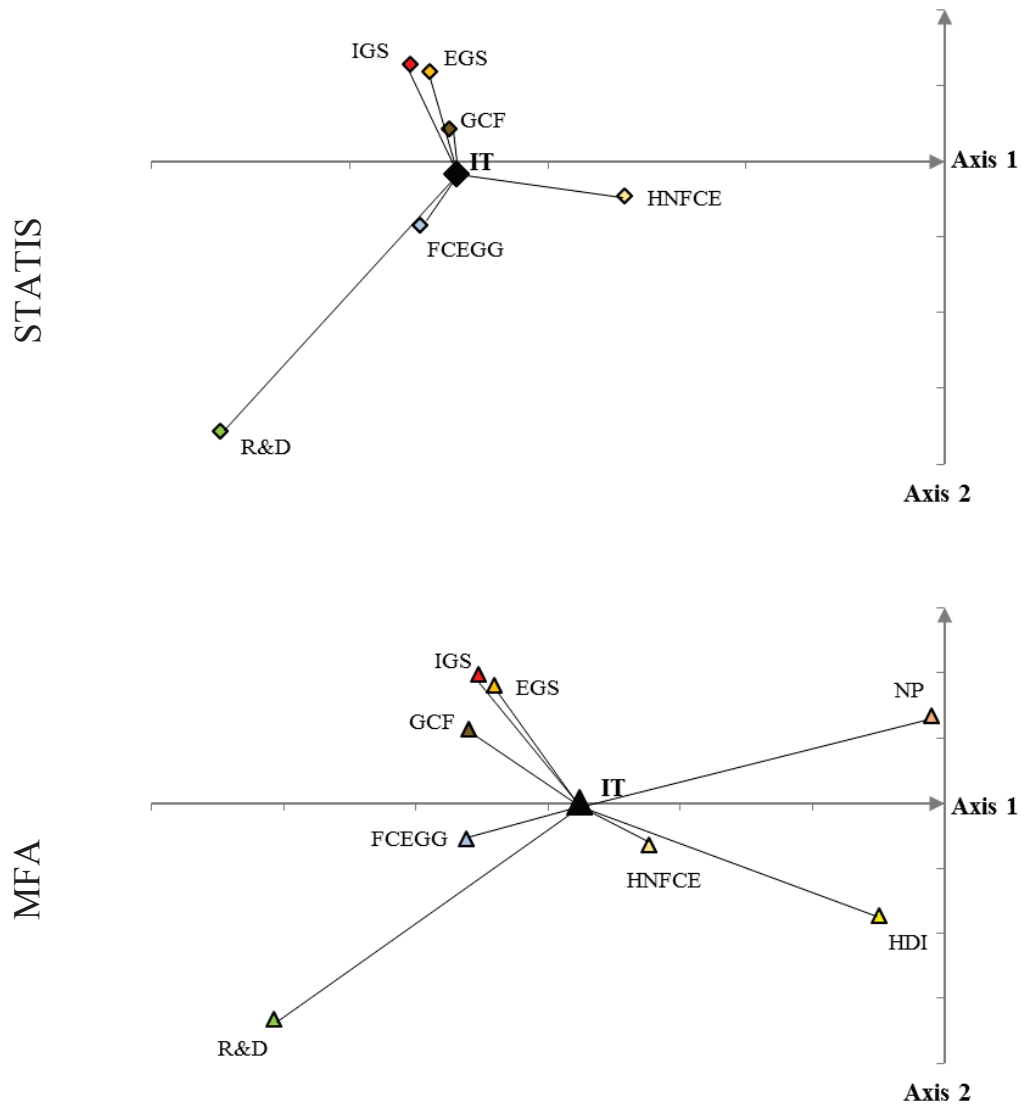
Table 4.70 – Compromise/consensus coordinates of economic growth and development for Italy

	Axis 1	Axis 2	
S T A T I S	-1.228	-0.176	IT
	-0.805	-0.467	HNFCE
	-1.247	0.427	GCF
	-1.321	-0.851	FCEGG
	-1.298	1.176	EGS
	-1.347	1.270	IGS
	-1.824	-3.579	R&D
M F A	-1.376	-0.003	IT
	-1.114	-0.325	HNFCE
	-1.799	0.561	GCF
	-1.807	-0.282	FCEGG
	-1.702	0.898	EGS
	-1.762	0.982	IGS
	-2.534	-1.660	R&D
	-0.047	0.667	NP
	-0.247	-0.864	HDI

Source: own elaboration

Figure 4.34 graphically illustrate Table 4.70.

Figure 4.34 – Comparison of STATIS and MFA: Projection of economic growth and development on Italy's factorial representation



Source: own elaboration

In Table 4.71 a synthesis of the interpretation for these two techniques is presented, regarding Italy's performance.

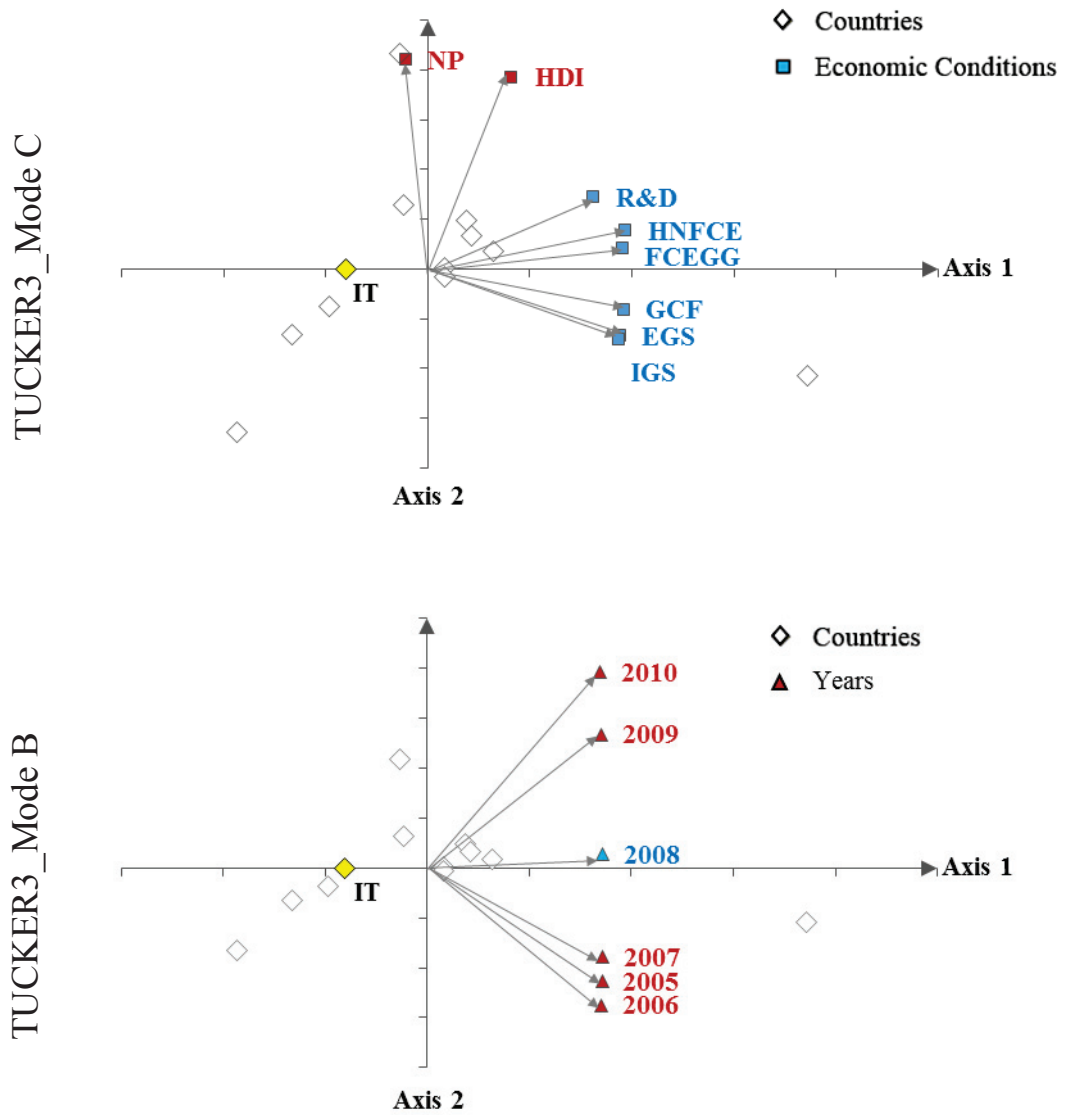
Table 4.71 – Italy’s Performance from STATIS and MFA point of view

ITALY	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was GCF. • there are three patterns of groups of variables (GCF, IGS and EGS), (FCEGG and R&D), and (HNFCE) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to Italy’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was HNFCE. • there are four patterns of groups of variables (GCF, IGS and EGS), and (HNFCE and HDI), (FCEGG and R&D) and (NP) with positive correlation within each pattern and negative correlation between patterns (FCEGG and R&D) and NP and between patterns (GCF, IGS and EGS), and (HNFCE and HDI). • the contributions most notoriously opposed were between NP and FCEGG and also between GCF and HNFCE, for the construction of the centroid, and thus to Italy’s centroid positioning.

Source: own elaboration

Figure 4.35 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Italy’s factorial representation.

Figure 4.35 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Italy’s factorial representation



Source: own elaboration

And Table 4.72 synthesizes the interpretation of the Figure 4.35 regarding Italy’s performance.

Table 4.72 – Italy’s Performance from TUCKER3 point of view

ITALY	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Italy's performance was more negatively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.73 the coordinates of Spain are presented for STATIS and MFA, respectively.

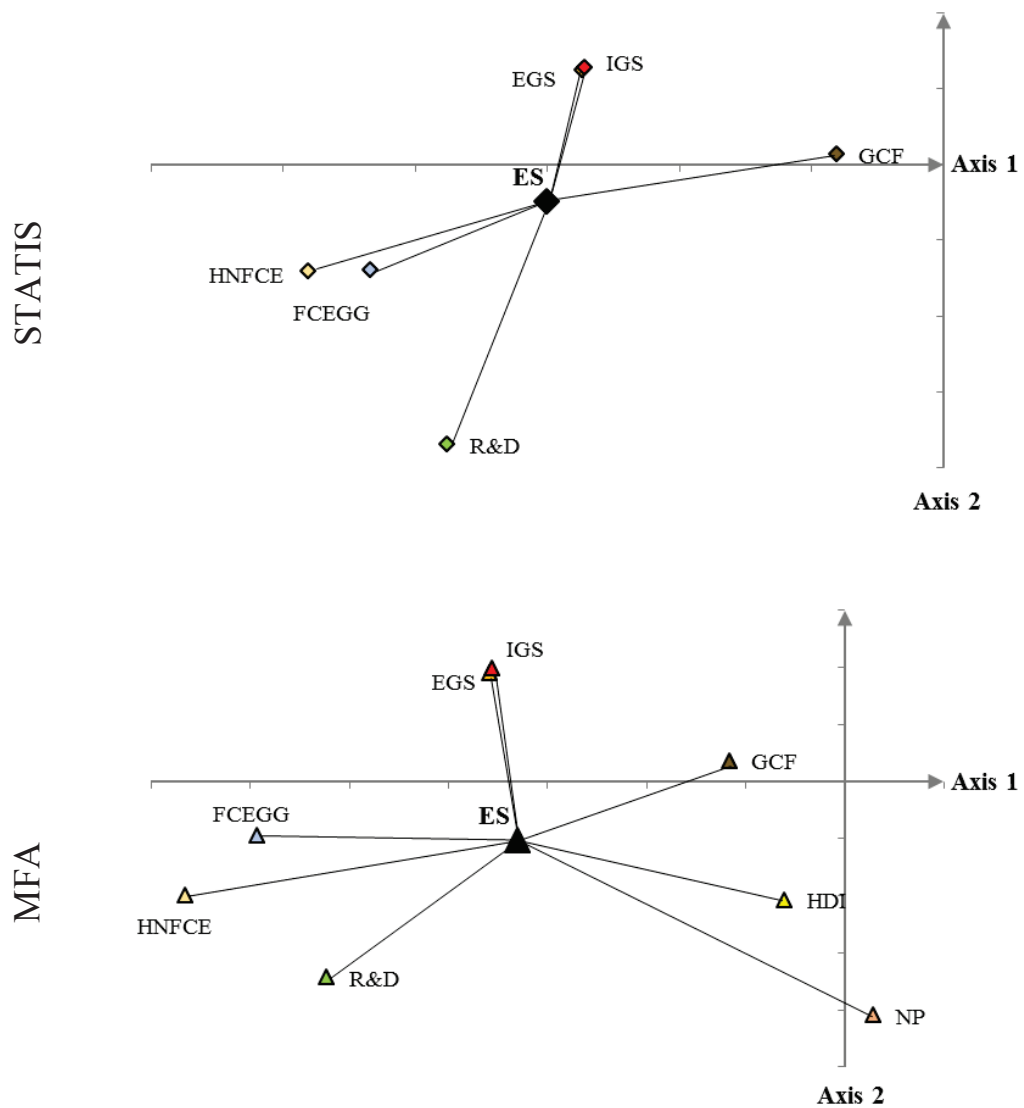
Table 4.73 – Compromise/consensus coordinates of economic growth and development for Spain

	Axis 1	Axis 2	
S T A T I S	-1.502	-0.497	ES
	-2.403	-1.405	HNFCE
	-0.404	0.130	GCF
	-2.169	-1.401	FCEGG
	-1.367	1.238	EGS
	-1.359	1.282	IGS
	-1.881	-3.693	R&D
M F A	-1.652	-0.518	ES
	-3.324	-0.995	HNFCE
	-0.581	0.182	GCF
	-2.966	-0.471	FCEGG
	-1.792	0.946	EGS
	-1.779	0.991	IGS
	-2.613	-1.716	R&D
	0.145	-2.047	NP
-0.305	-1.036	HDI	

Source: own elaboration

Figure 4.36 graphically illustrate Table 4.73.

Figure 4.36 – Comparison of STATIS and MFA: Projection of economic growth and development on Spain’s factorial representation



Source: own elaboration

Table 4.74 synthesizes the interpretation of these two techniques, regarding Spain’s performance.

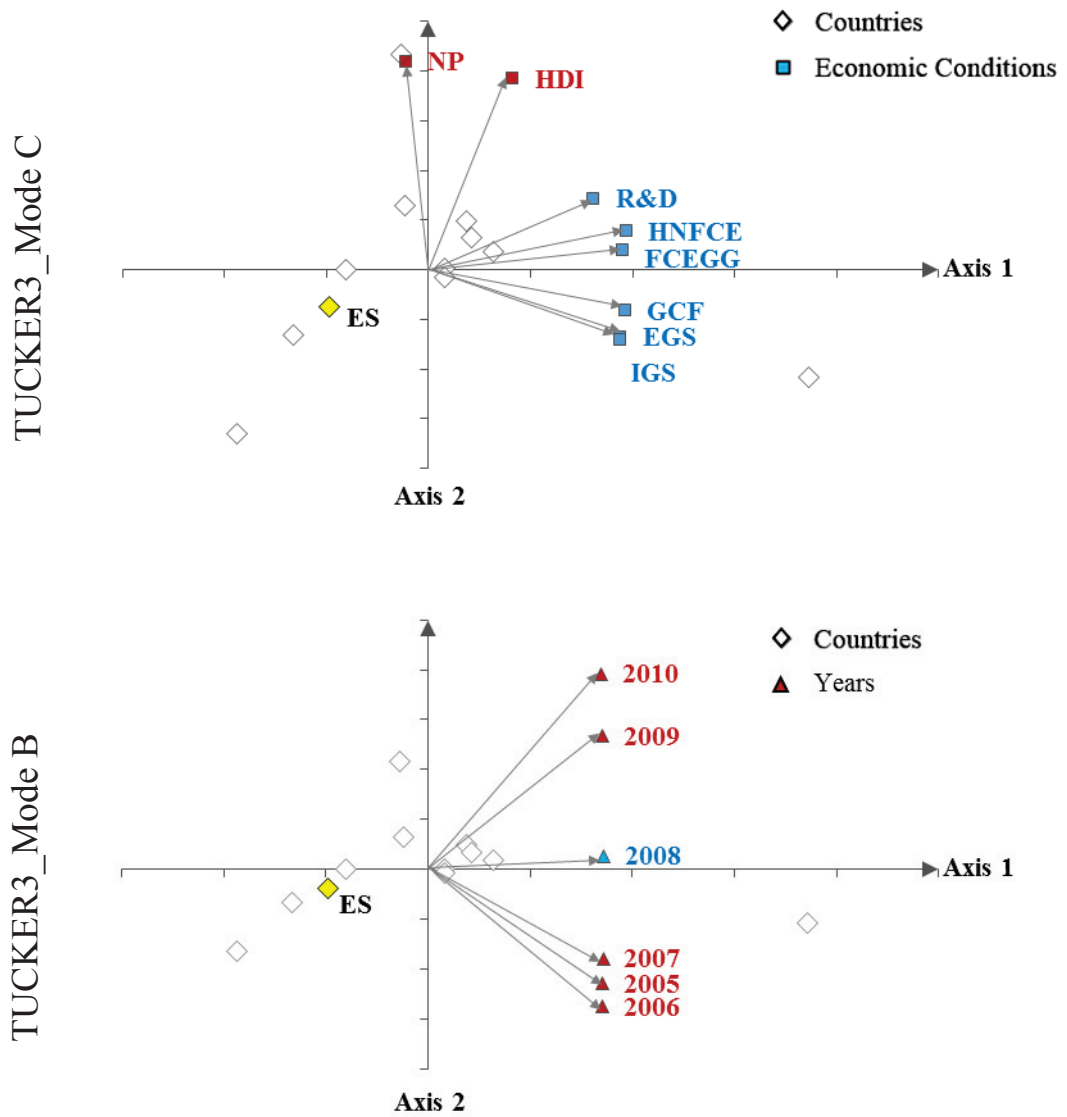
Table 4.74 – Spain’s Performance from STATIS and MFA point of view

SPAIN	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was EGS. • there are two patterns of groups of variables (GCF, IGS and EGS), and (HNFCE, FCEGG and R&D), with positive correlation within each pattern and negative correlation between patterns. • the contribution of the two patterns of groups of variables was opposed, for the construction of the centroid, and thus to Spain’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was NP, and the one that contributed the least was EGS. • there are three patterns of groups of variables (GCF, IGS and EGS), and (HNFCE, FCEGG and R&D), and (HDI and NP) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to Spain’s centroid positioning.

Source: own elaboration

Figure 4.37 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Spain’s factorial representation.

Figure 4.37 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Spain’s factorial representation



Source: own elaboration

And Table 4.75 synthesizes the interpretation of the Figure 4.37 regarding Spain’s performance.

Table 4.75 – Spain’s Performance from TUCKER3 point of view

SPAIN	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. Spain's performance was more negatively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

In Table 4.76 the coordinates of Greece are presented for STATIS and MFA, respectively.

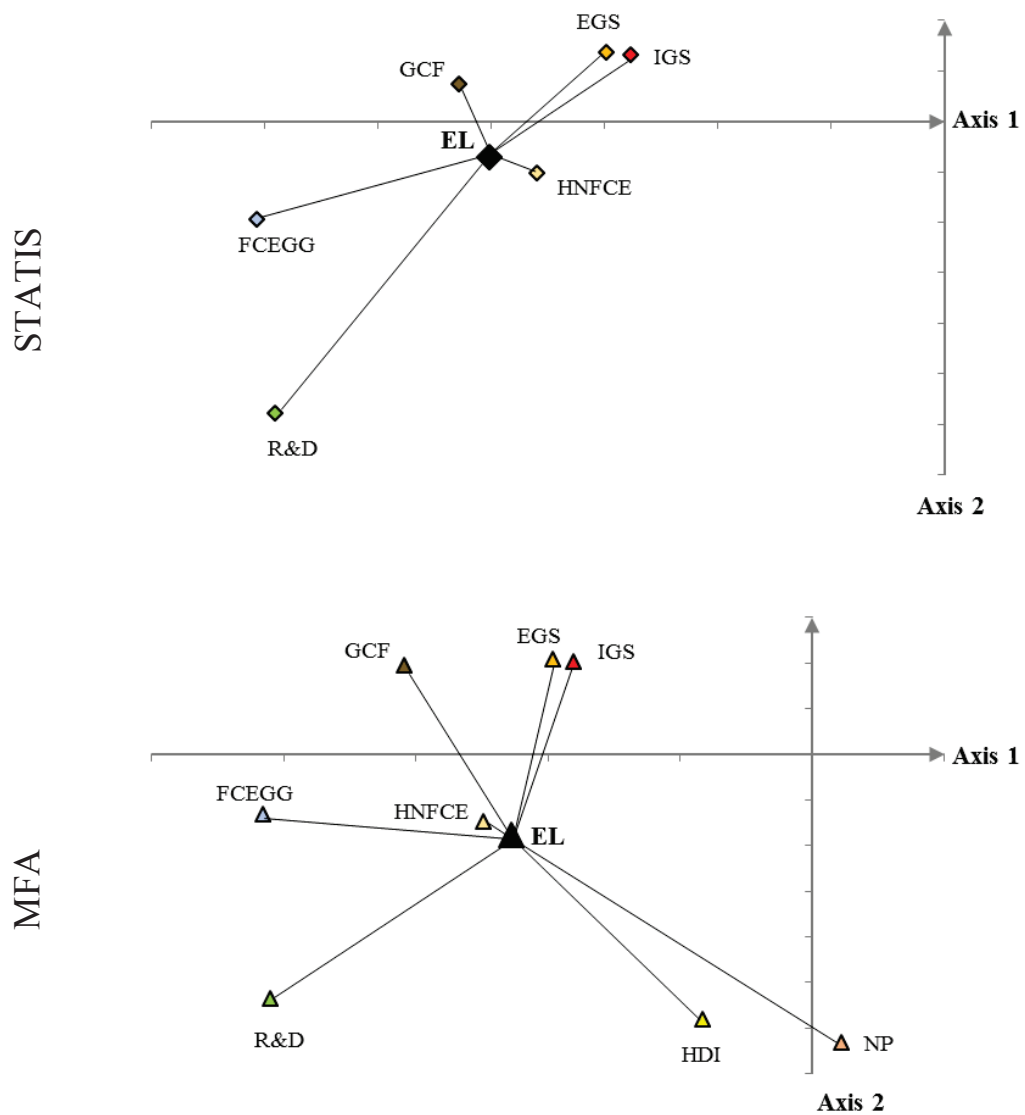
Table 4.76 – Compromise/consensus coordinates of economic growth and development for Greece

	Axis 1	Axis 2	
S T A T I S	-2.006	-0.720	EL
	-1.795	-1.026	HNFCE
	-2.143	0.719	GCF
	-3.035	-1.959	FCEGG
	-1.491	1.351	EGS
	-1.381	1.302	IGS
	-2.952	-5.795	R&D
M F A	-2.274	-0.894	EL
	-2.484	-0.747	HNFCE
	-3.088	0.963	GCF
	-4.150	-0.658	FCEGG
	-1.956	1.032	EGS
	-1.807	1.007	IGS
	-4.101	-2.688	R&D
	0.224	-3.159	NP
-0.829	-2.904	HDI	

Source: own elaboration

Figure 4.38 graphically illustrate Table 4.76.

Figure 4.38 – Comparison of STATIS and MFA: Projection of economic growth and development on Greece’s factorial representation



Source: own elaboration

Table 4.77 synthesizes the interpretation of these two techniques, regarding Greece’s performance.

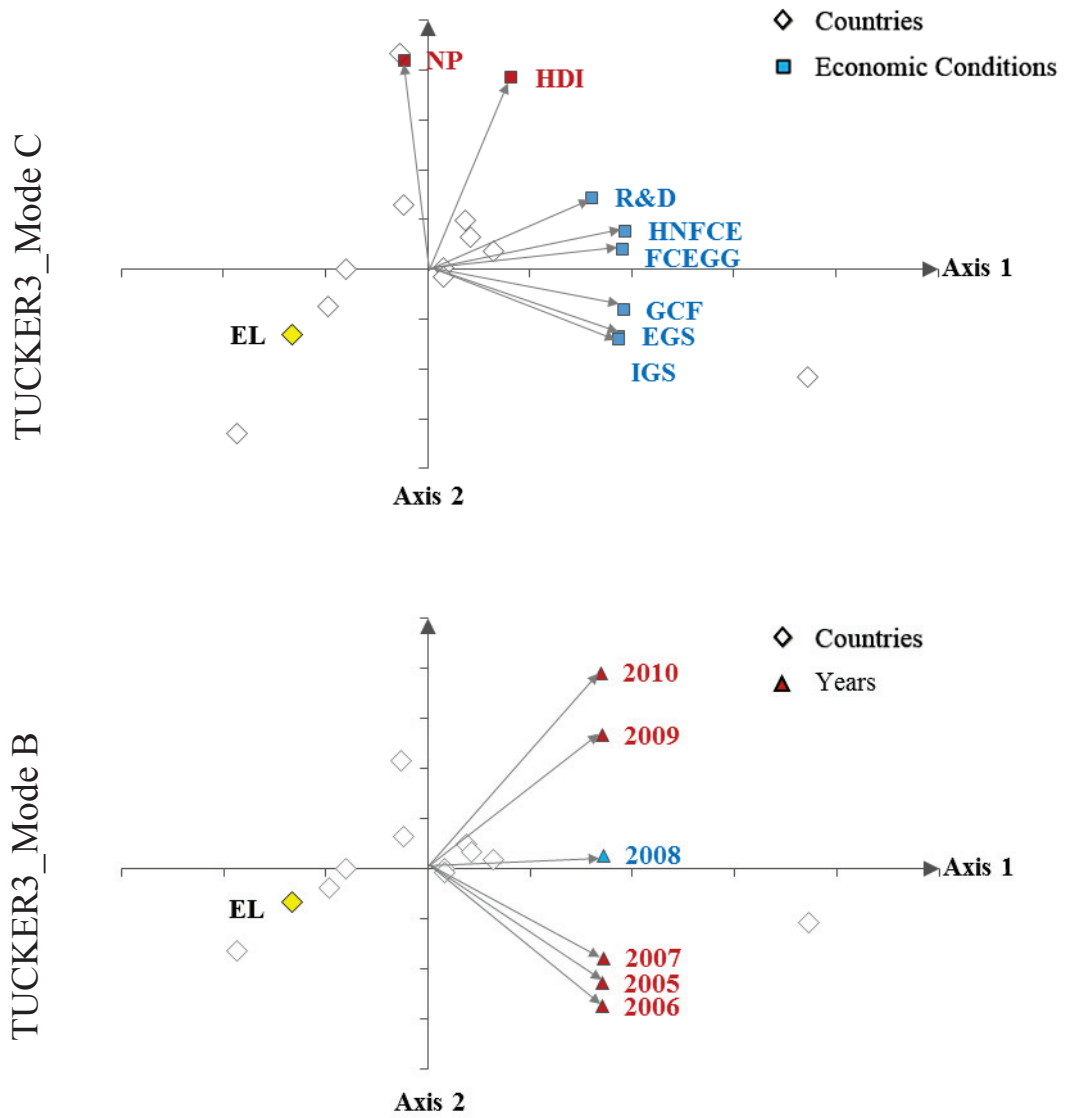
Table 4.77 – Greece’s Performance from STATIS and MFA point of view

GREECE	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was R&D, and the one that contributed the least was HNFCE. • there are two patterns of groups of variables (R&D, FCEGG and GCF), and (HNFCE, IGS and EGS), with positive correlation within each pattern and predominantly negative correlation between patterns. • the contribution of the two patterns of groups of variables was opposed, for the construction of the centroid, and thus to Greece’s centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was NP, and the one that contributed the least was HNFCE. • there are three patterns of groups of variables (GCF, IGS and EGS), and (HNFCE, FCEGG and R&D), and (HDI and NP) with positive correlation within each pattern and predominantly negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to Greece’s centroid positioning.

Source: own elaboration

Figure 4.39 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Greece’s factorial representation.

Figure 4.39 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Greece’s factorial representation



Source: own elaboration

And Table 4.78 synthesizes the interpretation of the Figure 4.39 regarding Greece’s performance.

Table 4.78 – Greece’s Performance from TUCKER3 point of view

GREECE	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Greece's performance was more negatively associated with year 2008 and with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

Finally, in Table 4.79 are presented the coordinates of Portugal for STATIS and MFA, respectively.

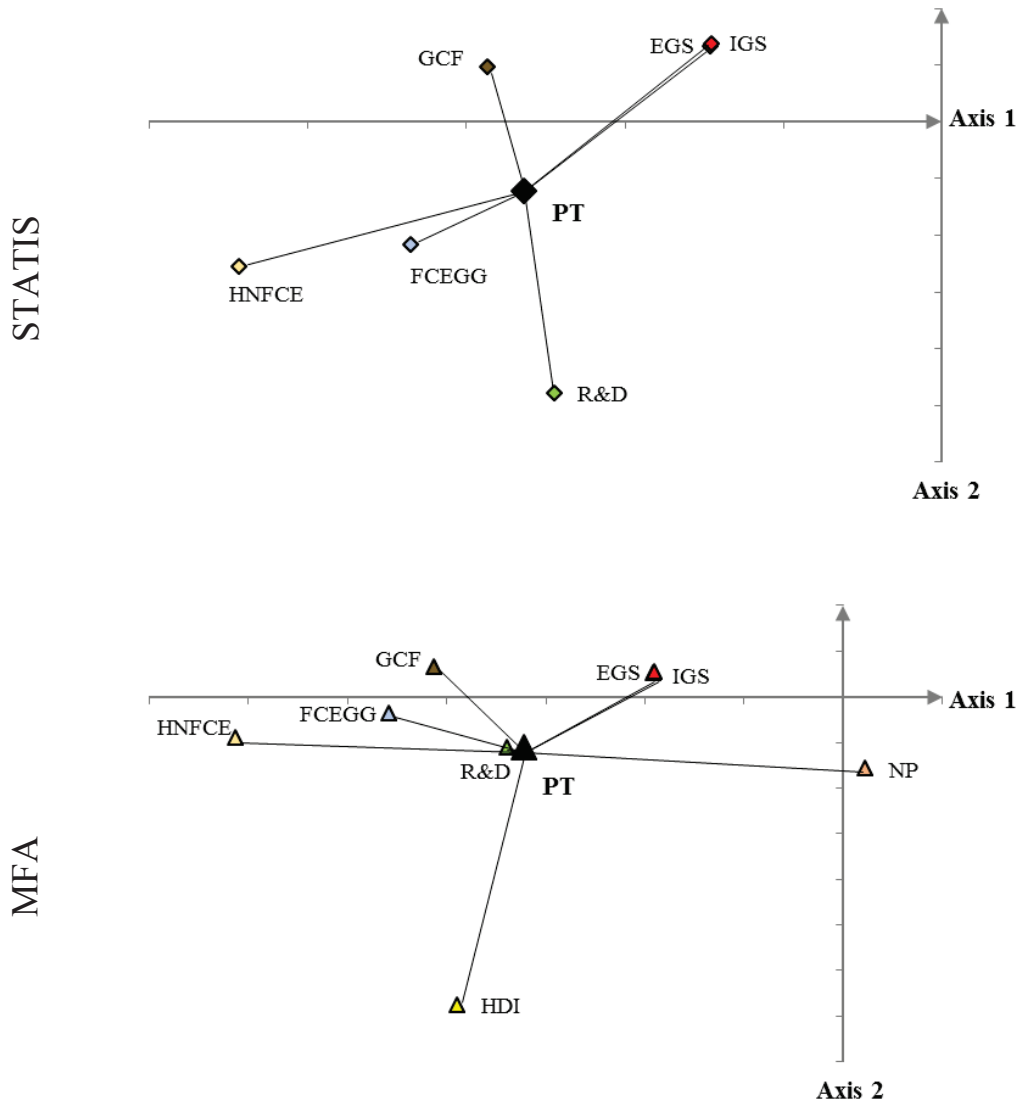
Table 4.79 – Compromise/consensus coordinates of economic growth and development for Portugal

	Axis 1	Axis 2	
S T A T I S	-2.636	-1.234	PT
	-4.430	-2.563	HNFCE
	-2.863	0.966	GCF
	-3.348	-2.162	FCEGG
	-1.461	1.323	EGS
	-1.449	1.367	IGS
	-2.442	-4.778	R&D
M F A	-3.214	-2.256	PT
	-6.129	-1.823	HNFCE
	-4.128	1.282	GCF
	-4.579	-0.723	FCEGG
	-1.916	1.011	EGS
	-1.896	1.058	IGS
	-3.393	-2.224	R&D
	0.223	-3.149	NP
	-3.895	-13.478	HDI

Source: own elaboration

Figure 4.40 graphically illustrates Table 4.79.

Figure 4.40 – Comparison of STATIS and MFA: Projection of economic growth and development on Portugal’s factorial representation



Source: own elaboration

Table 4.80 synthesizes the interpretation of these two techniques, regarding Portugal’s performance.

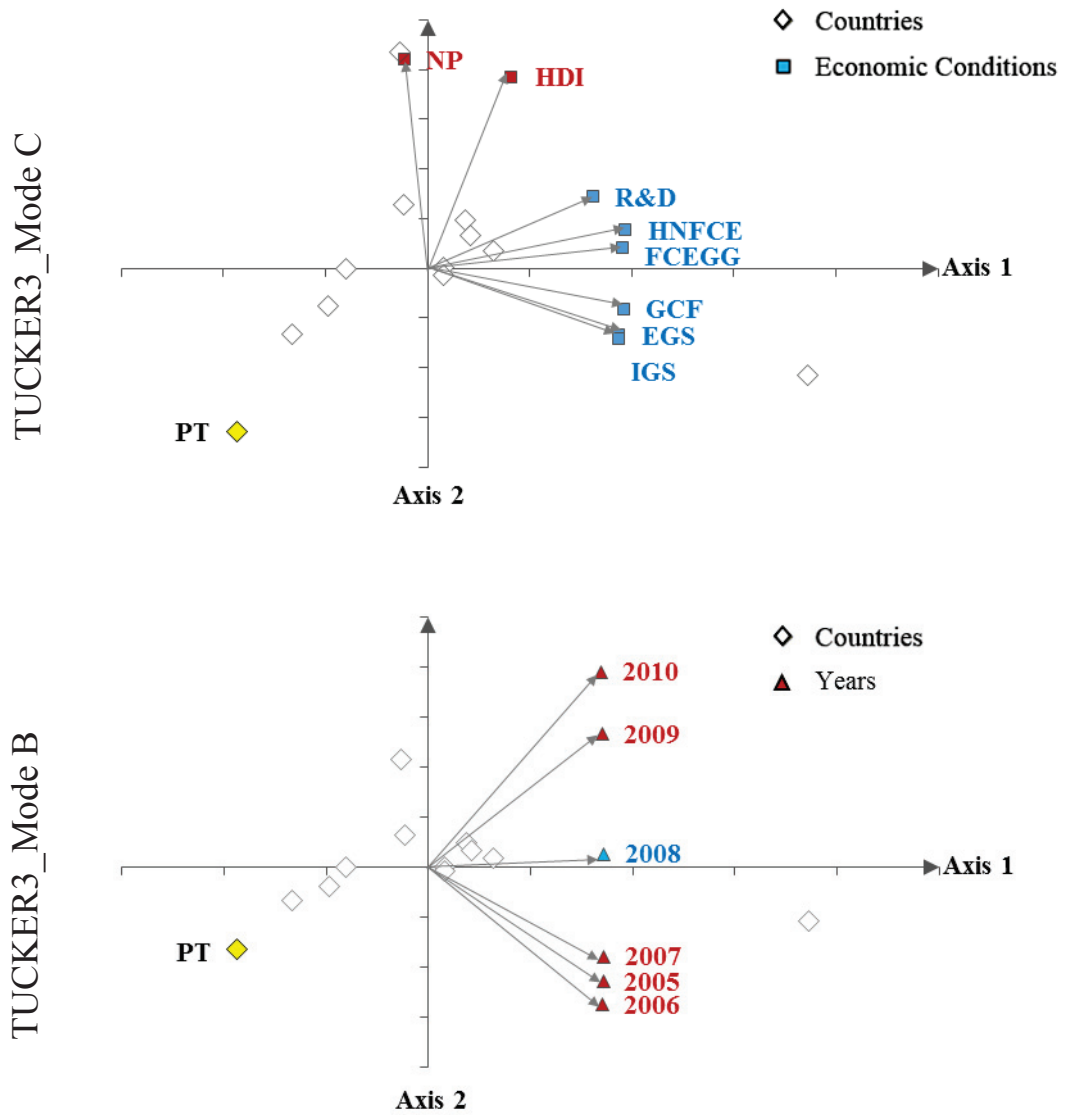
Table 4.80 – Portugal Performance from STATIS and MFA point of view

PORTUGAL	
STATIS	<ul style="list-style-type: none"> • the groups of variables that most contributed to the construction of the centroid was HNFCE, and the one that contributed the least was FCEGG. • there are two patterns of groups of variables (R&D, IGS and EGS), and (HNFCE, FCEGG and GCF), with positive correlation within each pattern and negative correlation between patterns. • the contribution of the two patterns of groups of variables was opposed, for the construction of the centroid, and thus to Portugal's centroid positioning.
MFA	<ul style="list-style-type: none"> • the group of variables that most contributed to the construction of the centroid was NP, and the one that contributed the least was R&D. • there are three patterns of groups of variables (NP, IGS and EGS), and (HNFCE, FCEGG, GCF and R&D), and (HDI) with positive correlation within each pattern and negative correlation between patterns. • the contribution of the three patterns of groups of variables was opposed, for the construction of the centroid, and thus to Portugal's centroid positioning.

Source: own elaboration

Figure 4.41 graphically illustrates TUCKER3 Mode C and Mode B for the projection of economic growth and development on Portugal factorial representation.

Figure 4.41 – TUCKER3 Mode C and Mode B: Projection of economic growth and development on Portugal’s factorial representation



Source: own elaboration

And Table 4.81 synthesizes the interpretation of the Figure 4.41 regarding Portugal performance.

Table 4.81 – Portugal Performance from TUCKER3 point of view

PORTUGAL	
TUCKER3 Mode C and Mode B	<ul style="list-style-type: none"> • This countries' performance was characterized by three sub-periods: (1)- 2005-2007, negatively associated with economic development through HDI and NP; (2)- 2008, linked to the effects of the crisis in economic growth through a positive relationship with R&D, HNFCE, FCEGG, GCF, IGS and; (3)- the period post crisis, 2009-2010 positively associated with economic development through HDI and NP. • GCF, IGS, and EGS had the greatest contribution to economic growth in particular in the period 2005-2007; • HNFCE, FCEGG, and R&D, in this order, contributed the most to economic growth 2008 forward. • Portugal's performance was more positively associated with year 2009-2010 and negatively with FCEGG, HNFCE, R&D, GCF, EGS, and IGS.

Source: own elaboration

4.5.5 2005-2010 interactions in Eurozone countries

The analysis of the elements of the Core matrix is the final and most important step. The core elements are further analyzed to explain the model, that is, those whose absolute value is greater, and thus have greater variability explained. The interpretation of these elements is carried out from the signals of each of the components of the three modes, symbolically and in the case of the first element of the core matrix:

$$[(+)g_{1,1,1}] \times \left[\begin{pmatrix} + \\ - \end{pmatrix} P \times \begin{pmatrix} + \\ - \end{pmatrix} Q \times \begin{pmatrix} + \\ - \end{pmatrix} R \right]$$

First element of the Core matrix: $(1,1,1) = 18.504$

It is necessary to analyze the elements of the first component of the mode A (countries), the first component of the mode B (years) and the first component of the C mode (Economic Conditions). Thus, there is a need to study the possible combinations of signals from elements of the first components of each mode, namely:

$$[(+)g_{1,1,1}] \times [(+)P_1 \times (+)Q_1 \times (+)R_1]$$

$$[(+)g_{1,1,1}] \times [(-)P_1 \times (+)Q_1 \times (+)R_1]$$

$$[(+)g_{1,1,1}] \times [(+)P_1 \times (-)Q_1 \times (+)R_1]$$

$$[(+)g_{1,1,1}] \times [(+)P_1 \times (+)Q_1 \times (-)R_1]$$

$$[(+)g_{1,1,1}] \times [(+)P_1 \times (-)Q_1 \times (-)R_1]$$

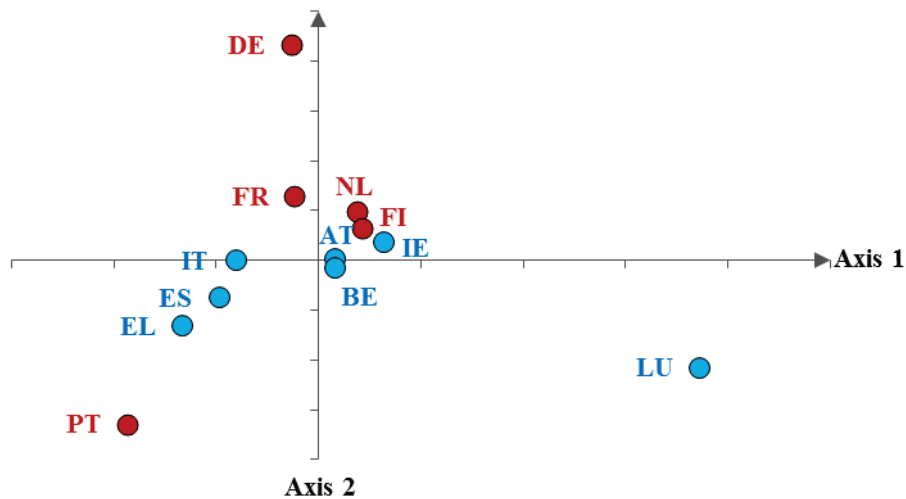
$$[(+)g_{1,1,1}] \times [(-)P_1 \times (-)Q_1 \times (+)R_1]$$

$$[(+)g_{1,1,1}] \times [(-)P_1 \times (+)Q_1 \times (-)R_1]$$

$$[(+)g_{1,1,1}] \times [(-)P_1 \times (-)Q_1 \times (-)R_1]$$

Through Figures 4.42, 4.43 and 4.44, you can see the positive and negative relations between countries and years, and the economic conditions in relation to the first components of the three modes.

Figure 4.42 – TUCKER3 Mode A axes 1-2 for the 1st element of the Core matrix



Source: own elaboration

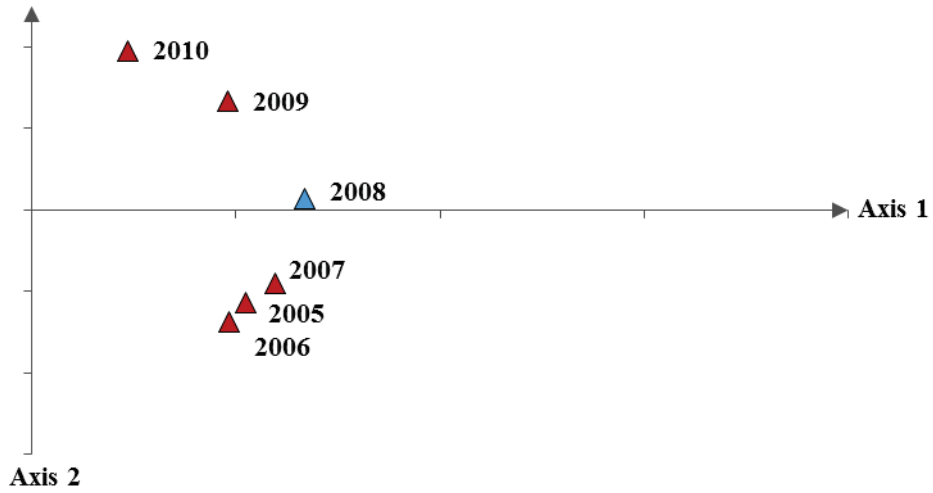
It is noted in relation to the first component, that there are six countries with a positive relationship and six countries with a negative correlation, as described in Table 4.82.

Table 4.82 – Signs of countries for the 1st element of the Core matrix

+	-
LU, AT, BE, FI, NL and IE	DE, FR, IT, ES, EL and PT

Source: own elaboration

Figure 4.43 – TUCKER3 Mode B axes 1-2 for the 1st element of the Core matrix



Source: own elaboration

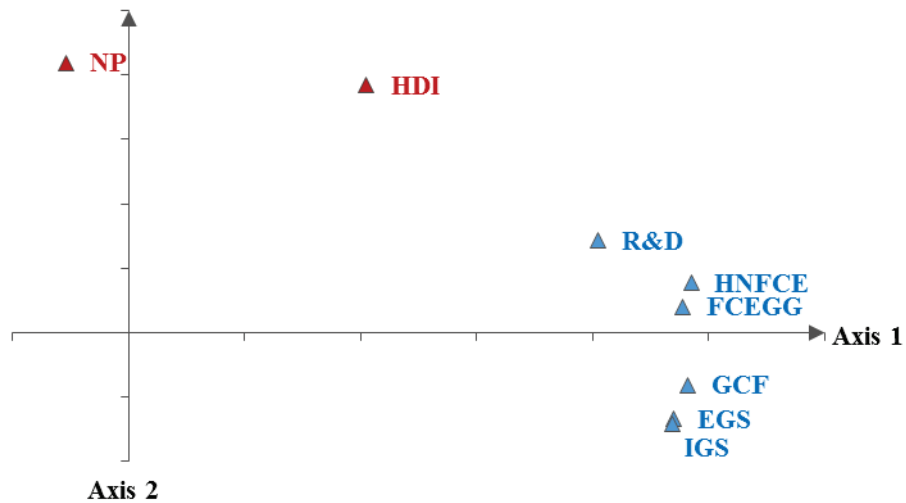
There is, on the first component, that all the years have a positive relationship (Table 4.83).

Table 4.83 – Signs of years for the 1st element of the Core matrix

+	-
2005, 2006, 2007, 2008, 2009 and 2010	

Source: own elaboration

Figure 4.44 – TUCKER3 Mode C axes 1-2 for the 1st element of the Core matrix



Source: own elaboration

It is noted in relation to the first component, seven of the eight economic conditions revealed a positive relationship (Table 4.84).

Table 4.84 – Signs of economic conditions for the 1st element of the Core matrix

+	-
EGS, IGS, GCF, R&D, HNFCE, FCEGG and HDI	NP

Source: own elaboration

Table 4.85 reflects the combinations detected in the analyzed signal components and the respective result of the interaction, whose signal is obtained by multiplying the signals of the respective elements. It is not necessary to analyze the negative signs, on the first component of the B-mode, since all the years show positive coordinates.

Table 4.85 – Interaction results for the 1st element of the Core matrix

COMBINATION OF SIGNS		INTERACTION RESULT
1	$[(+)g_{1,1,1}] \times [(+)P_1 \times (+)Q_1 \times (+)R_1]$	+
2	$[(+)g_{1,1,1}] \times [(-)P_1 \times (+)Q_1 \times (+)R_1]$	-
3	$[(+)g_{1,1,1}] \times [(+)P_1 \times (+)Q_1 \times (-)R_1]$	-
4	$[(+)g_{1,1,1}] \times [(-)P_1 \times (+)Q_1 \times (-)R_1]$	+

Source: own elaboration

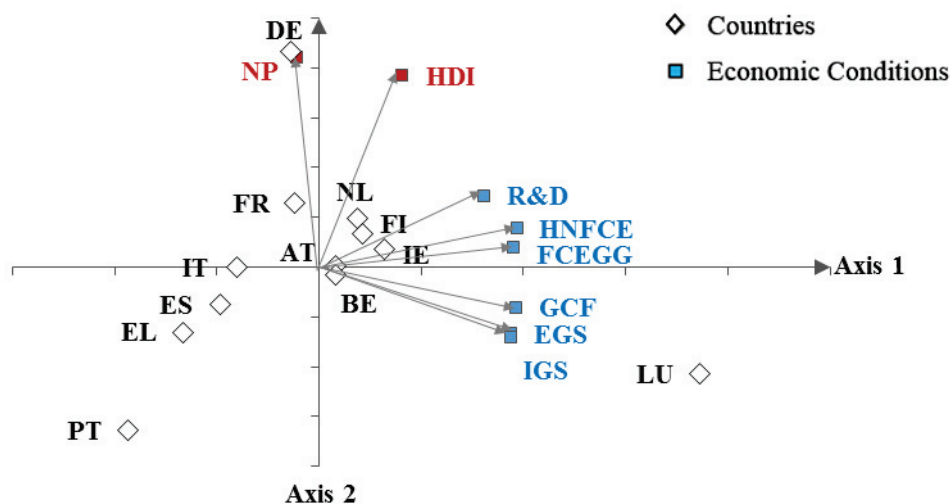
The interpretation of Table 4.85, allows to recognize, with 64.9% of explained variance, the following:

- For the period 2005-2010, the countries DE, FR, IT, ES, EL and PT have a negative interaction with EGS, IGS, GCF, R & D, HNFCE, FCEGG and HDI;

- For the period 2005-2010, countries LU, AT, BE, FI, NL and IE have a positive interaction with EGS, IGS, GCF, R & D, HNFCE, FCEGG and HDI;
- For the period 2005-2010, the countries DE, FR, IT, ES, EL and PT have a positive interaction with NP;
- For 2005-2010, the countries LU, AT, BE, FI, NL and IE have a negative interaction with NP.

This information from the three modes can also be represented through conjoint factor structures (biplot) which will support the interpretation of the main elements of the Core matrix (Figure 4.45).

Figure 4.45 – BIPLLOT: countries and economic conditions in the 1st element of the Core matrix



Source: own elaboration

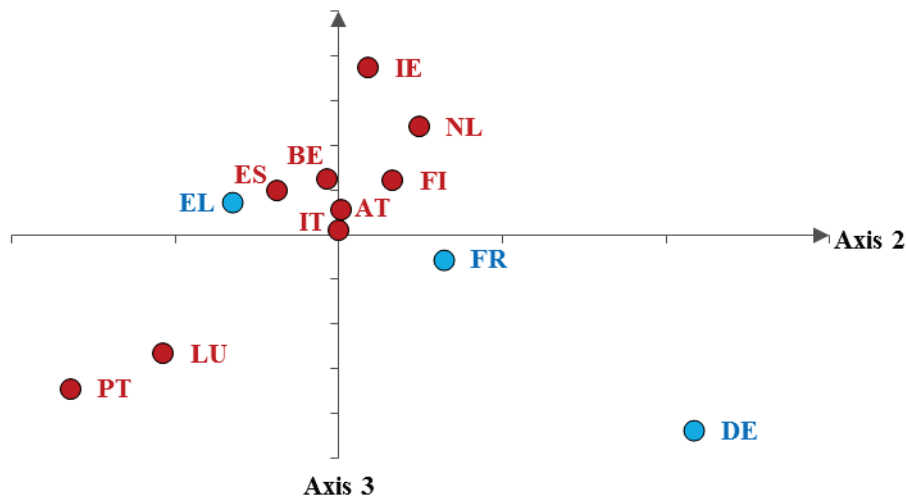
For the 1st element, it is shown that not all solutions obtained with the biplot will coincide with the interaction analysis interpretation, and sometime can be opposite. For example with the biplot representation we would not interpret that ES, EL, and PT, have positive interaction with NP. This interaction concept is specific of TUCKER3.

Second Core array element: (2,1,2) = 10.066

It is necessary to analyze the elements of the second component of the mode A (countries), the first component of the mode B (years) and the second component of the C mode (Economic Conditions).

Through Figures 4.46, 4.47, and 4.48, you can see the positive and negative relations between countries, the years and the economic conditions for the selected components for the three modes.

Figure 4.46 – TUCKER3 Mode A axes 2-3 for the 2nd element of the Core matrix



Source: own elaboration

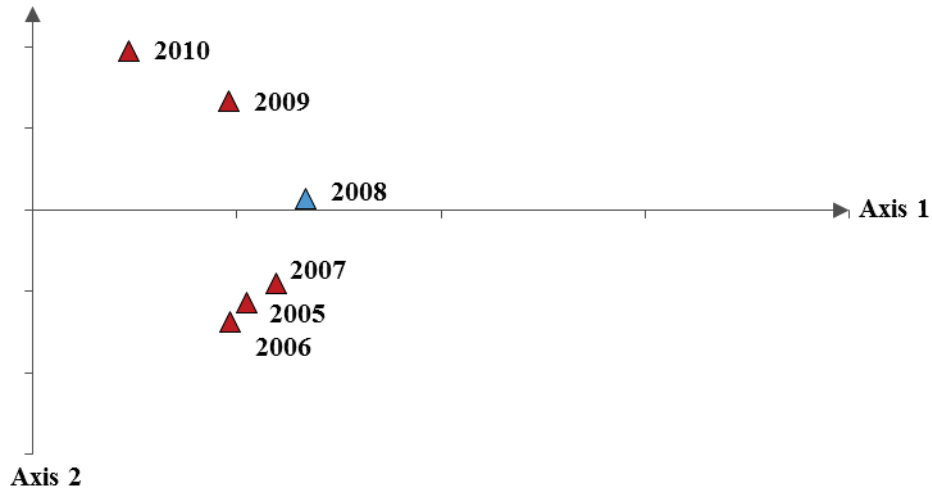
It is noted in relation to the first component, that there are eight countries with a positive relationship and four countries with a negative correlation, as described in Table 4.86.

Table 4.86 – Signs of countries for the 2nd element of the Core matrix

+	-
AT, BE, FI, NL, IT, ES, EL and IE	DE, FR, LU and PT

Source: own elaboration

Figure 4.47 – TUCKER3 Mode B axes 1-2 for the 2nd element of the Core matrix



Source: own elaboration

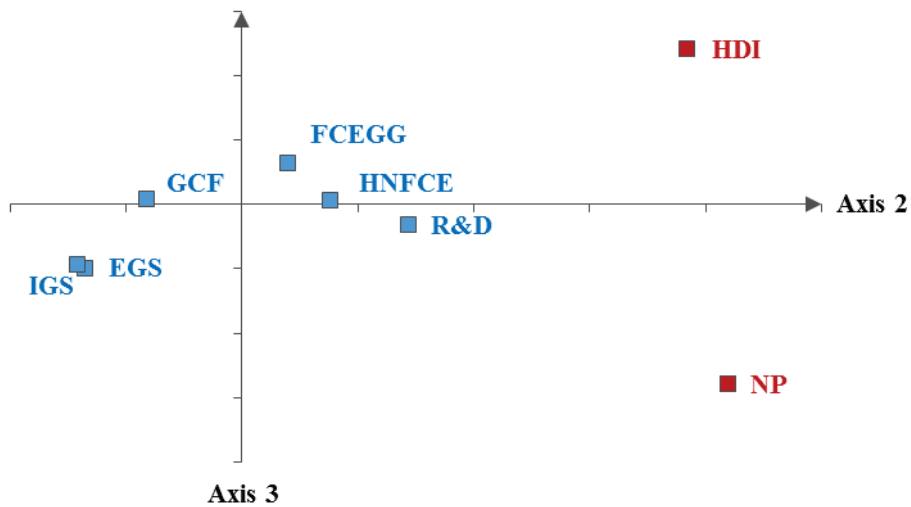
It is noted also in relation to the first axis, that all year’s show a positive relationship (Table 4.87).

Table 4.87 – Signs of years for the 2nd element of the Core matrix

+	-
2005,2006, 2007, 2008, 2009 and 2010	

Source: own elaboration

Figure 4.48 – TUCKER3 Mode C axes 2-3 for the 2nd element of the Core matrix



Source: own elaboration

It is also noted with respect to the first axis that five of the eight economic conditions revealed a positive relationship (Table 4.88).

Table 4.88 – Signs of the economic conditions for the 2nd element of the Core matrix

+	-
FCEGG, HNFCE, R&D, HDI and NP	IGS, EGS and GCF

Source: own elaboration

Table 4.89 summarizes the detected combinations in the signs of the components here analyzed and the result of the respective interaction.

Table 4.89 – Interaction results for the 2nd element of the Core matrix

	COMBINATION OF SIGNS	INTERACTION RESULT
1	$[(+)g_{2,1,2}] \times [(+)P_2 \times (+)Q_1 \times (+)R_2]$	+
2	$[(+)g_{2,1,2}] \times [(-)P_2 \times (+)Q_1 \times (+)R_2]$	-
3	$[(+)g_{2,1,2}] \times [(+)P_2 \times (+)Q_1 \times (-)R_2]$	-
4	$[(+)g_{2,1,2}] \times [(-)P_2 \times (+)Q_1 \times (-)R_2]$	+

Source: own elaboration

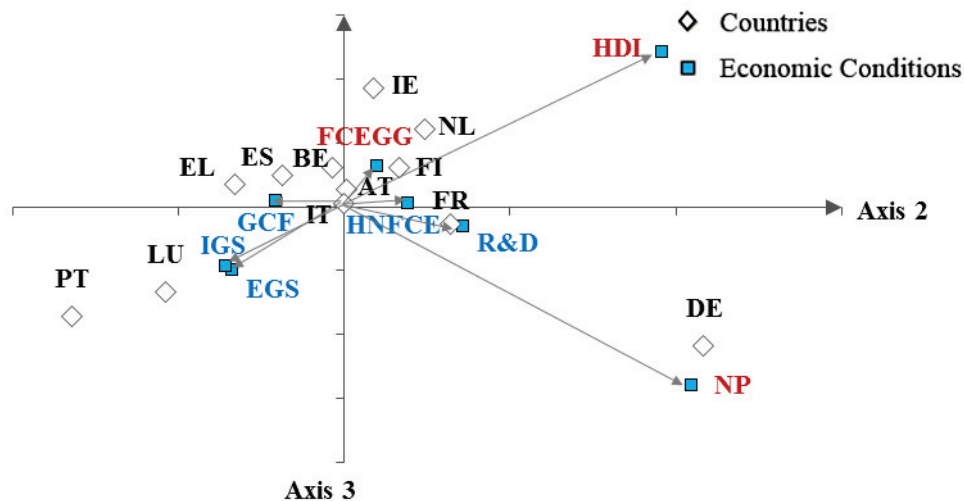
The interpretation of Table 4.89, allows us to say, with 19.2% of explained variance, the following:

- For the period 2005-2010, the countries DE, FR, LU and PT have a negative interaction with HDI, FCEGG, HNFCE, R&D and NP;

- For the period 2005-2010, the countries IE, NL, FI, AT, IT, FR and DE have a positive interaction with FCEGG, HNFCE, R&D, HDI and NP;
- For the period 2005-2010, the countries DE, FR, LU and PT have a positive interaction with GCF, IGS and EGS;
- For the period 2005-2010, the countries IE, NL, FI, AT, IT, FR and DE have a negative interaction with GCF, IGS and EGS.

This information from the three modes can also be represented through conjoint factor structures (biplot) which will support the interpretation of the main elements of the Core matrix (Figure 4.49).

Figure 4.49 – BIPLLOT: countries and economic conditions in the 2nd element of the Core matrix



Source: own elaboration

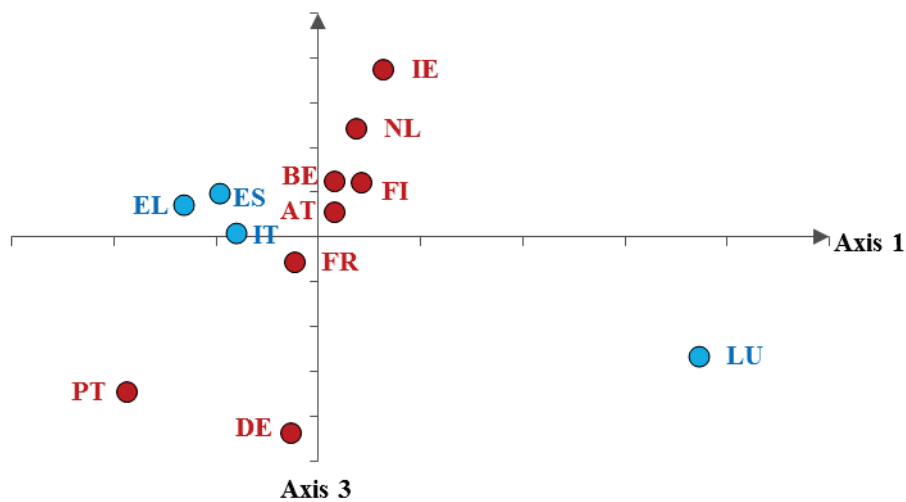
Once more, now for the 2nd element, it is shown that not all solutions obtained with the biplot coincide with the interaction analysis interpretation. For example with the latter representation we would not interpret that DE and FR have a negative interaction with NP.

Third element of the Core matrix: $(3,1,3) = 6.205$

It is necessary to analyze the elements of the third component mode A (countries), the first component of the mode B (years) and the third component of C mode (Economic Conditions).

Through Figures 4.50, 4.51 and 4.52, is possible to evidence the positive and negative relations of the countries, years and economic conditions for the selected components for the three modes.

Figure 4.50 – TUCKER3 Mode A axes 1-3 for the 3rd element of the Core matrix



Source: own elaboration

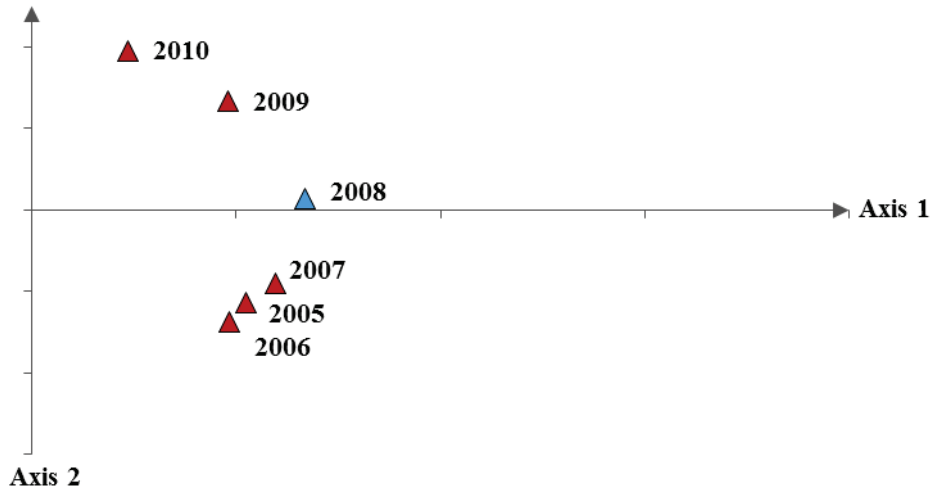
It is observed, in relation to the first axis, that there are eight countries with a positive relationship and four countries with a negative correlation, as described in Table 4.90.

Table 4.90 – Signs of the countries for the 3rd element of the Core matrix

+	-
AT, BE, FI, NL, IT, ES, EL and IE	DE, FR, LU and PT

Source: own elaboration

Figure 4.51 – TUCKER3 Mode B axes 1-2 for the 3rd element of the Core matrix



Source: own elaboration

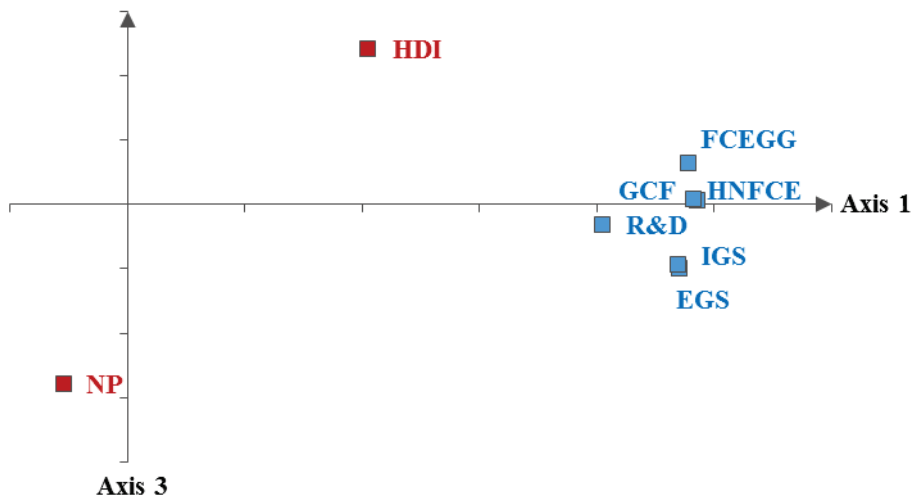
Also, it appears that in relation to the first axis, all the years show a positive relationship (Table 4.91).

Table 4.91 – Signs of the years for the 3rd element of the Core matrix

+	-
2005,2006, 2007, 2008, 2009 and 2010	

Source: own elaboration

Figure 4.52 – TUCKER3 Mode C axes 1-3 for the 3rd element of the Core matrix



Source: own elaboration

It is also verified relatively to the first axis, that four of the eight economic conditions revealed a positive relationship (Table 4.92).

Table 4.92 – Signs of economic conditions for the 3rd element of the Core matrix

+	-
FCEGG, HNFCE, GCF and HDI	NP, IGS, EGS and R&D

Source: own elaboration

Table 4.93 summarizes the detected combinations in the signs of the components here analyzed and the result of the respective interaction.

Table 4.93 – Interaction results for the 3rd element of the Core matrix

COMBINATION OF SIGNS		INTERACTION RESULT
1	$[(+)g_{3,1,3}] \times [(+)P_3 \times (+)Q_1 \times (+)R_3]$	+
2	$[(+)g_{3,1,3}] \times [(-)P_3 \times (+)Q_1 \times (+)R_3]$	-
3	$[(+)g_{3,1,3}] \times [(+)P_3 \times (+)Q_1 \times (-)R_3]$	-
4	$[(+)g_{3,1,3}] \times [(-)P_3 \times (+)Q_1 \times (-)R_3]$	+

Source: own elaboration

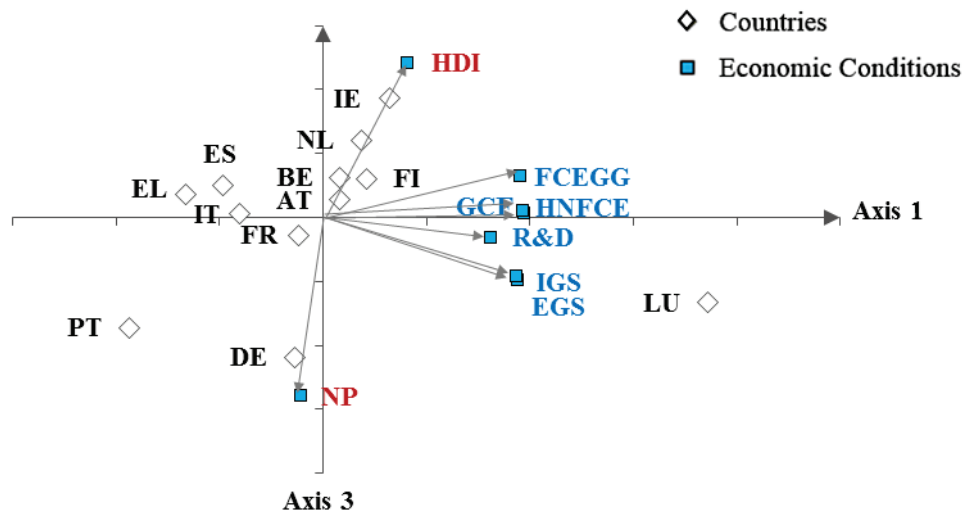
The interpretation of Table 4.93, allows us to say, with only 7.3% of the explained variance, the following:

- For the period 2005-2010, the countries AT, BE, FI, NL, IT, ES, EL and IE have a positive interaction with FCEGG, HNFCE, GCF and HDI;

- For the period 2005-2010, the countries DE, FR, LU and PT have a negative interaction with FCEGG, HNFCE, GCF and HDI;
- For the period 2005-2010, countries AT, BE, FI, NL, IT, ES, EL and IE have a negative interaction with NP, IGS, EGS and R&D;
- For the period 2005-2010, the countries DE, FR, LU and PT have a positive interaction with NP, IGS, EGS and R&D.

This information from the three modes can also be represented through conjoint factor structures (biplot) which will support the interpretation of the main elements of the Core matrix (Figure 4.53).

Figure 4.53 –BIPLOT: countries and economic conditions in the 3rd element of the Core matrix



Source: own elaboration

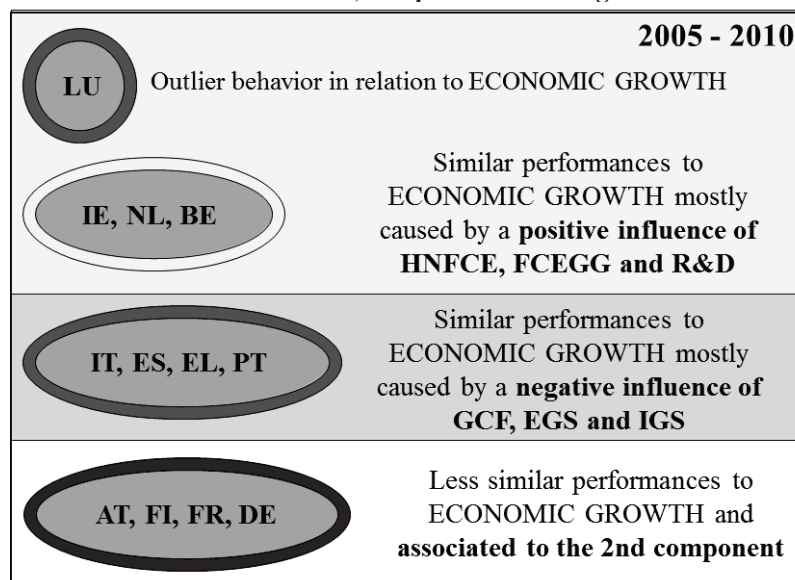
This biplot interpretation coincides with the conclusions from the interaction analysis.

4.6 Discussion

4.6.1 Economic growth and development in the Eurozone

According to the STATIS method the analysis of compromise space pointed out four groups of Eurozone countries in a relatively unstable factorial structure for the period 2005-2010. One of these groups (AT, FI, FR, and DE), associated with the second component (not interpretable because the second component variables NP and HDI did not contribute to the stable common structure). The other three groups of countries are related to the first component, easily interpreted as ECONOMIC GROWTH represented as a gradient of GDP per capita, which decreases from right to left. Figure 4.54 summarizes not only the main influences on economic growth but also the similarities in the type of growth. The similarities came from the analysis of the STATIS interstructure and the influences came from the analysis of the original variables correlation with the components of compromise factorial space.

Figure 4.54 – STATIS: interstructure, compromise and original variables correlation

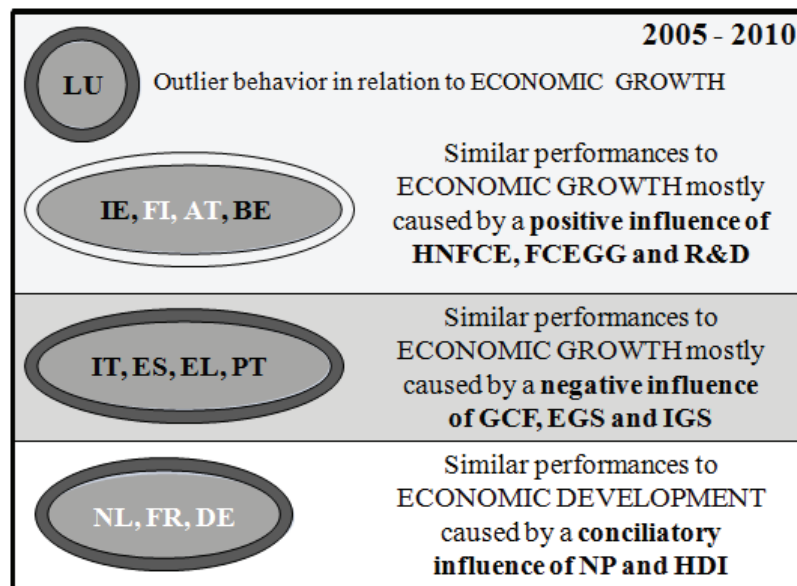


Source: own elaboration

According to the MFA method the analysis of consensus space also pointed out four groups of Eurozone countries in a stable factorial structure for the period 2005-2010. Three of these groups are connected with the first component, interpreted as ECONOMIC

GROWTH, as in the STATIS method. The fourth group (NL, FR, and DE) is associated with the second component which is interpreted as ECONOMIC DEVELOPMENT, represented as a gradient of NP and HDI, which increases from bottom to top. Figure 4.55 summarizes not only the main influences as well the similarities in the type of growth and development. The similarities came from the analysis of the MFA interstructure and the influences came from the analysis of the original variables correlation with the components of consensus factorial space.

Figure 4.55 – MFA: interstructure, consensus and original variables correlation

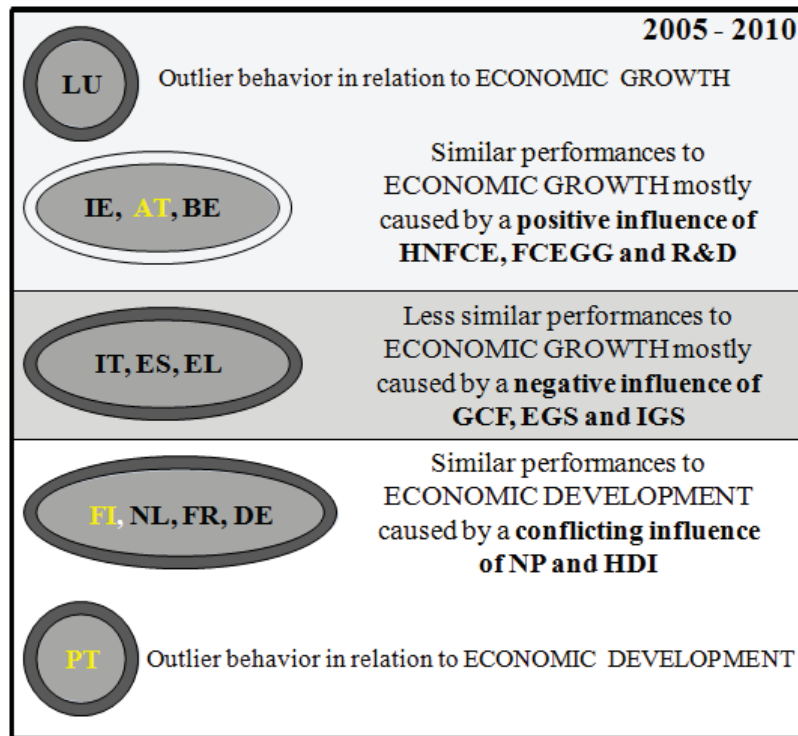


Source: own elaboration

According to the TUCKER3 method the analysis of Mode A factorial space pointed out five groups of Eurozone countries in a stable factorial structure for the period 2005-2010. Three of these groups are connected with first component, interpreted as ECONOMIC GROWTH, as in the STATIS and MFA methods. The fourth (FI, NL, FR, and DE) and fifth groups (PT) are associated with the second component which is interpreted as ECONOMIC DEVELOPMENT, represented as a gradient of NP and HDI, as in the MFA method. Figure 4.56 summarizes not only the main influences but also the similarities in the types of growth and development. The similarities came from the analysis of the Mode

C factorial space and the influences came from the analysis of the original variables correlation with the Mode A factorial space.

Figure 4.56 – TUCKER3: Mode C and original variables correlation with Mode A



Source: own elaboration

Figures 4.54, 4.55 and 4.56 capture the main changes in economic growth and in economic development in the Eurozone for 2005-2010 performance. Through STATIS, MFA, and TUCKER3 methods it is possible to recognize distinct five facts:

1. In the three methods there are countries that are similarly characterized;
2. R&D is associated with economic growth;

3. The general perspective of STATIS, with an unstable factorial structure, only analyzes economic growth. Regardless of this general view it is still possible to evaluate the main influences on economic growth as well the similarities in the type of growth in the twelve Eurozone countries;
4. The MFA method, with a more stable factorial structure, distinguishes economic growth of economic development and includes the possibility of evaluating FI, AT, FR, and DE. Particularizing the role of development in the similarities and in the main influences of the economic identities for the twelve Eurozone countries;
5. The TUCKER3 method, also with a appropriate factorial structure due to the Model fit obtained, examines with more detail the differences and the influences of the economic identities, which become more realistic for the twelve Eurozone countries.

4.6.2 Stability and change effects on trends over 2005-2010

The visualization of the period 2005-2010, with regard to the mechanisms of stability and change, starts from the correlation between the years in terms of the groups of variables and allowed not only the understanding of the relations with the axes of the compromise/consensus for STATIS, MFA, and Mode A of TUCKER3, but also of the relations between the groups themselves.

However in Mode B of TUCKER3 it is possible, through the spatial interpretation of the positioning of the years, to identify difference in patterns in a direct way. More concretely, years 2005, 2006, and 2007 positioned on the fourth quadrant are very different from the ones on the first quadrant – 2008, 2009, and 2010. It is also true that 2005, 2006, 2007, 2009, and 2010 are linked to the second component and 2008 is related to the first component, which proves that 2008 is the year of materialization of change. This is consistent with the occurrence of the global financial crisis and its repercussion on the Eurozone as well as in the rest of the world.

Nevertheless the factorial structure of Mode C of TUCKER3 also performs the positioning of the groups of variables for each country. And, in this sense one can observe the different performances of the 12 Eurozone countries as presented in a synthetic manner in Tables 4.94 to 4.96.

Table 4.94 – LU, IE, NL and FI performances synthetizes

	LUXEMBOURG	IRELAND	THE NETHERLANDS	FINLAND
S T A T I S	<ul style="list-style-type: none"> •R&D is associated with state expenditures (FCEGG) and showed more variability in the economic growth performance; •Private investment (GCF), associated with exports (EGS), showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and the trade balance. 	<ul style="list-style-type: none"> •Household consumption (HNFCE), showed more variability in the economic growth performance ; •The State expenditures (FCEGG) showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between State expenditures and private investment. 	<ul style="list-style-type: none"> •State expenditures (FCEGG) showed more variability in the economic growth performance; •R&D is highly associated with private investment (GCF) and showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between State expenditures (FCEGG) and all other groups. 	<ul style="list-style-type: none"> •R&D, associated with state expenditures (FCEGG), showed more variability in the economic growth performance; •Household consumption (HNFCE), highly associated with state expenditures (FCEGG), showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and GCF.
M F A	<ul style="list-style-type: none"> •HDI is correlated with NP and showed more variability in the economic development performance; •State expenditures (FCEGG), highly associated with household consumption (HNFCE), showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of : (1)- trade balance and private investment (EGS, IGS and GCF); (2)- state expenditures, household consumption and R&D; •Policy options for economic development had the effect of NP and HDI. 	<ul style="list-style-type: none"> •HDI showed more variability in the economic development performance; •The trade balance, showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF) and household consumption (HNFCE); (2)- balance of trade, state expenditures (FCEGG) and R&D; •Policy options for economic development had the effect of NP and HDI. 	<ul style="list-style-type: none"> •State expenditures (FCEGG) showed more variability in the economic growth performance; •R&D is highly associated with private investment (GCF) and showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- state expenditures (FCEGG); (2)- household consumption (HNFCE), R&D, private investment (GCF) and balance of trade; •Policy options for economic development had opposite effects of NP and HDI. 	<ul style="list-style-type: none"> •R&D is correlated with state expenditures (FCEGG), and showed more variability in the economic development performance; •Household consumption (HNFCE), highly associated with state expenditures (FCEGG), showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- R&D, state expenditures (FCEGG), household consumption (HNFCE); (2)- trade balance; (3)- GCF; •Policy options for economic development had opposite effects of NP and HDI.
T U C K E R 3	<ul style="list-style-type: none"> •Regarding economic growth in 2005-2007, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •Regarding economic growth in 2008-2010, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •Regarding economic growth in 2008-2010, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •Regarding economic growth in 2008-2010, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance .

Source: own elaboration

Table 4.95 – AT, BE, DE and FR performances synthesized

	AUSTRIA	BELGIUM	GERMANY	FRANCE
S T A T I S	<ul style="list-style-type: none"> •R&D, is negatively correlated with state expenditures (FCEGG), showed more variability in the economic growth performance; •Private investment (GCF) showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between state expenditures (FCEGG) and household consumption (HNFCE). 	<ul style="list-style-type: none"> •Estate expenditures (FCEGG) and showed more variability in the economic growth performance; •Private investment (GCF) is associated with R&D, and showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and estate expenditures (FCEGG). 	<ul style="list-style-type: none"> •R&D and showed more variability in the economic growth performance; •Exports (EGS), showed less variability in economic growth performance of EGS; •Options for economic growth denoted opposite policies between R&D and private investment (GCF). 	<ul style="list-style-type: none"> •Imports (IGS) is highly correlated with exports (EGS), and showed more variability in the economic growth performance; •Private investment (GCF) showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and Imports (IGS).
M F A	<ul style="list-style-type: none"> •R&D, is negatively correlated with state expenditures (FCEGG), and showed more variability in the economic growth performance; •HDI showed less variability in economic development performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF), household consumption (HNFCE) and R&D; (2)- trade balance and state expenditures (FCEGG); •Policy options for economic development had effects of NP and HDI. 	<ul style="list-style-type: none"> •Estate expenditures (FCEGG) and showed more variability in the economic growth performance; •Imports (IGS), showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- Estate expenditures (FCEGG) and imports (IGS); (2)- household consumption (HNFCE), R&D, private investment (GCF) and exports (EGS); •Policy options for economic development had the opposed effects of NP and HDI. 	<ul style="list-style-type: none"> •NP showed more variability in the economic development performance; •Exports (EGS) is highly associated with imports (IGS), and showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- state expenditures (FCEGG), private investment (GCF) and trade balance; (2)- household consumption (HNFCE) and R&D; •Policy options for economic development had the effect of NP and HDI. 	<ul style="list-style-type: none"> •NP showed more variability in the economic development performance; •HDI showed less variability in economic development performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF) and trade balance; (2)- household consumption (HNFCE), state expenditures (FCEGG) and R&D; •Policy options for economic development had the effect of NP and HDI.
T U C K E R 3	<ul style="list-style-type: none"> •Regarding economic growth in 2008-2010, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •Regarding economic growth in 2005-2007, policy options revealed positive influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •In the time period 2008-2010, policy options revealed positive in relation to the economic development performance pointed out by HDI, and NP . 	<ul style="list-style-type: none"> •In the time period 2008-2010, policy options revealed positive in relation to the economic development performance pointed out by HDI, and NP .

Source: own elaboration

Table 4.96 – PT, IT, EL and ES performances synthetizes

	PORTUGAL	ITALY	GREECE	SPAIN
S T A T I S	<ul style="list-style-type: none"> •Household consumption (HNFCE) showed more variability in the economic growth performance; •Estate expenditures (FCEGG) showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and private investment (GCF). 	<ul style="list-style-type: none"> •R&D is associated with estate expenditures (FCEGG), and showed more variability in the economic growth; •Private investment (GCF) is associated with balance of trade, and showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and Private investment (GCF). 	<ul style="list-style-type: none"> •R&D is associated with estate expenditures (FCEGG), and showed more variability in the economic growth performance; •Household consumption (HNFCE) showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between R&D and balance of trade. 	<ul style="list-style-type: none"> •R&D is associated with estate expenditures (FCEGG), and showed more variability in the economic growth performance; •Exports (EGS) is highly associated with imports (IGS), and showed less variability in economic growth performance; •Options for economic growth denoted opposite policies between private investment (GCF) and household consumption (HNFCE).
M F A	<ul style="list-style-type: none"> •NP showed more variability in the economic development performance; •R&D showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- balance of trade; (2)- household consumption (HNFCE), estate expenditures (FCEGG), private investment (GCF) and R&D; •Policy options for economic development had the effects of NP and HDI. 	<ul style="list-style-type: none"> •R&D is associated with estate expenditures (FCEGG), and showed more variability in the economic growth performance; •Household consumption (HNFCE), showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF) and balance of trade; (2)- household consumption (HNFCE); (3)- estate expenditures (FCEGG) and R&D; •Policy options for economic development had the effects of NP and HDI. 	<ul style="list-style-type: none"> •NP is correlated with HDI and showed more variability in the economic development performance; •Household consumption (HNFCE) showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF) and balance of trade; (2)- household consumption (HNFCE), estate expenditures (FCEGG) and R&D; •Policy options for economic development had the effects of NP and HDI. 	<ul style="list-style-type: none"> •HDI is correlated with NP and showed more variability in the economic development performance; •Exports (EGS) is highly associated with imports (IGS), and showed less variability in economic growth performance; •Policy options indicate for economic growth the influence of: (1)- private investment (GCF) and balance of trade; (2)- household consumption (HNFCE); (3)- Estate expenditures (FCEGG) and R&D; •Policy options for economic development had the opposed effects of NP and HDI.
T U C K E R 3	<ul style="list-style-type: none"> •Regarding economic growth in 2005-2007, policy options revealed negative influence of private investment (GCF), R&D, household consumption (HNFCE), and estate expenditures (FCEGG) and balance of trade. 	<ul style="list-style-type: none"> •Regarding economic growth in 2008-2010, policy options revealed negative influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance. 	<ul style="list-style-type: none"> •Regarding economic growth in 2005-2007, policy options revealed negative influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance . 	<ul style="list-style-type: none"> •Regarding economic growth in 2005-2007, policy options revealed negative influence of private investment (GCF), R&D, household consumption (HNFCE), estate expenditures (FCEGG), and trade balance .

Source: own elaboration

4.6.3 2005-2010 interactions in Eurozone countries

The Eurozone economies are increasingly interconnected. In this sense, interaction occurs when there is involvement of two or more countries where the action of one causes a reaction in one another. The greater interaction between different countries promotes greater synergistic effects and economic dependence between different growth and development structures. In this context to analyze the level of synergistic interactions results in identifying the level of dependency (or synergy) of the euro countries and this leads to different degrees of growth and development impacts.

Table 4.97 describes the positive and negative interactions more significant in the performance of the euro area countries for the period 2005-2010.

Table 4.97 – Interactions in Eurozone

	Positive Interactions	Negative Interactions
64.9%	LU, AT, BE, FI, NL and IE <i>versus</i> EGS, IGS, GCF, R&D, HNFCE, FCEGG and HDI	DE, FR, IT, ES, EL and PT <i>versus</i> EGS, IGS, GCF, R&D, HNFCE, FCEGG and HDI
	DE, FR, IT, ES, EL and PT <i>versus</i> NP	LU, AT, BE, FI, NL and IE <i>versus</i> NP
19.2%	AT, BE, FI, NL, IT, ES, EL and IE <i>versus</i> FCEGG, HNFCE, R&D, HDI and NP	DE, FR, LU and PT <i>versus</i> FCEGG, HNFCE, R&D, HDI and NP
	DE, FR, LU and PT <i>versus</i> IGS, EGS and GCF	AT, BE, FI, NL, IT, ES, EL and IE <i>versus</i> IGS, EGS and GCF
7.3%	AT, BE, FI, NL, IT, ES, EL and IE <i>versus</i> FCEGG, HNFCE, GCF and HDI	DE, FR, LU and PT <i>versus</i> FCEGG, HNFCE, GCF and HDI
	DE, FR, LU and PT <i>versus</i> NP, IGS, EGS and R&D	AT, BE, FI, NL, IT, ES, EL and IE <i>versus</i> NP, IGS, EGS and R&D

Source: own elaboration

With 64.9% of information captured from the original data, about the interactions on Eurozone countries, over 2005-2010, it can be stated that:

- The Benelux countries (BE, NL and LU) associated with AT, FI and IE revealed positive synergistic effects mostly on economic growth which also produced influence on economic development through HDI. On the other hand, were negatively affected on NP;
- The southern countries (IT, ES, EL and PT) benefited from a positive synergistic effect from DE and FR (the two strongest economies) on NP. On the other hand, were negatively affected mostly on economic growth by DE and FR. These levels of dependency also influenced negatively the economic development through HDI;

With only 19.2% of information captured from the original data, about the interactions on Eurozone countries, over 2005-2010, it can be stated that:

- AT, BE, FI, NL, IT, ES, EL, and IE benefited from a positive synergistic effect of HNFCE, FCEGG, R&D, NP and HDI and suffered a negative synergistic effect on GCF, EGS and IGS;
- DE, FR, LU and PT benefited from a positive synergistic effect on GCF, EGS and IGS and suffered a negative synergistic effect of HNFCE, FCEGG, R&D, NP and HDI.

4.6.4 A comparison between the results of STATIS, MFA, and TUCKER3

The spatial configuration from the three methods STATIS, MFA and TUCKER3, captured the evolution in performance of the euro area countries which occurred between 2005-2010 and enabled the recognition of two distinct realities, perfectly supported by the observed reality and literature known as: BEFORE THE CRISIS and the 2008 CRISIS. Nonetheless the STATIS method produced a compromise space where the number of patents (NP) and the Human Development Index (HDI) were poorly represented, inhibiting any interpretation about these two groups of variables, and restricting the characterization of the countries to the first component designated by ECONOMIC GROWTH.

In MFA there were changes in the groups of countries provided by STATIS in terms of association with the first and second component. That is a natural consequence from their different component construction that in STATIS “imposes” the compromise in the first component and, where MFA promotes a consensus with a “bidimensional” distribution of inertia to the two components. The MFA method did not have any limitation in its interpretation for it relies on both components to characterize and interpret the results, thus it was possible to use all groups of variables in the interpretation and its results were more “complete” in comparison with STATIS, encompassing ECONOMIC GROWTH and ECONOMIC DEVELOPMENT. The main contributors for the construction of the first component were the variables related with Gross Domestic Product and R&D investment, and for the second component were the Human Development Index (HDI) and the number of patents (NP). In this sense, for countries that have data more adequately accommodated by factorial structures with representation on the second component, MFA is better suited.

The TUCKER3 method, due to the different conceptualization of this three-way method which is symmetric, provided a different approach from the ones on STATIS and MFA. It does not possess a compromise or consensus space which represents the common structures provided by the previous methods, yet it is possible to represent the groups of variables through Mode C. This way it is possible to represent the groups of variables in space and identify the existing relations in its own fashion. The same approach can be taken when representing the countries by means of mode A. This representation can lead

to the identification of groups of countries with similar positioning and associations with the components.

The STATIS and MFA methods incorporate the circle of correlations to study the relations between the variables and the way these contribute to the stability or divergence of the compromise or consensus respectively. In TUCKER3 the circle of correlations is not a common feature, but if one wishes to study these relations it is possible using Mode A. The result is no more than the same representation as the one resulting from the concatenated supermatrix of MFA, as its configuration corresponds to that of the unfolding of Mode A.

CHAPTER 5

**CONCLUSIONS, LIMITATIONS AND
SUGGESTIONS FOR FURTHER RESEARCH**

Regardless of the limitations found in this study and the suggestions for future research, the results obtained have led to a number of conclusions that are confined to four areas:

- I. In the context of economic growth and development in the Eurozone;
- II. From the perspective of stability and change effects on trajectories over 2005-2010;
- III. 2005-2010 interactions in Eurozone countries;
- IV. Relevance in the use of three-way methods.

5.1 Conclusions

5.1.1 In the context of economic growth and development in the Eurozone

1. From STATIS point of view it is possible only to evaluate the main influences on economic growth and the similarities in the type of growth in the twelve Eurozone countries.
2. The MFA method distinguishes economic growth of economic development and includes the possibility of evaluating both concepts for the twelve Eurozone countries. It particularizes the role of development in the similarities and in the main influences of the social-economic identities.
3. The TUCKER3 method examines with more detail the similarities and the influences of the economic entities, highlighting the interactions of the twelve Eurozone countries.
4. Traditionally ECONOMIC GROWTH is evaluated and quantified by the rate of variation of GDP. On the other hand ECONOMIC DEVELOPMENT normally corresponds to society's ability to satisfy its needs and to attain a determined level of well-being. Despite these facts both being of use for any country's policy maker, ECONOMIC GROWTH is a necessary condition but not sufficient to

attain ECONOMIC DEVELOPMENT. The analysis of the period 2005-2010 showed some discrepancies between Eurozone countries. In other words some countries have attained different performances in ECONOMIC GROWTH quite divergent from the expected ECONOMIC DEVELOPMENT.

5. The southern countries are not a consistent group in the sense that their exchanges of elements to other groups does not connote with their usual profiles.

5.1.2 A perspective of stability and change over 2005-2010 and its effects on the countries' trends

The behavior of the twelve Eurozone countries in the period 2005-2010 can be summarized in two areas – STABILITY and CHANGE:

6. The period 2005-2010, with regard to the mechanisms of STABILITY with focus on common structures (visible via the STATIS and MFA methods) and CHANGE with focus on behavior differences (perceived by the TUCKER3 model) allowed not only the understanding of the relations with the Eurozone countries, but also of the relations between the groups of countries.
7. With TUCKER3 it was possible to identify CHANGE in performances in a way consistent with the occurrence of the global financial crisis and its interactions on the Eurozone.
8. Macroeconomic conditions are relevant determinants of economic performance, as stable macroeconomic environment favors growth, through reduction of uncertainty, whereas macroeconomic instability has a negative impact on growth through its effects on productivity and investment, hindering ECONOMIC GROWTH.

9. The economic policy of the member states' governments came into play to correct deficits of investment by the private sector in research and development.
10. Human development and patent production are more frequently associated with ECONOMIC DEVELOPMENT and investment on research and development with ECONOMIC GROWTH; nonetheless they can interact in self-reinforcing processes.
11. Investment in innovation and R&D activities can play a major role in increasing economic progress and, in this sense, have been the main drivers for certain countries' economic performance.

5.1.3 2005-2010 main interactions in Eurozone countries

12. Benelux countries (BE, NL and LU) associated with AT, FI and IE revealed positive synergistic effects mostly on ECONOMIC GROWTH which also produced influence on ECONOMIC DEVELOPMENT through HDI. On the other hand, were negatively affected on NP.
13. The southern countries (IT, ES, EL and PT) benefited from a positive synergistic effect from DE and FR (the two strongest economies) on NP. On the other hand, were negatively affected mostly on ECONOMIC GROWTH by DE and FR. These levels of dependency also influenced negatively the ECONOMIC DEVELOPMENT through HDI.

5.1.4 Relevance in the use of the three-way methods

14. Despite the methodological differences between the three methods, they show common elements related to: the adequacy of the data, the representations of the individuals, variables, conditions, and trajectories of individuals through the conditions.
15. Regardless of the STATIS method revealing a more simplistic analysis, the compromise/consensus positions and the trajectories of individuals obtained by the STATIS method are quite close to those obtained by MFA. Both the STATIS and MFA infrastructure axes have been interpreted thanks to the correlations between them and the initial variables. The advantage of MFA compared to STATIS is evidenced by the retention of information on the second component allowing the interpretation of data that is not confined on the first component.
16. The results allowed the implementation of aspects related to matrix computation, data processing, parameter estimation, and selection of the best model, determining the adjustment and generation of representations of the different modes and combination of these with TUCKER3. The advantage of MFA with regard to the TUCKER3 model is defined by the production of the contributions of the representations of individuals and variables to the inertia of each component. On the other hand, the TUCKER3 model provides an advantage over MFA with respect to the trajectories and interactions, which favors more and diverse interpretation.
17. The major aim of applying three-way data analysis is to unravel complex patterns of dependencies between the observations. In fact, the identification of benefits by the three methods are the features which suggest considering them as complementary. Therefore, the approach of the three-way methods will allow, in our opinion, not only to point out useful details for designing a more realistic economic and social diagnosis of the Eurozone for the period 2005-2010, but also to demonstrate the advantage of the information derived from three-way methods, as a complement to traditional methods of analysis.

5.2 Research limitations

The main limitations of this research can be summarized as follows:

1. The data concerning the Human Development Index used in this research had constraints on its comparability, which was only possible to overcome from 2005 onwards;
2. The different notation of the French and Anglo-Saxon school causes difficulties in presenting a standard methodological description and in the inter-comparability between methods.

5.3 Suggestions for further research

For further research, we present three suggestions:

1. The first is, of course, is the inclusion of more aggregates of socio-economic variables;
2. The second involves replicating this methodological process for a longer time period;
3. To use the same approach to sample containing countries in different stages of economic growth and development.

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APPENDIX

APPENDIX I

Three-mode G supermatrix central array generated from the three component matrices before selecting component number for the A, B, and C modes.

	Mode C_Axis 1				Mode C_Axis 2				Mode C_Axis 3				Mode C_Axis 4				Mode C_Axis 5				Mode C_Axis 6				Mode C_Axis 7			
	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4	Mode B_Axis 1	Mode B_Axis 2	Mode B_Axis 3	Mode B_Axis 4
Mode A_Axis 1	18.504	-0.134	0.005	0.015	-0.004	-0.287	0.075	0.096	-0.025	-0.216	0.066	0.140	0.036	-0.064	-0.094	-0.147	0.083	-0.010	0.084	-0.013	0.119	0.126	-0.007	0.019	-0.124	-0.010	0.033	
Mode A_Axis 2	0.009	0.062	0.160	-0.054	10.066	0.148	0.082	0.005	-0.046	0.088	-0.042	-0.005	0.003	-0.043	0.179	0.078	-0.012	-0.094	-0.093	0.074	-0.030	0.038	0.124	0.051	-0.009	0.083	-0.040	0.027
Mode A_Axis 3	0.015	-0.704	-0.218	-0.112	0.032	0.153	-0.162	-0.022	6.205	0.045	-0.198	-0.069	-0.294	0.134	0.102	0.104	-0.055	0.536	-0.072	0.032	0.072	0.724	0.161	0.183	0.006	-0.028	0.303	-0.170
Mode A_Axis 4	-0.002	0.748	0.051	0.085	-0.016	0.153	-0.018	0.023	0.429	0.120	-0.036	-0.005	5.012	0.011	0.103	0.084	0.012	-0.553	-0.019	0.030	-0.187	-0.475	-0.020	0.028	-0.020	-0.109	-0.191	0.033
Mode A_Axis 5	-0.009	0.601	0.095	-0.020	-0.010	-0.078	0.049	0.009	0.212	0.167	-0.015	0.014	-0.240	-0.390	0.091	-0.020	2.732	-0.197	-0.076	0.008	-0.576	-0.763	0.089	0.042	-0.140	0.076	-0.154	0.049
Mode A_Axis 6	-0.001	0.714	0.047	-0.067	-0.012	-0.161	0.079	-0.093	0.226	0.029	0.064	-0.102	-0.272	-0.332	0.117	0.062	-0.522	-0.664	0.037	0.072	0.222	-0.735	0.061	-0.026	-1.000	-0.232	-0.151	0.115
Mode A_Axis 7	0.004	0.587	-0.050	-0.003	-0.007	0.057	0.084	0.019	0.130	0.166	0.047	0.009	-0.177	-0.152	-0.015	0.049	-0.338	-0.108	-0.003	0.053	-0.238	-0.519	0.092	0.083	1.186	-0.092	-0.102	0.045
Mode A_Axis 8	0.009	-0.119	0.174	-0.014	-0.009	0.130	0.032	0.087	0.010	0.073	0.082	0.052	-0.026	-0.065	-0.044	-0.026	-0.240	0.019	-0.036	0.065	-1.007	-0.034	-0.061	0.106	-0.238	0.307	-0.098	-0.045
Mode A_Axis 9	-0.010	-0.058	-0.157	0.072	0.013	-0.208	0.087	-0.038	0.007	-0.261	0.085	-0.021	0.011	0.028	0.002	-0.068	0.009	-0.018	0.183	-0.087	-0.210	0.007	0.062	-0.125	0.015	-0.343	0.043	0.014

APPENDIX 2

Summary of Analyses
Overall Fitted and Residual Sum of Squares

Plot Label	Model Size	Sum of Components	Difference in Prop.Fit	SS(Res)	df	Prop. SS(Fit)	Prop. SS(Res)	Number of Iterations	
1	1	1x1x1	3 *	.6487	185.49358	552	.6487	.3513	9
2	2	1x2x2	5		185.27920	541	.6491	.3509	13
3	3	1x3x3	7		185.22700	532	.6492	.3508	24
4	4	1x4x4	9		185.19855	525	.6492	.3508	28
5	5	1x5x5	11		185.19800	520	.6492	.3508	29
6	6	2x1x2	5 *	.1919	84.18848	535	.8405	.1595	13
7	7	2x2x1	5		183.18077	537	.6530	.3470	7
8	8	2x2x2	6 *	.0003	84.04405	528	.8408	.1592	15
9	9	2x2x3	7		83.92046	521	.8410	.1590	17
10	A	2x2x4	8		83.90335	516	.8411	.1589	21
11	B	2x3x2	7		84.00781	523	.8409	.1591	15
12	C	2x3x3	8		83.87501	514	.8411	.1589	32
13	D	2x3x4	9		83.80325	507	.8413	.1587	27
14	E	2x3x5	10		83.78474	502	.8413	.1587	31
15	F	2x4x2	8		84.00138	520	.8409	.1591	15
16	G	2x4x3	9		83.86069	509	.8412	.1588	50 M
17	H	2x4x4	10		83.76491	500	.8413	.1587	43
18	I	2x4x5	11		83.73345	493	.8414	.1586	44
19	J	2x5x3	10		83.85519	506	.8412	.1588	45
20	K	2x5x4	11		83.75381	495	.8414	.1586	42
21	L	2x5x5	12		83.71815	486	.8414	.1586	42
22	M	3x1x3	7 *	.0732	45.38181	520	.9140	.0860	29
23	N	3x2x2	7		81.75331	517	.8451	.1549	13
24	O	3x2x3	8 *	.0012	44.76455	508	.9152	.0848	28
25	P	3x2x4	9		43.98387	501	.9167	.0833	30
26	Q	3x2x5	10		43.94628	496	.9168	.0832	29
27	R	3x3x1	7		183.04503	524	.6533	.3467	13
28	S	3x3x2	8		81.68662	510	.8453	.1547	15
29	T	3x3x3	9		44.60027	498	.9155	.0845	30
30	U	3x3x4	10		43.78999	488	.9171	.0829	32
31	V	3x3x5	11		43.64174	480	.9173	.0827	33
32	W	3x4x2	9		81.66531	505	.8453	.1547	17
33	X	3x4x3	10		44.56885	490	.9156	.0844	31
34	Y	3x4x4	11		43.70686	477	.9172	.0828	32
35	Z	3x4x5	12		43.54617	466	.9175	.0825	39
36	a	3x5x2	10		81.65800	502	.8453	.1547	17
37	b	3x5x3	11		44.56208	484	.9156	.0844	31
38	c	3x5x4	12		43.69209	468	.9172	.0828	33
39	d	3x5x5	13		43.51864	454	.9176	.0824	38
40	e	4x1x4	9 *	.0469	19.98384	507	.9621	.0379	33
41	f	4x2x2	8		81.59538	508	.8454	.1546	13
42	g	4x2x3	9		42.81344	497	.9189	.0811	28
43	h	4x2x4	10 *	.0022	18.81464	488	.9644	.0356	32
44	i	4x2x5	11		17.54637	481	.9668	.0332	30
45	j	4x3x2	9		81.50507	499	.8456	.1544	30
46	k	4x3x3	10		42.64319	484	.9192	.0808	28
47	l	4x3x4	11		18.55359	471	.9649	.0351	32
48	m	4x3x5	12		17.26077	460	.9673	.0327	30
49	n	4x4x1	9		183.00595	513	.6534	.3466	18
50	o	4x4x2	10		81.48095	492	.8457	.1543	48
51	p	4x4x3	11		42.58782	473	.9193	.0807	28
52	q	4x4x4	12		18.49004	456	.9650	.0350	32
53	r	4x4x5	13		17.16206	441	.9675	.0325	30
54	s	4x5x2	11		81.46425	487	.8457	.1543	50 M
55	t	4x5x3	12		42.56600	464	.9194	.0806	27
56	u	4x5x4	13		18.46969	443	.9650	.0350	32
57	v	4x5x5	14		17.11196	424	.9676	.0324	30
58	w	5x1x5	11 *	.0133	11.78227	496	.9777	.0223	32
59	x	5x2x3	10		42.62578	488	.9193	.0807	28
60	y	5x2x4	11		17.13771	477	.9675	.0325	32
61	z	5x2x5	12 *	.0035	9.92379	468	.9812	.0188	32
62	1	5x3x2	10		81.39777	490	.8458	.1542	35
63	2	5x3x3	11		42.42717	472	.9196	.0804	30
64	3	5x3x4	12		16.83558	456	.9681	.0319	32
65	4	5x3x5	13 *	.0006	9.62202	442	.9818	.0182	32
66	5	5x4x2	11		81.35918	481	.8459	.1541	42
67	6	5x4x3	12		42.34574	458	.9198	.0802	29
68	7	5x4x4	13		16.73971	437	.9683	.0317	32
69	8	5x4x5	14 *	.0002	9.53008	418	.9819	.0181	32
70	9	5x5x1	11		182.98933	504	.6534	.3466	20
71	A	5x5x2	12		81.33749	474	.8459	.1541	39
72	B	5x5x3	13		42.31285	446	.9199	.0801	29
73	C	5x5x4	14		16.71084	420	.9683	.0317	32
74	D	5x5x5	15 *	.0001	9.48924	396	.9820	.0180	32

Note: * = best solution for a given value of the sum of number of components
M = maximum number of iterations was reached.