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CARDBOARDING MIXED REALITY WITH DÜRER MACHINES

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Abstract

Mixed reality is a direct application of 15 to 17th century principles of anamorphosis. The inherent physicality of anamorphosis, materialized through the Dürer perspective machine makes it an excellent teaching tool for students of digital art interested in mixed reality, whose initial core competencies may be extremely varied. The needs of these students are not met by the mere ephemera of learning how to use a particular piece of software; such knowledge is both transient and limiting of imaginative possibilities. We discuss a didactic strategy of *cardboarding*, i.e., a process of deliberate rudimentarization, to expose the inner workings of opaque chains of digital processes, to both clarify the elements of these chains and create *loci* for artistic intervention at their points of connection.

Keywords

Anamorphoses
Optical Illusion
Mixed Reality
Perspective
Dürer Machines

1. INTRODUCTION

In teaching the principles of virtual, augmented, mixed, merged, or whatever new prefix to reality happens to come into the lexicon next week, it is useful to be grounded in principles neither ephemeral nor incidental. The present paper is based on didactic considerations arrived at in the course of teaching beginning Ph.D. students in the Doctorate in Digital Media Arts (DMAD) at the author's institution (Univ. Aberta), as well as previous experience in teaching the same principles to a varied audience of artists and school teachers. I will argue that anamorphosis is the core concept around which these subjects are erected, and that its inherent physicality, expressed through the Dürer perspective machine, makes it a common ground for students with diverse backgrounds and core competencies in art, programming, and mathematics. I shall also picture this specific problem as an instance of the didactic concept of *cardboarding*, which has a more general character.

2. MIXED REALITY AS RENAISSANCE HI-TECH

One should begin to teach a concept in its simplest form. According to one definition, (Milgram and Kishino 1994), Mixed Reality (MR) it is "the merging of real and virtual worlds to produce new environments and visualizations where physical and digital objects co-exist and interact in real time". For a manageable start, let's reduce this to something simpler: "the seamless integration of real and virtual objects in the same environment".

If we drop the requirement that these virtual elements be "digital", then a rudimentary type of mixed reality goes back at least as far as the early 1400s, with Brunelleschi's famous experimental representation of the octagonal Baptistery seen from the portal of the Florence Cathedral. This wasn't a mere perspective drawing. It was explicitly presented as an optical illusion—an anamorphosis—that could be overlaid with the actual view of the Baptistery, through the hi-tech marvels of painted panel, mirror, and peephole, for a direct experience in merging painting and reality (Kemp 1990). Thus is anamorphosis, as the optical mixing of real and virtual elements, at the heart of perspective, both historically and conceptually. By the 17th century the subject was exhaustively and playfully fleshed out in in François Niceron's *La Perspective Curieuse ou Magie Artificielle des Effets Merveilleux de l'Optique*. Not only is conical anamorphosis here put to use in the service of optical illusion but also reflection, refraction, and, in an appendix by Mersenne, binocular vision and the sense of depth (Niceron 1652). The arguable zenith of anamorphic *trompe l'oeil* was to be achieved at the close of that same century in Andrea Pozzo's frescoes on the vaulted ceiling of the Church of Sant'Ignazio at Rome, painted between 1685 and 1694. Here, illusionary columns merge so well with the physical architecture that one hesitates to draw the line between the two. This is the very definition of "seamless integration", hence is it not mixed reality? True, the picture is static, and must be seen from a fixed observation point, but there is no awkward helmet or glasses to wear (unless its prescription glasses), no garish clash of real and illusionary elements, and as for the aesthetics, it would be kindly not to compare it to the artistic achievements of more recent creations, be them the budget handheld illusions of Pokemon Go or the high-end demos of Microsoft HoloLens. Such comparison reminds us that industrial progress—especially in the digital age—is

first and foremost about the triad of "cheaper, faster, and more abundant". There is a gain in efficiency often payed for by a certain loss of charm. This kind of progress is nothing to sniff at. "Cheap" matters: Pozzo's masterful illusion depends heavily upon the space at his disposal, and few of us could afford such a canvas as the ceiling of a church.[1] "Fast and plentiful" matters: we get sixty frames per second on a set of VR glasses instead of a single static "frame" that would take years and a fortune to paint—and if a fresh illusion is created many times per second, the point of view can shift, the illusion can move and truly be interacted with. Cheap, Fast, and plentiful, matters. It opens the door to new forms of creativity, even if the Pozzo's are few and far between—weren't they always? A more serious problem concerns us. Not a loss of charm or of artistry, but of knowledge.

1
Or, for that matter, a slot in the large *rotundas* of 19th century panorama craze (Huhtamo 2013), another iteration of mixed reality closer to our days and expressive of the technology of its time.

3. KNOBS ON BLACK BOXES

Having granted that MR is a hi-tech elaboration of 15-17th century principles, one might still question what its more primitive implementations can offer in didactical terms to students of digital art. I think the answer is best given by first indulging in a more general discussion. It is a recurrent downside of industrial progress that the powerful tools at our disposal tend to lead, by their very efficiency, to a paradoxical loss in knowledge. One recalls Adam Smith praising the gains of division of labour while lamenting the dimming of the worker's mind, who goes from craftsman to mere cog. Automation often begins by turning humans themselves into automatons.

Take for example the task of drawing a horizontal plane anamorphosis, a kind of optical illusion common in street art and in table-top illusions. You will find many easy tutorials on the web to achieve these table-top illusions, usually under the designation of "3D art". These tutorials mostly consist of selecting menus and click-and-dragging handles in photoshop. They allow the student to achieve his aim without in any way understanding the basis of what is achieved; hence, they circumvent knowledge rather than facilitate its acquisition. Consider the steps of a typical tutorial:

1) Take a photo of a cube sitting on a table. Load it into Photoshop.
2) Select the perspective grid tool. Fit a perspective grid to the photo of the table so as to cover the image of the cube.
3) Rectify the perspective grid by pulling on handles, deforming the photo along with it.
4) Print the rectified image. This image will produce the illusion of a "3D" cube when photographed from the original spot.

This is rather unsatisfying. At no point does the student need to know what a perspective grid is, how to draw one, how to rectify it, or have any idea why the sequence of steps leads to an optical illusion. The elementary operations involved—say, the smooth deformation of the gridded picture—do not correspond to any physically realizable operations. Further, not only is its practical scope limited to small scale table-top anamorphoses, it offers no conceptual basis to inquire beyond that scope. You learn how to turn knobs on a black box, devoid of any theoretical knowledge of what goes on inside it. This is the difference between achieving *competency* and achieving *understanding*. *Compe-*

tencies, easily measured and therefore so loved by the accountancy obsessed bureaucrats of education, is something that can be achieved even by machines. *Understanding* is something altogether different.

This problem of *savoir-faire sans savoir* (knowing how-to-do without knowing) has been addressed philosophically by Stiegler (Stiegler 2010). It has recently been addressed artistically by Rodriguez (Rodriguez 2016), who expressed similar preoccupations to my own. My purpose here is to address it didactically. I would relate it tangentially to the growing bureaucratization of society (Graeber 2015) and of the thought-process itself. I contend that most digital creation tools, by the opaqueness concurrent with their efficiency, enhance a bureaucratic frame of mind. A knowledge of doing through menu clicking, not only is not real knowledge, it is not even real *doing*. What we learn is how to send a properly formatted petition for a black box to do *for us* — a bureaucratic incantation. This feeling often remains even when bypassing the UI and looking at actual code. As argued in Papert and Turkle (1991), reliance on black boxes in programming traps the student into limited modes of thought and expression. Object-oriented programming often embodies, through encapsulation and abstraction, the very essence of black boxing, and message passing to methods has often all the intellectual charm of carefully filling out forms. Like all bureaucracies, OOP starts from the assumption of a state of chaos that must be controlled, focusing on preventing any individual programmer in a large team from doing too much damage to things he does not fully understand (on this view see (Graham 2017)). This is arguably a perversion of OOPs original aims.

It was to counter this frame of mind which I took, in the teaching experiment I will discuss, a deliberate attitude of radical *unboxing*, that is, of exposing the hidden clockwork of complex technology through a two-pronged strategy. First, to state a clear theoretical principle that is both simple and has inherent physicality. Second, by *cardboarding* this principle, that is, by expressing and exploring its physicality by the simplest technological means. In both steps there is an insistence in a process of materialization — favouring physical machinery over software implementations. Only after a process is thoroughly understood in this way is the student allowed to optimize it by taking recourse to higher technology. We do not, after all, wish to forego current technology, we just insist on seeing its clockwork first. The simple rule is: you are not allowed to use a machine you do not yet understand.

3.1. Google's Cardboarding of VR as a case study

Having explored this didactic process for a few years, only very recently did I start to call it *cardboarding*. The term is of course an allusion to Google Cardboard — not the VR platform per se, but the simple yet ingenious VR viewer made of elementary, fully exposed parts — cardboard frame, lens element, magnet — that is a great example of didactic rudimentarization. Intended as a cheap viewer accessible to all, this simple yet brilliant idea made virtual reality a physically understandable process by literally carboarding a technology that is usually presented as a slick, encapsulated whole. By exposing its gears, it made the abstract concept of VR into an entailment of individual elements — a conceptual and physical chain, where the form of the frame, the optics of the lens element, the principles of stereography, can be considered first separately and then in their interaction. Also, by separating those elements, it created points for interven-

tion, both in the elements and in the links of their entailment. Fascinating discussions ensued online, with users tinkering with the design, altering it, asking if they could do without the lenses, for instance, and hence learning their purpose, discussing virtual images and focusing distances, or wondering if they might change the lenses, or grind their own, or improvise them with water-filled enclosures cut from plastic bottles; or if the magnetic button could be improved or done away with; or how the stereography might be altered by distance to the eye, which of course leads to a need to understand stereography itself. The process of tinkering with variations on the cardboard machinery has arguably been more intellectually and artistically stimulating than any of the VR apps available for use with the device. Instead of one more consumer forum petitioning for features it created a forum of doers engaged in a creative process.

4. CARDBOARDING MIXED REALITY THROUGH ANAMORPHOSIS

In order to *unbox* Mixed Reality for my students, I focused on conical anamorphosis as the concept that unifies the whole subject, being the geometric foundation and historical precursor of MR methods. By anamorphosis I do not mean the confusingly vague dictionary definition. In a recent paper (Araújo 2015) I have defined the term precisely:

> **Let O be the observer's viewpoint, S a compact surface, and X a tridimensional topologically closed set. X defines a cone of vision $C_O(X)$ with vertex at O. Let $C_{(O,S)}(X)$ be the topological closure of $C_O(X) \cap S$. We say that $C_{(O,S)}(X)$ is the *anamorphosis* of X onto S relative to O and that $C_{(O,S)}(X) \setminus (C_O(X) \cap S)$ is the set of vanishing points of the *anamorphosis*.**

This is very concise; it even defines vanishing points within the context of anamorphosis, with no need of defining perspective, and frames anamorphosis as a game of compactification, i.e., of making closed and bounded images of unbounded objects (closed and bounded is closely related to computable). As it is, however, it is too abstract for our needs. We need to translate this for students with varied backgrounds in art, mathematics and computation. We start by stating a basic principle of monocular vision which we call the *principle of radial occlusion:*

> ***(R.O.): For an eye at point O, points P and Q are indistinguishable if they are on the same ray from O.***

This sounds simple to the point of triviality: a thing hides another if it is "in front of it". The proof that it isn't trivial is that it isn't generally true; it is violated by refraction at the interface between optical media, e.g., by a lens. Since the human eye has a lens element, the principle clearly has a limited scope of validity. Still, it applies approximately enough to be usefully descriptive of our default interaction with light. When it does apply, it suggests a simple method for creating an optical simulacrum of a 3D object via a drawing on a 2D surface: a conical anamorphosis. We can state matters in this way:

> **Given a 3D object X (say, a cube), and point O in space (called the observer's point, or the viewpoint), the points of X define a cone of rays stemming from O. If we put a surface S between O and the object X, the intersection of this cone with the surface S will create a 2D region (a drawing) on that surface. I say that this drawing is the anamorphosis of X on S with relation to the point O. Since the principle of radial occlusion can be applied to each of the rays of the cone, then this anamorphosis should be optically indistinguishable from the original object X, when seen from point O.**

Now we are ready for "cardboarding" this concept. The inherent physicality of anamorphosis can be made evident by the use—or rather the subversion—of Dürer machines.

We have all seen these simple devices under the guise of "perspective machines", though few of us took the trouble of actually implementing one. They are always briefly mentioned in perspective courses and then promptly forgotten. In my course I insist on calling them anamorphosis machines rather than "perspective machines", both because it is more exact and because I haven't defined perspective yet. This is a deliberate omission. Students have so many preconceptions about perspective that the term is better avoided at the start lest it interfere with the understanding of anamorphosis. We define it later in a general way that encompasses also the curvilinear perspectives.

The Dürer anamorphosis machine can be implemented very simply with a picture frame and a fixed point O (e.g., a nail) to which a thread it attached and which represents the position of the eye (see Figure 1). The machine operates thus: A point P is chosen on the object to be drawn (e.g., a lute). The thread is pulled straight from O to P; it crosses the plane of the frame on a point Q. By the principle of radial occlusion points P and Q are indistinguishable when seen from O. Hence if we repeat this process for enough points P_i we obtain a drawing on the plane of the frame that should be indistinguishable from the original lute when seen from O. This picture is a finite approximation of the anamorphosis of the lute onto the plane, relative to O.

Of course, this anamorphosis onto the vertical plane of the frame is what we usually call a perspective. For our purposes this is a terrible choice of surface, since again it would call up all the misconceptions the adult student holds about perspective and obscure the lessons in the blindness of familiarity—we see what we expect to see. To make the student truly realize the illusionary nature of the construction we subvert it in a way that shakes his expectations: we choose the horizontal plane of projection—the table—for anamorphic surface. This is analogous to the artist's trick of turning a drawing upside down to see it anew.

We point out to the student that Dürer's machine can be run both back and forth, that is, that the principle of radial occlusion also says that point P on the object is equivalent to a point R on the table, and therefore the machine can create an optical illusion on the table by running the thread forward rather than backwards from P. The image thus created on the horizontal plane is still an anamorphosis, that must be optically equivalent both to the original object and to the anamorphosis on the vertical plane (Figure 1). The student can verify this prediction by actual construction of the anamorphosis point by point. For an object made up of straight line segments (say, a cube) the construction can be made exact in a finite number of steps. Then the student can verify the efficacy of the optical illusion by looking at it from O (monocularly, of course—looking through

a cardboard tube helps) and comparing the anamorphosis to the actual object both in superposition and side-by-side. After direct checking with the naked eye, taking a photo from O (Figure 2) helps leisurely and detailed verification of the result, of its character of "mixed reality", and also of its limitations. For instance, by shooting with a large aperture to reduce depth-of-field, one violates radial occlusion through refraction, and the drawing will defocus away from the focal plane.

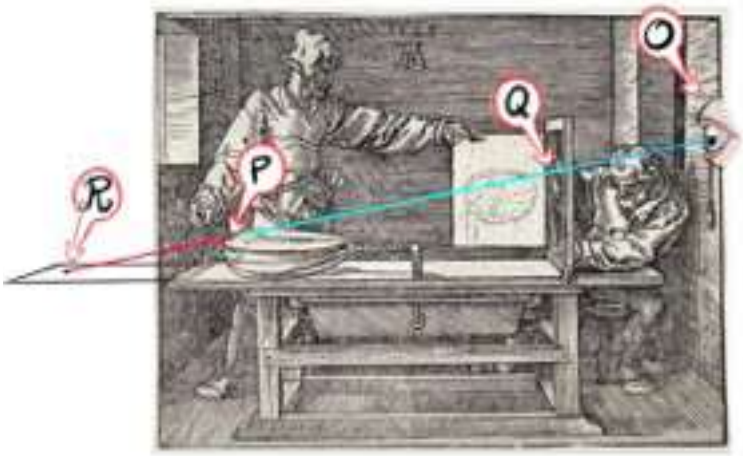


Fig. 1
Perspective Machine by Albrecht Dürer (defacement by the author).

Notice our ruse: Whether a plane is "vertical" or "horizontal" is of course irrelevant for the construction; but in choosing the horizontal plane of the table we tricked the student into placing the object at a larger angle from the perpendicular to O. What we mean colloquially by a "linear perspective" is simply a plane anamorphosis that happens to be drawn at a "small" angle to the perpendicular from O. By increasing this angle we make the illusion *fragile*, unforgiving of even small deviations of the eye from point O. This in turn makes both the illusionistic effect more impressive when correctly achieved, and the importance of the viewpoint O as a real point in physical space perfectly evident to the student in a way that mere words could not. There is more didactic value in a few moments of wobbling the head near O and watching the illusion oscillate wildly than in any amount of lecturing (present writing included) on the importance of respecting the viewpoint. Having seen the optical illusion work under these unusual and striking circumstances in the horizontal plane, the student realizes the illusory nature of ordinary perspective drawings in a new way. This role of the viewpoint is always payed lip service to by teachers of perspective and just as ignored in practice as Dürer machines, because it is never made physical. The premature focus on perspective diagrams and constructions leads to its dematerialization, converting it from a spatial point to a dot on a diagram. One forgets that perspective diagrams are machines that gain efficiency through abstractions. Using them too soon is typical black-boxing. I have argued elsewhere (Araújo 2017) that some pervasive misconceptions regarding Leonardo's paradox are much owed to premature focus on diagrammatic construction rather than physical experiment. Next we remark to the student that nothing in the definition of anamorphosis obliges S to be a plane. The illusion should work on any curved surface. The student is therefore encouraged to experiment the Dürer machine on different surfaces: the cylinder, the sphere, etc. (Figure 3).

The student is now ready to raise the level of abstraction, based upon the concepts he has experimented with. He can be made to reason that when the images of parallel spatial lines terminate within the projection surface, they must do so

on common limit points—their vanishing points—and that these points can be determined very physically, by merely pointing a finger (or our thread) from O, parallel to the spatial lines, and finding the point of S that the finger points at. Thus we materialize vanishing points. We further note that defining them at the level of anamorphosis, as objects depending only on X, O, and S, means that they will be equally defined for all central perspectives (including the curvilinear ones) once we finally get around to defining what a perspective is.

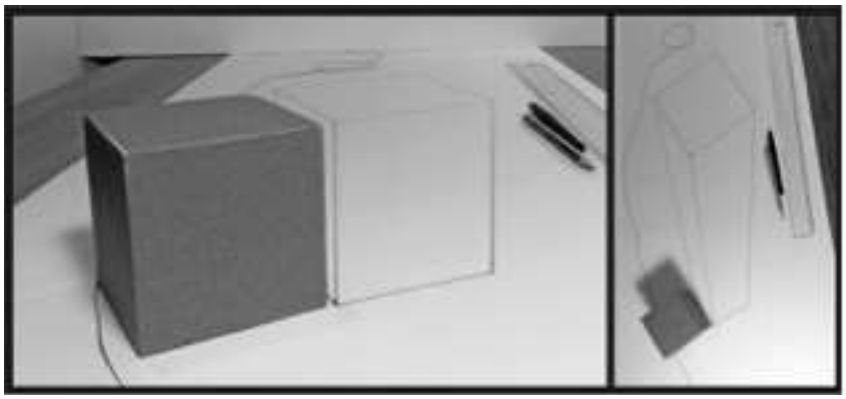


Fig. 2
Plane anamorphosis of a cube (student work by Manuel Flores). Notice convergence of images of vertical edges to a vanishing point (on the ground, exactly beneath the observer's point where the thread is fixed).

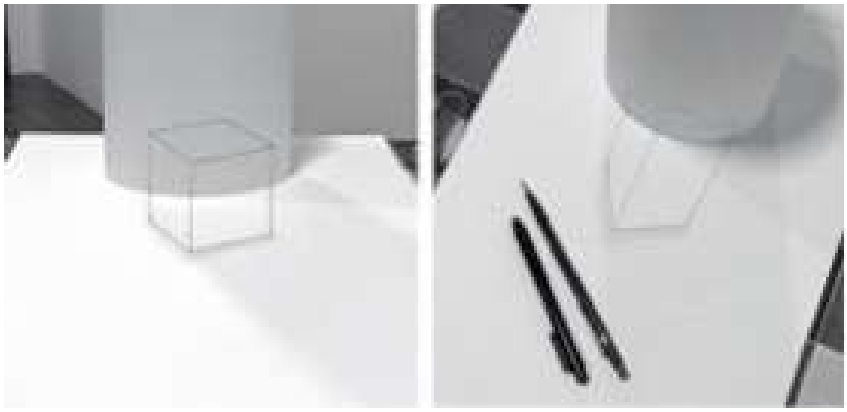


Fig. 3
Anamorphosis of a cube on a cylinder and a plane. Student work by Hugo Silva.

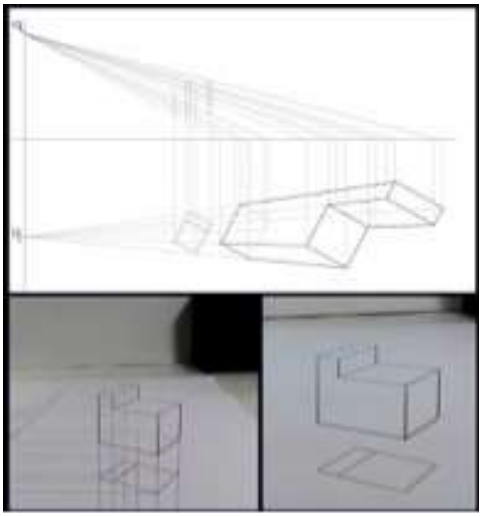
We now encourage the student to verify that lines do converge in this way on the drawings he has experimented with. Habituation has inured most people to the wonder of vanishing points on the "perspective plane" but they are always surprised by realizing that in the construction of Figure 2 the images of the vertical edges of the cube converge exactly under his eye, and intrigued to find such convergence in anamorphosis over more general surfaces, where the lines project into curves.

Now the student is safe for a step up in the process of de-materialization and efficiency: virtualizing Dürer machines through orthogonal diagrams (Figure 4 (left)) with full understanding of the physical process they represent. A real object is no longer required; instead an imagined object has its top and lateral orthogonal views designed on the page and the thread is replaced by drawn lines stemming from the top and side views of point O (points O_T and O_S respectively). This is simply a drawing of the action of the physical machine, and if the work is done in 1:1 scale the final drawing obtained on the top view is already the anamorphosis, so the optical illusion can be visualized by placing the eye directly above point O_T, raised up from the sheet by the height of point O_S.

These diagrams are then generalized for projecting on cylinders, ruled surfaces, spheres, etc., each case requiring specific geometric reasoning to execute the intersections and to translate the points thus obtained into a physical anamorphosis. For example, in the cylindrical case, one can unroll the cylinder diagrammati-

cally, project the intersections and then cut out the resulting drawing and roll it back onto a physical cylinder (Figure 4 (right)). At this point one can optionally use the Geogebra software to good effect, to optimize geometric operations already well understood. Being an implementation of ruler-and-compass euclidean geometry, Geogebra has the advantage of aligning well with the geometric operations that a human can physically perform.

At this point we may enhance the mixed reality effect by addressing binocular vision. We can draw two anamorphoses of the same object, from two different points O_L and O_R, corresponding to the point of view of the left and right eyes respectively. By drawing the left drawing in blue and the right one in red and using classic red-blue 3D glasses to present to each eye only its respective view, an anaglyphic anamorphosis is created. The reader has surely seen an anaglyph before, drawn of course on a frontal plane. It is very different to see an anaglyphic anamorphic cube floating in 3D on top of your table, surrounded by ordinary office equipment, and watch it wobble as you shift your eyes close to the observation spot. The sensation can only be described as mixed reality—the more vivid for being devoid of screens or cumbersome apparatus. This experience cannot be properly provided on this printed page, so I recommend it as an exercise to the reader.



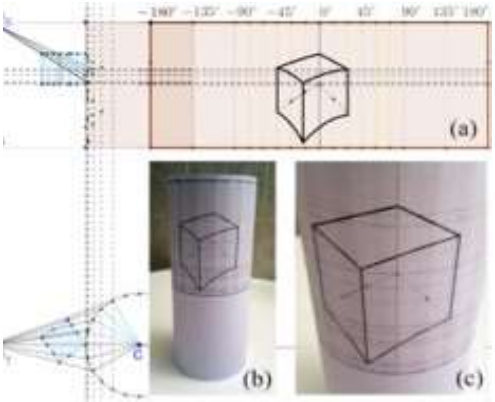


Fig. 4—Left
Plane anamorphosis of L-shaped object, drawn from orthographic views, to be viewed from above O_T with the eye at the height determined by O_S. (Student work by Manuel Flores).

Fig. 4—Right
From orthographic views one obtains a cylindrical perspective of a cube on a rectangle (*a*), that can be cut and folded (*b*) onto a cylindrical anamorphosis, to be seen from the spatial point at a height of $|GO_S|$ over O_T (*c*).

Then we move to immersive anamorphic surfaces, such as cylinders or spheres with the viewer now placed at the axis or at the center respectively. Projection diagrams can be developed for these cases with more refined descriptive geometry, analytic maps can be obtained, line images can be classified (sine curves for cylinders, great circles for spheres, etc.), and finally code can at last be written, as a mechanization of a properly understood process. Students can choose among these directions the ones that accord to their inclinations, backgrounds and artistic needs (space limitations dictate brevity here, but the process again is one of progressive experimentation as before).

Let me also speak briefly of colour. We've considered up to now only "wireframe" objects. Colouring them gives substance and solidity to the optical illusion, and motivates a discussion of the psychophysical aspects of color (CIE color matching, metamerism, Grassman's laws, etc) well grounded in experiment. A judicious use of gouache, markers, and mobile apps allows for rudimentary color matching experiments that make students think on how a color sensation can be measured. Alas, it is common to study color by discussing color spaces as abstract entities and forget to relate them to experiments of eye and brain; which is like

teaching geography by discussing how maps interrelate while ignoring actual land. I find it useful to define a notion of color anamorphosis to point out that just as a 2D drawing mimics a 3D body by exploiting a limitation of the eye (radial occlusion) we can exploit another limitation of eye and brain — metamerism — that allows a color sensation caused by a body with a certain power profile under a given illuminant (say, an apple under the sun) to be mimicked by a different body / illuminant combination (say a dab of paint under an incandescent bulb), and that just as before the trick only worked when seen from O, this new ruse only works when seen from O under the prescribed illuminant. These abstract concepts can be *cardboarded*, by a sequence of experiments that first attribute values (Araújo n.d.), then hue and saturation, in a very material way, to the planes of the wireframe anamorphoses already produced, developing the sophistication of the illusion side-by-side with that of the reasoning.

At last, we reach the notion of perspective. I have stated elsewhere (Araújo 2015) a general notion of central perspective as an entailment of conical anamorphosis followed by a flattening map (with some technical properties not relevant here) that maps the anamorphosis to a planar drawing. This includes the curvilinear perspectives, linear perspective being the special case in which the flattening is just the identity map modulo change of scale. This too is explored by hand, with simple instruments, mainly ruler and compass, before actual software is used. Having established anamorphosis above, one treats all the major curvilinear perspectives with great speed by just studying their flattening maps, as in cartography. Again, all work is done by hand and only then does software enter the picture. Students are encouraged to compare their hand drawn cylindrical/spherical perspectives to the ones they can produce at the click of a button (see Figure 5), and to gain intuition regarding the scope of each curvilinear perspective, to predict, to interpret and to plan for an effect.

4.1. Locus of interaction and locus for intervention

An essential part of the didactic experiment of *cardboarding* is the interplay between analog and digital work. Students are encouraged to try things out in their preferred software tools in parallel with the required *cardboarding* exercises (but not *instead of*). This leads to a freeform play that varies according to student. I will relate two instances. Once, while showing that lines project to sinusoids in cylindrical perspective I proposed the simple experiment of physically immersing a tilted cylinder in water and unrolling it to reveal a sinusoidal watermark. Not content with dipping toilet paper cores into water bowls, a student insisted on using a *sketch up* plugin that unrolls developable surfaces. This, I suppose, could be a dire indication that today's students identify real space with their digital canvas and trust it more than their own eyes; but it can also be seen as physical experiment inspiring digital exploration, or as a healthy sceptic testing whether the software indeed performs as the mind determines it should.

In another instance, I provided a class with an example of a 360 degree spherical perspective of a room, drawn with ruler and compass. A student took it upon himself to examine this drawing, model the room in 3DS MAX, and render it through a spherical lens, to compare the result with the original hand drawing (Figure 5 (a,b,c)). We discussed the reasons for several disparities: differences in shadows motivated a discussion of digital light sources of various types; the appearance of diverging, fuzzy lines near the edge of the digital render led to a

discussion of the limitations of finite approximations versus exact mathematical projections, dispelling the notion that we can trust exactness to arise from mere computational force. Conversely, the computer checked human fallibility: the student noted the doors would not align, whence I found that I had absentmindedly mistaken a faded wall line for the line of the door and drawn it at the wrong position. So digital and analog kept each other in check. This kind of interplay can be enhanced by the use of VR panoramas. A spherical perspective drawing can be done in ruler and compass and then fed to a VR panorama display (available at Flickr, Google, etc.) to be checked for accuracy of the resulting illusion. VR panorama rendering works by mapping spherical perspectives onto planar anamorphoses (Figure 5 (d,e)), hence providing an excellent feedback loop between analog and digital work.

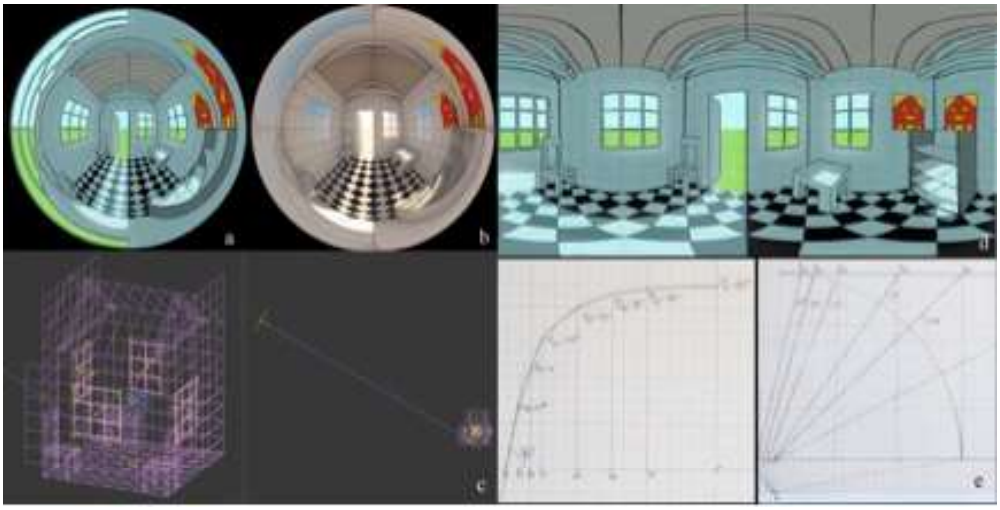


Fig. 5
a) Drawing by the author of a 360-degree spherical perspective with the ruler, compass, and nail method (Araújo 2015) (digitally colored). Student Hugo Silva reconstructed the scene in 3dsMAX (c) by observation of the initial drawing and then rendered the model with a simulated spherical lens (b). Notice the error on the door in the author's original drawing. e) ruler and compass construction of the same view in equirectangular perspective (d) which can then be viewed online as a VR panorama (see (Araújo n.d.)). This back-and-forth between material and digital methods helps clarify both the workings of the perspective and the functioning of the rendering software.

Such interplay enhances the understanding of both the digital and the material processes. One can think of the computer and the human as machines with different primitive operations. For a programmer, understanding how to draw a spherical perspective line means expressing it in the computer's set of primitives, obtaining the parametrizations that, plotted pixel-by-pixel, will render the perspective image. For the human artist, understanding the same plot means reducing it to human-executable *primitive operations*, i.e., to a reasonable number (not in the thousands, but in the dozens) of actions executable by a human hand and eye aided by simple instruments. Both formulations of the task carry their own significance and illuminate the problem in their own way. To restrict ourselves to brute-force solutions requiring hundreds of computations is to block ourselves from the better part of human understanding. And to black box the problem away—hiding the multitude of operations under a method call—is not only to lose access to a chunk of knowledge but, as argued by Papert and Turkle (Papert and Turkle 1991), to lose *closeness to the object* and with it access to modes of reasoning that presuppose such closeness. In this sense I would argue for cardboarding not only as a learning tool, but as a stimulus to creativity. Unboxing digital tools and exposing them as chains of simpler physical and conceptual elements creates loci for artistic intervention—tinkering spots—both within each element and at the connections between elements (as we have seen regarding the google cardboard viewer). Consider the example of perspective. The artist can render a spherical perspective at the flick of a button. But if we unbox this process to show the linkage of the anamorphosis and the flattening, we expose these elements to inquiry and manipulation. Each element exposed is

a new locus for intervention. The artist who exposes this entailment is no longer confined to merely composing his picture elements and choosing a perspective from a menu — he can start by composing the space itself. He can imagine his own flattening to specification, or not specify it at all. Some of my students, having learned how to plot cylindrical perspectives, soon felt free to disrespect its rules whenever required for good effect — once you know what a rule does you know when and how to break it; or to change it: you can consider different surfaces for the anamorphosis step. Or you can act at the very link between the two steps and question why must we have this entailment at all? Can you reverse it, fully or half-way? VR panoramas are such a reversal, from curvilinear perspective back to anamorphosis, and a new place for analog intervention into a digital process. Can you do away with the entailment altogether, and have a perspective without anamorphosis? (yes) Without radial occlusion? (yes) What would it look like? What would that mean for hidden faces algorithms? Through such questioning, the scope of the imagination is enlarged. Finally, this also fosters a better dialog between artist and developer, in a field were such dialog is necessary and often difficult — an artist without knowledge of the principles does not know what to ask for, since he knows not what is possible. He will request fanciful features that are at once impracticable and not daring enough. But one who knows his tools widens the range of fruitful speculation.

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