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NOTE

REMARK TO THE PAPER OF S. SAMKO, "A NOTE ON RIESZ FRACTIONAL INTEGRALS IN THE LIMITING CASE $\alpha(x)p(x) \equiv n$", FROM FCAA, VOL. 16, NO 2, 2013

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Abstract

We improve the formulation of the main statement in the paper "Note on Riesz fractional integrals in the limiting case $\alpha(x)p(x) \equiv n$", published in this journal, *Fract. Calc. Appl. Anal.*, Vol. **16**, No 2 (2013), DOI: 10.2478/s13540-013-0023-x;
http://link.springer.com/article/10.2478/s13540-013-0023-x.

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Key Words and Phrases: fractional integral, Riesz potential, variable exponent Lebesgue space, variable order, BMO (boundedness of the maximal operator)

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Theorem 3.1 in the above cited paper has a cumbersome formulation where the exponents $\alpha(x)$ and $p(x)$ are related to each other by the relation $\alpha(x)p(x) \equiv n$ and the assumptions on $p(x)$ were formulated weaker than for $\alpha(x)$, while they are immediately inherited from the above relation.

The formulation of Theorem 3.1 in that paper should be replaced by the following.

THEOREM 3.1. *Let Ω be a bounded open set and p is log-continuous and $1 < p_- \le p(x) \le p_+ < \infty$. Let also $\alpha(x)$ be Hölder continuous (of an*

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arbitrarily small order). If $\alpha(x)p(x) \geq n$, then the Riesz potential operator is bounded from $L^{p(\cdot)}(\Omega)$ to $BMO(\Omega)$.

Clearly, some disadvantage of this reformulation is that at the points $x \in \Omega$ where it may be $\alpha(x)p(x) > n$, we expect that the Riesz fractional integral behaves better than just a BMO function, being there locally Hölder continuous of order $\alpha(x) - \dfrac{n}{p(x)}$. However, the advantage is that with this reformulation we should not require that $p(x)$ must be Hölder continuous.

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