

Workshop on Inverse Obstacle Problems

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by

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Model-based inverse problems in underwater acoustics

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Outline

- Model-Based Processing (MBP)
- the problem
- the inversion methods
- real world examples
- conclusions



The problem

- **Implicit Model Conditioning**

Data: $y(\mathbf{r}_o) = g(\mathbf{r}_o) + n$

Model: $\tilde{g}(\mathbf{r})$

$$\hat{\mathbf{r}}_o = \min_{\mathbf{r}} \mathcal{F}[y(\mathbf{r}_o), \tilde{g}(\mathbf{r})]$$

- **Model with random components**

Data: $y(\mathbf{r}_o) = g(\mathbf{r}_o) + n$

Model: $m(\mathbf{r}) = \alpha \tilde{g}(\mathbf{r})$

α and $\mathbf{r}$ random



Forward models

$$\nabla^2 p + \frac{\omega^2}{c^2(z)} p = 0$$

- Ray solution

$$\tau = \int_{\Gamma} \frac{ds(z, r)}{c(z)}$$

$$p = p_o \sum_{i=1}^I \frac{1}{4\pi s_i} \exp(j\omega\tau_i)$$



Forward models(cont.)

- Normal-mode solution

$$p = p_o \sum_{m=1}^M \frac{\phi_m(z_o)\phi_m(z)}{\sqrt{k_m r}} \exp (ik_m r - \alpha_m r)$$

$$\frac{d^2\phi_m}{dz^2} + \left[\frac{\omega^2}{c^2(z)} - k_m^2 \right] \phi_m = 0$$



Perturbation inversion example

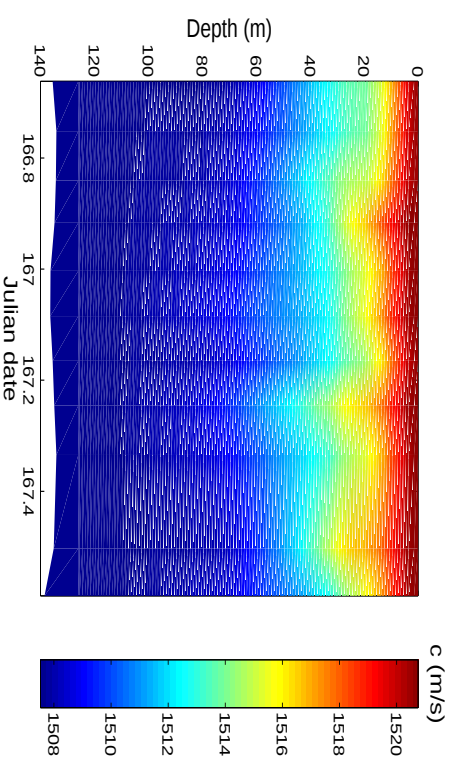
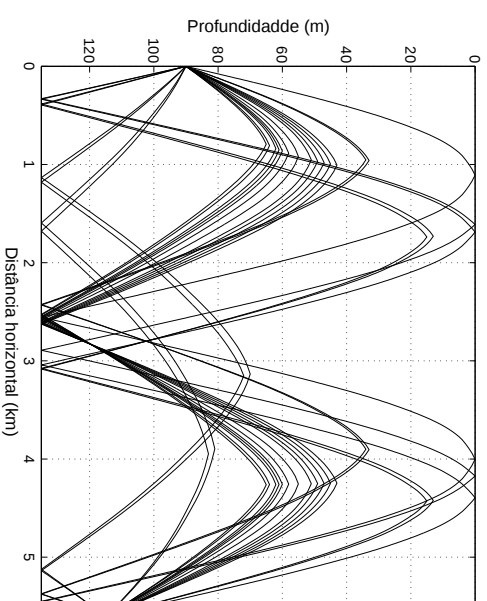
INTIMATE sea trial June 96 - Nazaré, Portugal

$$\Delta\tau \approx -\int_{\Gamma} \frac{\delta c(z)}{c_o^2(z)} ds$$

$$\Delta\tau = \mathbf{E}\delta\mathbf{c} + \mathbf{n}$$

$$\delta\mathbf{c} = \Psi\alpha$$

$$\alpha = [\Psi^t\mathbf{E}^t\mathbf{E}\Psi]^{-1}\Psi^t\mathbf{E}^t\Delta\tau$$





Forward model-based example

Data model:

$$y(t, \mathbf{r}_o) = x(t, \mathbf{r}_o) + n(t)$$

$$x(t, \mathbf{r}_o) = g(t, \mathbf{r}_o) * s(t)$$

Mean square error estimator:

$$\hat{\mathbf{r}}_o = \min_{\mathbf{r}} E[\|\mathbf{r} - \mathbf{r}_o\|^2]$$

The Matched-Filter (or correlator-receiver):

$$H(\omega, \mathbf{r}_o) = H_o \frac{G^*_\omega(\omega, \mathbf{r}_o) S^*_\omega(\omega)}{P_{nn}(\omega)} e^{-j\omega\tau}$$



Forward model-based example (cont.)

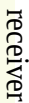
$\Rightarrow$ **Neyman-Pearson detector** (WGN)

$\Rightarrow$ maximizes the **Signal-to-Noise Ratio (SNR)**

$$\rho(t, \mathbf{r}) = \frac{1}{2\pi} \frac{\left| \int_{\Omega} H(\omega, \mathbf{r}) G(\omega, \mathbf{r}) S(\omega) e^{j\omega t} d\omega \right|^2}{\int_{\Omega} |H(\omega, \mathbf{r})|^2 P_{nn}(\omega) d\omega}$$

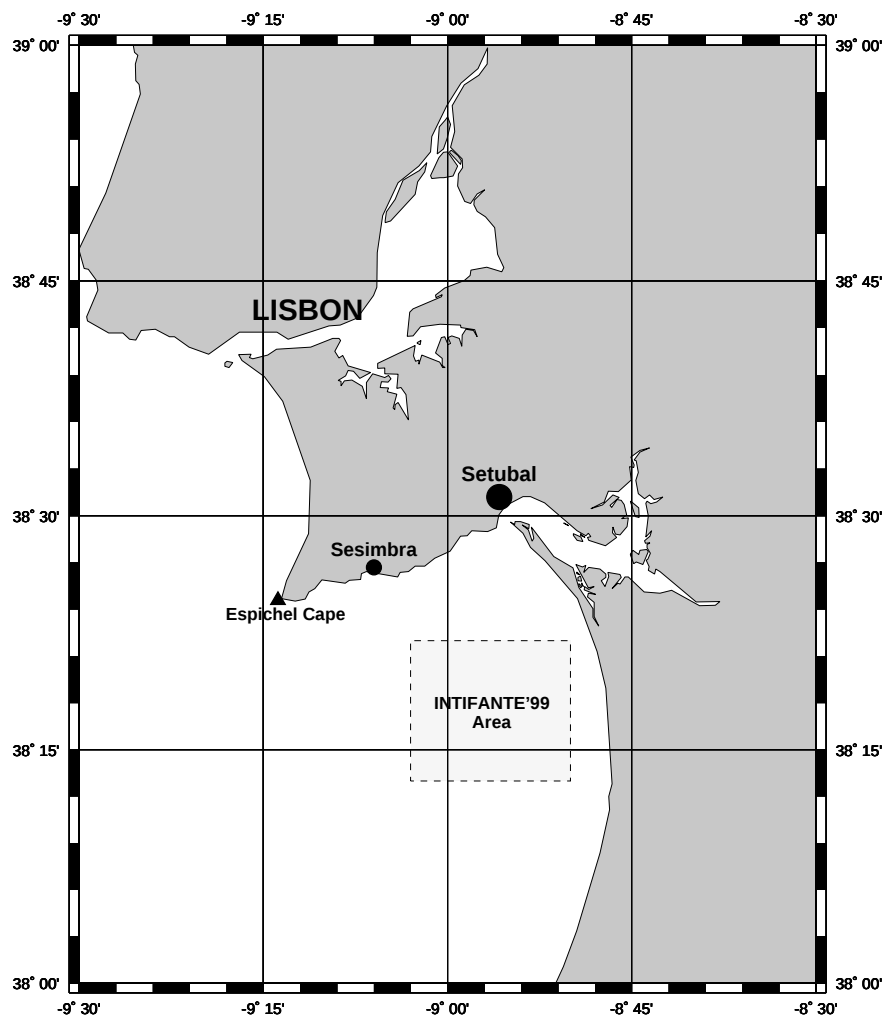
or the SNR estimate

$$\hat{\rho}(t, \mathbf{r}) = E \left[\frac{\left| \int_{\Omega} \tilde{G}^*(\omega, \mathbf{r}) S^*(\omega) Y(\omega, \mathbf{r}_o) e^{j\omega(t-\tau)} d\omega \right|^2}{\int_{\Omega} |\tilde{G}^*(\omega, \mathbf{r}) S^*(\omega)|^2 P_{nn}(\omega) d\omega} \right]$$

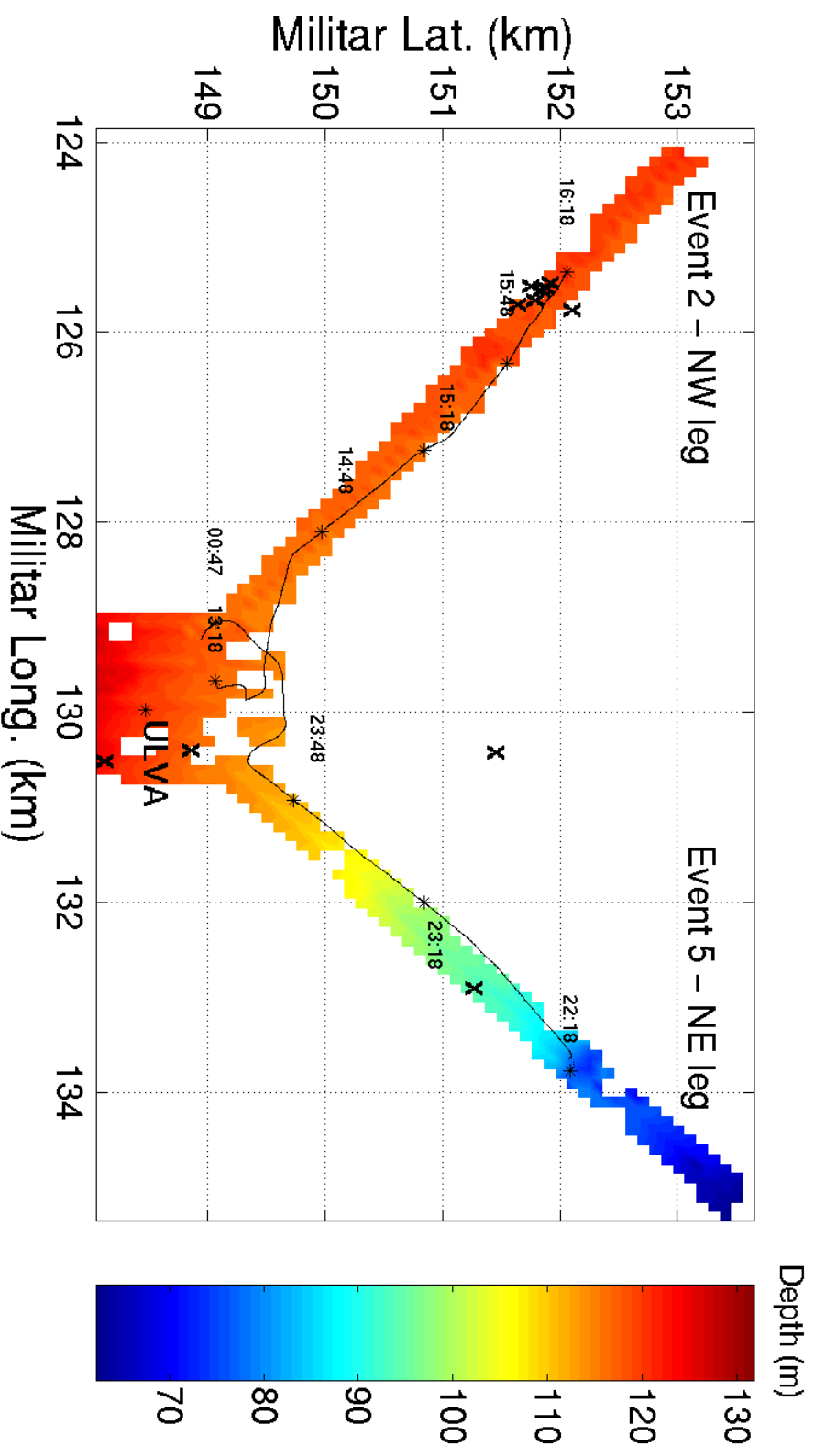




INTIFANTE'00 Experimental Site



Events 2 and 5: bathymetry and runs

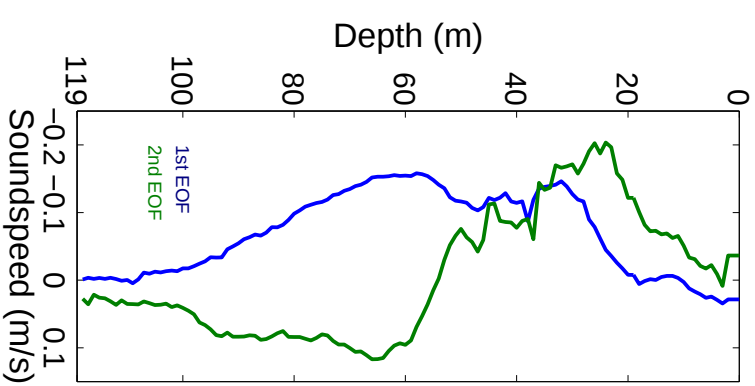
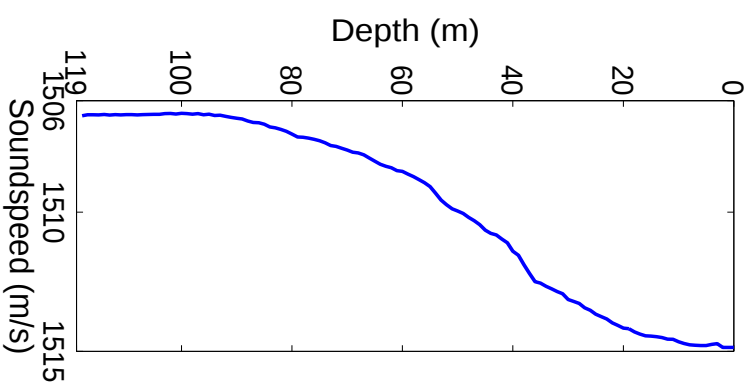




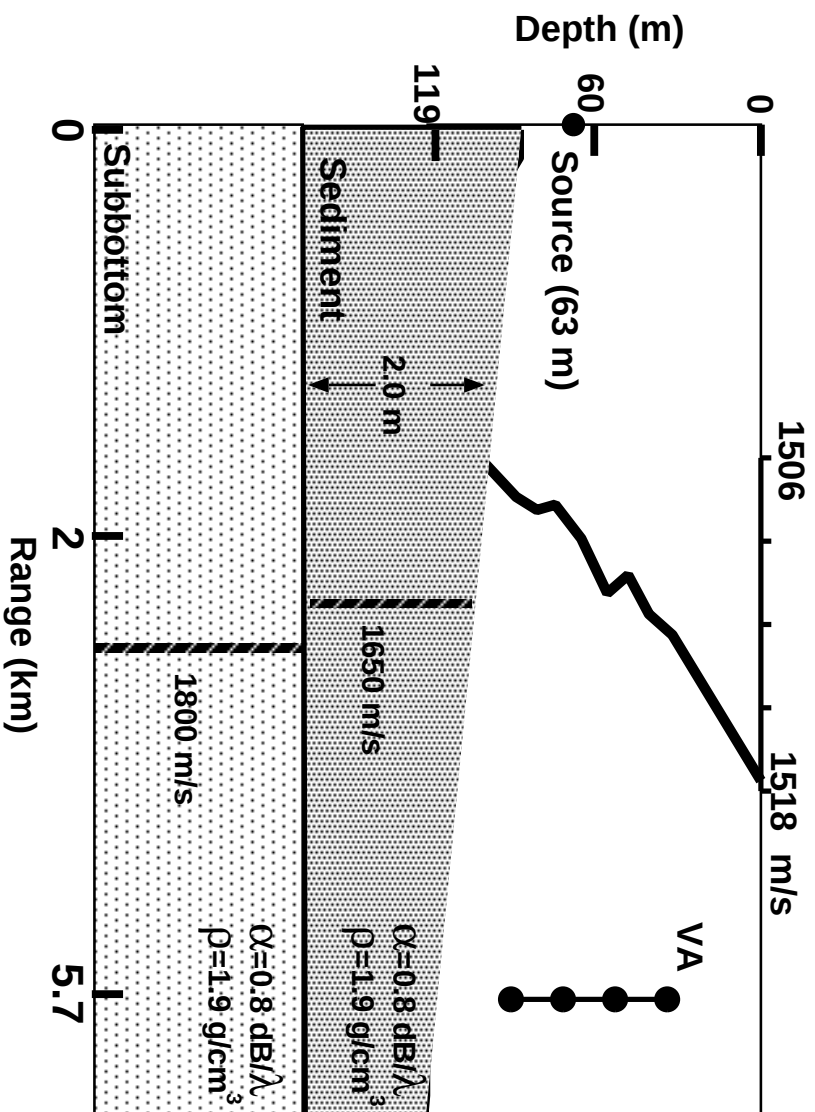
Empirical Orthogonal Functions

$$\mathbf{c}_{\text{EOF}} = \hat{\mathbf{c}} + \sum_{n=1}^N \alpha_n \mathbf{u}_n$$

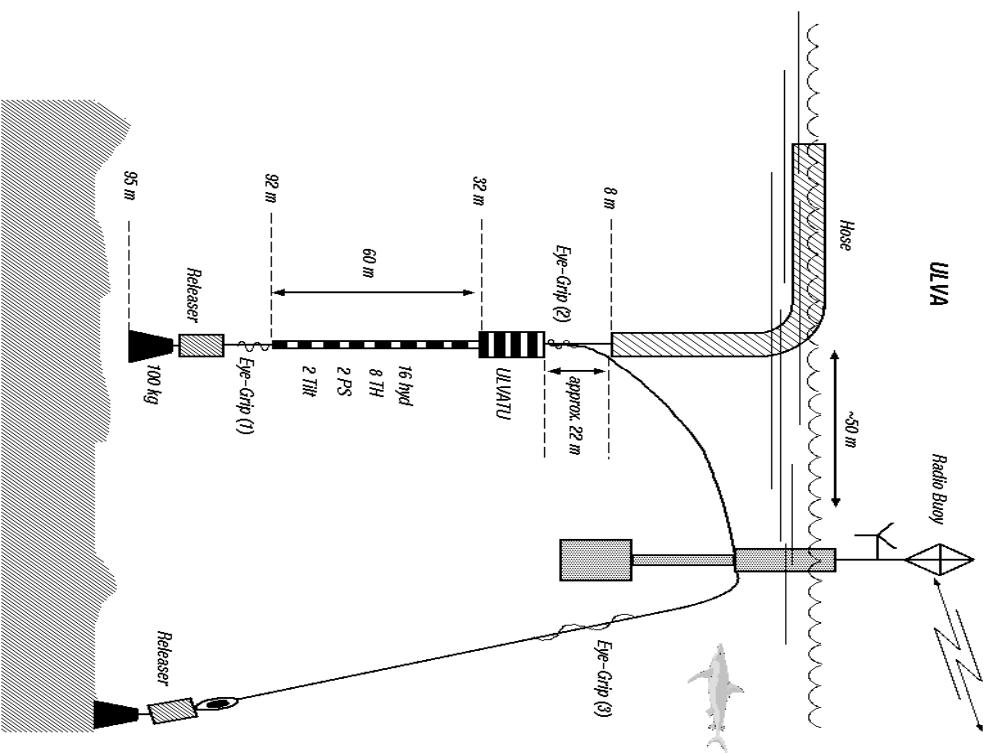
$$\hat{N} = \min_N \left\{ \frac{\sum_{n=1}^N \lambda_n^2}{\sum_{m=1}^M \lambda_m^2} > 0.8 \right\}$$



Physical model - NW/NE track

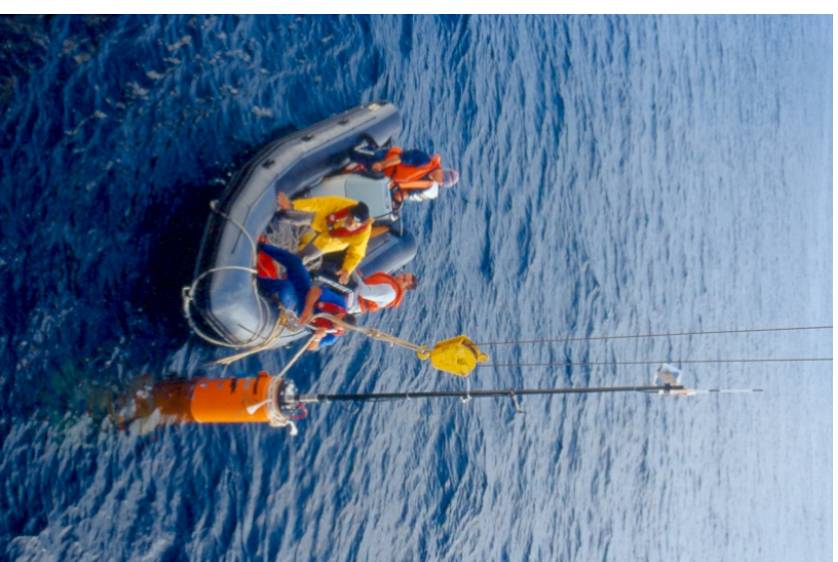
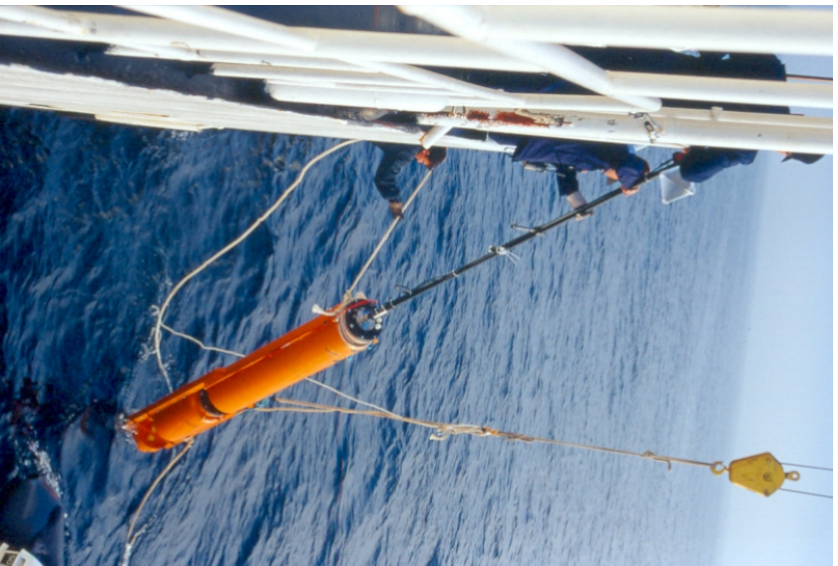


Vertical Line Array



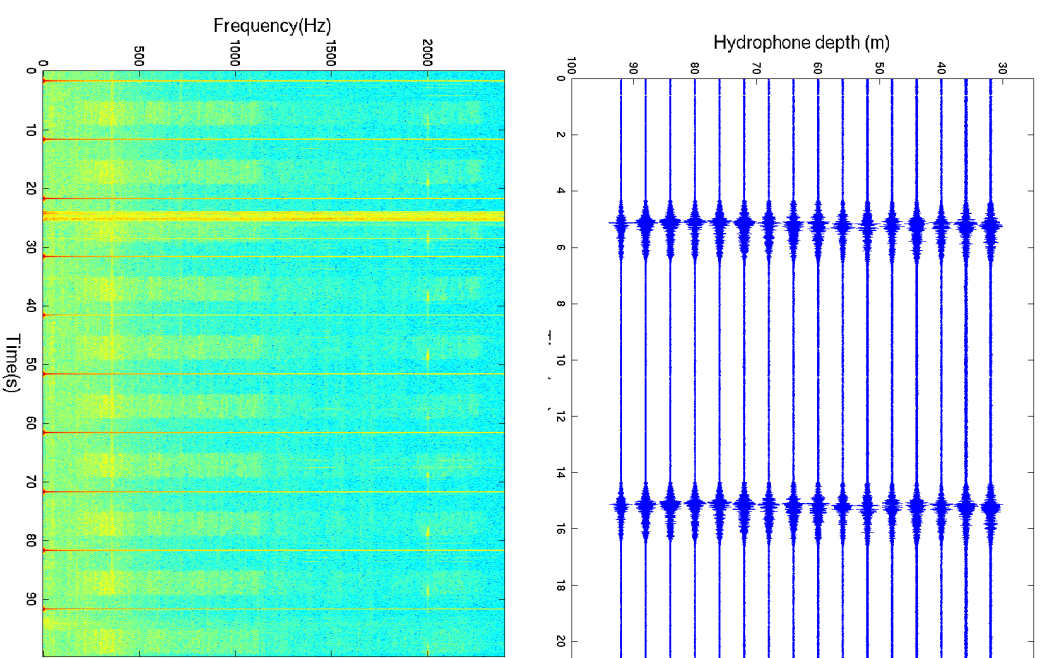


Radio Buoy: deployment and setup



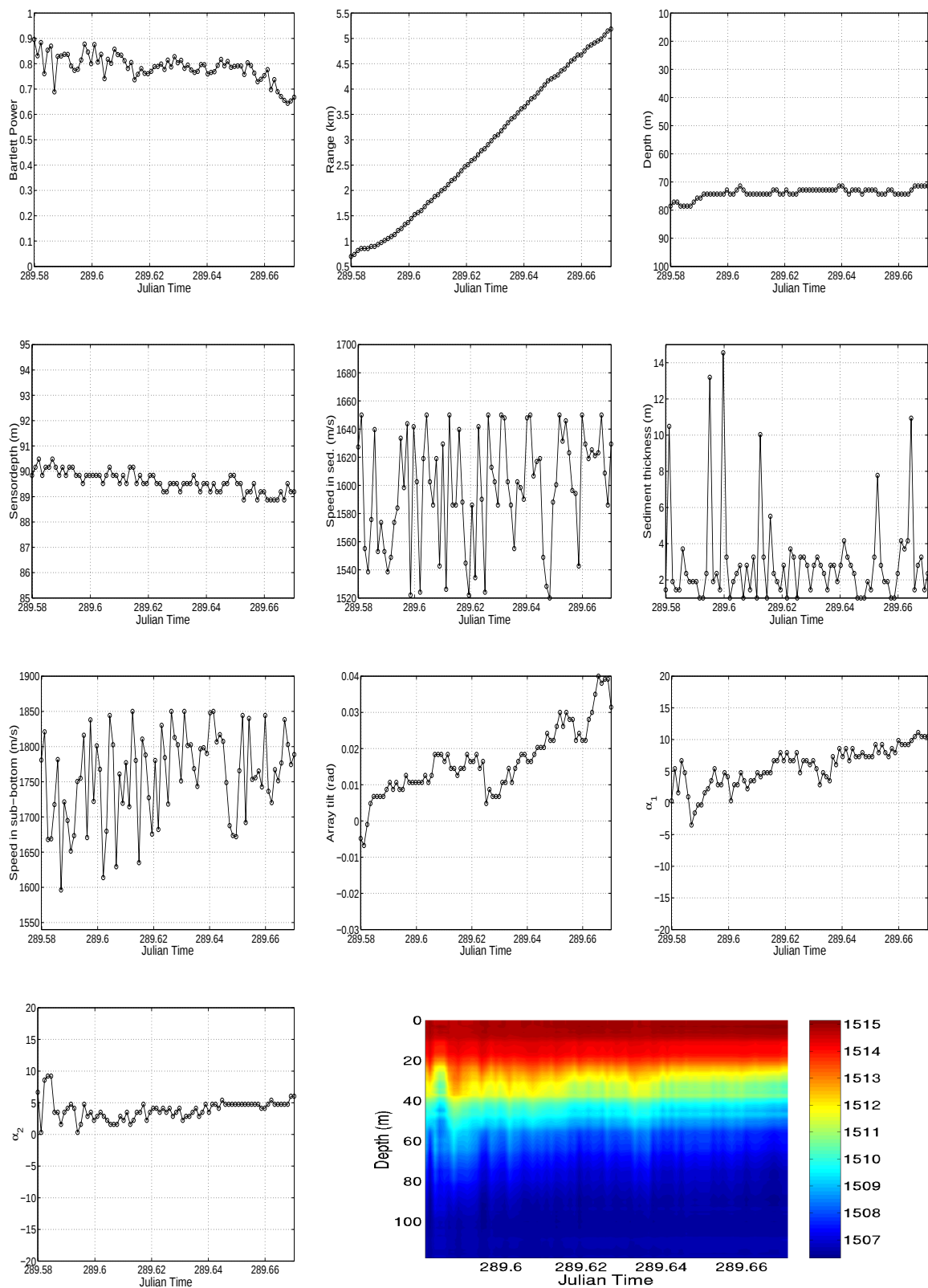


Sound source and emitted signals



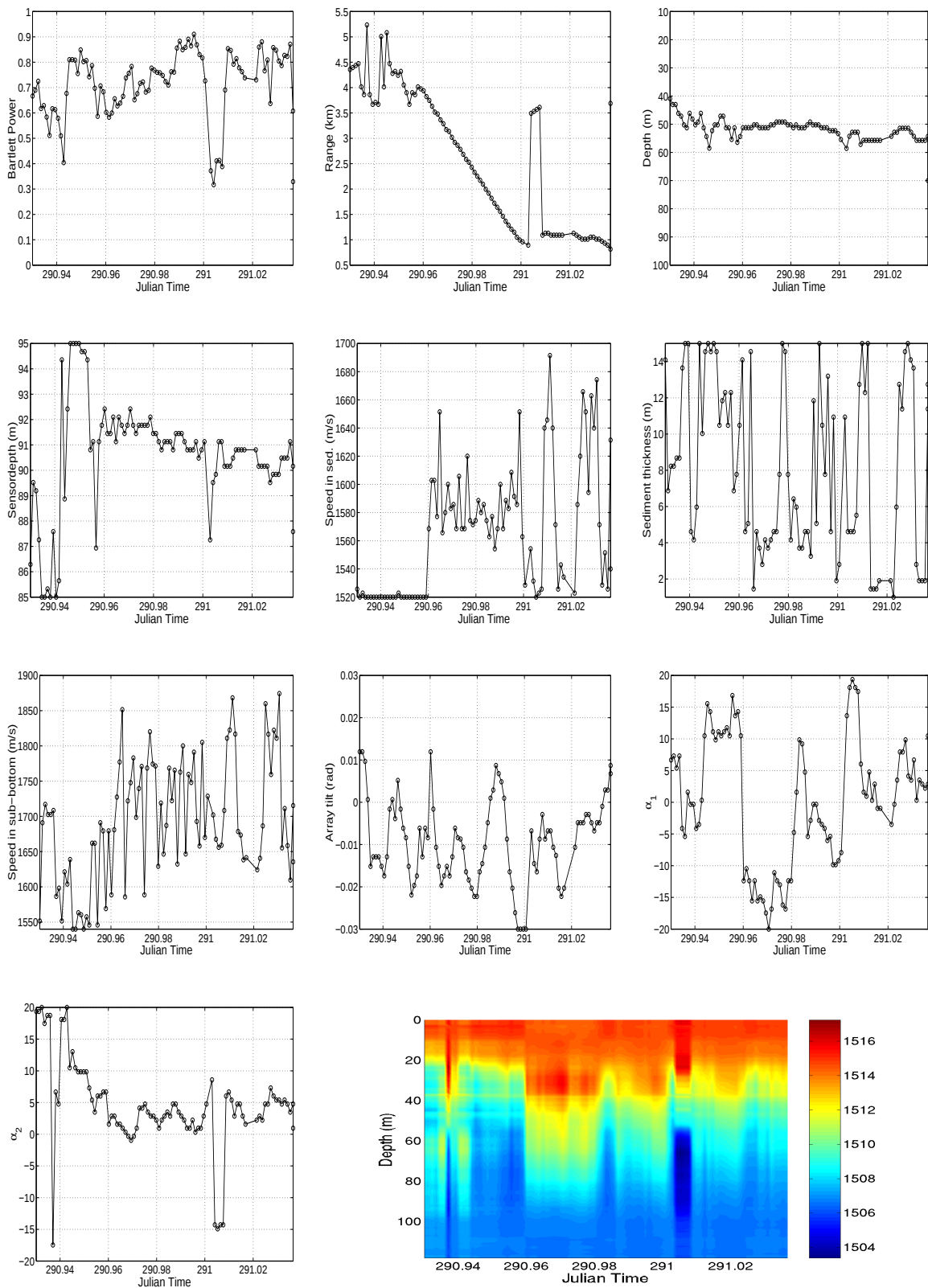


Inversion results for Event 2



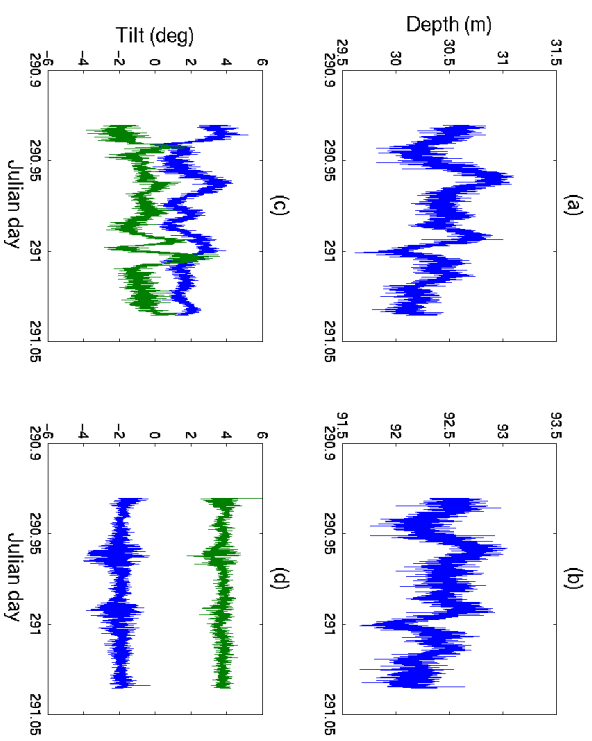
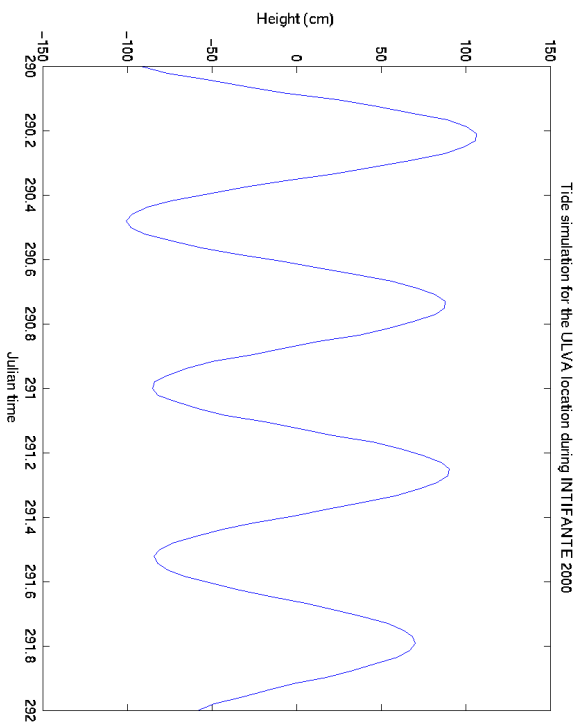


Inversion results for Event 5





Tide evolution and VA moving





Inverse modelling example

Training

The pair $\{r_j, x(r_j)\}$ allows for calculating coefficients c_{ij} of the neural net (RBF) network parameterized by functions ϕ

Data Inversion

Given $y(\mathbf{r}_o)$ calculate $\hat{\mathbf{r}}_o = \sum_{j=1}^N c_{ij} \phi(\|y(\mathbf{r}_o) - x(r_j)\|)$



Inverse modelling example(cont.)

ADVENTURE BANK sea trial 1994 - Sicily

